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International Mathematics

for the Middle Years

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Education

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The material below is found in the Companion Website which is included on the Interactive Student CD as both an archived version and a fully featured live version.

Activities and Investigations

1:02A	Coldest temperatures
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Drag and Drops

Chapter 1: Mathematical terms 1, Converting fractions to decimals, Simplifying ratios, Rates, Significant figures

Chapter 4: Mathematical terms 4, Addition and subtraction in algebra, Multiplying and dividing in algebra, Addition and subtraction of algebraic fractions, Multiplication and division of algebraic fractions, Grouping symbols, Simplifying expressions with grouping symbols, Factorising using common factors

Chapter 5: Mathematical terms 5, Number properties, Language and symbols, Special sets of numbers

Chapter 6: Mathematical terms 6, Index laws, Negative indices

Chapter 7: Mathematical terms 7, Equations, Solving inequations

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Chapter 9: Mathematical terms 9, Substitution, Formulae, Equations from formulae

Chapter 10: Mathematical terms 10, Angles 1, Angles 2, Angles 3, Triangles, Angle sum of a quadrilateral, Properties of quadrilaterals, Angle sum of polygons

Chapter 12: Mathematical terms 12, Perimeter 1, Perimeter 2, Area of sectors and composite figures, Surface area of a prism

Chapter 13: Mathematical terms 13

Chapter 14: Mathematical terms 14, Rolling two dice, Pack of cards

Animations

Chapter 5: Pythagoras' Theorem

Chapter Review Questions

These can be used as a diagnostic tool or for revision. They include multiple-choice, pattern-matching and fill-in-the-gaps-style questions.

Destinations

Links to useful websites which relate directly to the chapter content.

Features of *International Mathematics for the Middle Years*

International Mathematics for the Middle Years is organised with the international student in mind. Examples and exercises are not restricted to a particular syllabus and so provide students with a global perspective.

Each edition has a review section for students who may have gaps in the Mathematics they have studied previously. Sections on the language of Mathematics and terminology will help students for whom English is a second language.

Areas of Interaction are given for each chapter and Assessment Grids for Investigations provide teachers with aids to assessing Analysis and Reasoning, Communication, and Reflection and Evaluation as part of the International Baccalaureate Middle Years Program (IBMYP). The Assessment Grids will also assist students in addressing these criteria and will enhance students' understanding of the subject content.

How is *International Mathematics for the Middle Years* organised?

As well as the student coursebook, additional support for both students and teachers is provided:

- Interactive Student CD — **free** with each coursebook
- Companion Website
- Teacher's Resource — printout and CD.

Coursebook

Chapter-opening pages summarise the key content and present the learning outcomes addressed in each chapter.

Areas of Interaction references are included in the chapter-opening pages to make reporting easier. For example, Homo faber.

Prep Quizzes review skills needed to complete a topic. These anticipate problems and save time in the long run. These quizzes offer an excellent way to start a lesson.

Well-graded exercises — Within each exercise, **levels of difficulty** are indicated by the colour of the question number.

1 green . . . foundation 4 blue . . . core 9 red . . . extension

2 a An equilateral triangle has a side of length 4.68 m. What is its perimeter?

7 Solve the following equations.

a $\frac{x}{2} + \frac{x}{3} = 5$ b $\frac{p}{6} + \frac{p}{2} = 8$

8 a A radio on sale for $50 is to be reduced in price by 30%. Later, the discounted price is increased by 30%. What is the final price? By what percentage (to the nearest per cent) must the first discounted price be increased to give the original price?

Worked examples are used extensively and are easy for students to identify.

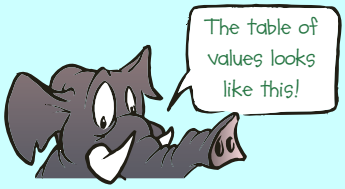
worked examples

1 Express the following in scientific notation.

a 243 b 60 000 c 93 800 000

Important rules and concepts are clearly highlighted at regular intervals throughout the text.

Cartoons are used to give students friendly advice or tips.



Foundation Worksheets provide alternative exercises for students who need to consolidate earlier work or who need additional work at an easier level. Students can access these on the CD by clicking on the Foundation Worksheet icons. These can also be copied from the Teacher's Resource CD or from the Teacher's Resource Centre on the Companion Website.

Foundation Worksheet 4:01A

Grouping symbols

1 a $(3 + 2) \times 10$
2 a $(8 - 2) \times 3$
3 a $10 - (4 + 3)$

Challenge activities and worksheets provide more difficult investigations and exercises. They can be used to extend more able students.

Fun Spots provide amusement and interest, while often reinforcing course work. They encourage creativity and divergent thinking, and show that Mathematics is enjoyable.

Investigations and **Practical Activities** encourage students to seek knowledge and develop research skills. They are an essential part of any Mathematics course. Where applicable, investigations are accompanied by a set of assessment criteria to assist teachers in assessing criteria B, C and D as prescribed by the MYP.

Diagnostic Tests at the end of each chapter test students' achievement of outcomes. More importantly, they indicate the weaknesses that need to be addressed by going back to the section in the text or on the CD listed beside the test question.

Assignments are provided at the end of each chapter. Where there are two assignments, the first revises the content of the chapter, while the second concentrates on developing the student's ability to work mathematically.

The **See** cross-references direct students to other sections of the coursebook relevant to a particular section.

The **Algebra Card** (see p xx) is used to practise basic algebra skills. Corresponding terms in columns can be added, subtracted, multiplied or divided by each other or by other numbers. This is a great way to start a lesson.

The Language of Mathematics

Within the coursebook, Mathematics literacy is addressed in three specific ways:

ID Cards (see pp xiv–xix) review the language of Mathematics by asking students to identify common terms, shapes and symbols. They should be used as often as possible, either at the beginning of a lesson or as part of a test or examination.

Mathematical Terms met during the chapter are defined at the end of each chapter. These terms are also tested in a **Drag and Drop** interactive that follows this section.

Reading Mathematics help students to develop maths literacy skills and provide opportunities for students to communicate mathematical ideas. They present Mathematics in the context of everyday experiences.

An **Answers** section provides answers to all the exercises in the coursebook, including the ID Cards.

Interactive Student CD

This is provided at the back of the coursebook and is an important part of the total learning package.

Bookmarks and links allow easy navigation within and between the different electronic components of the CD that contains:

- A copy of the student coursebook.
- Appendixes A–D for enrichment and review work, linked from the coursebook.
- Printable copies of the Foundation Worksheets and Challenge Worksheets, linked from the coursebook.
- An archived, offline version of the Companion Website, including:
  - Chapter Review Questions and Quick Quizzes
  - All the Technology Applications: activities and investigations and drag-and-drops
  - Destinations (links to useful websites)

All these items are clearly linked from the coursebook via the Companion Website.

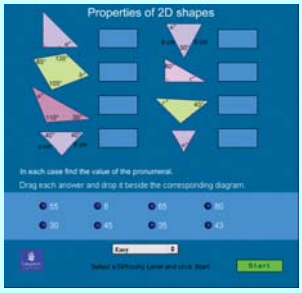
- A link to the live Companion Website.

Companion Website

The Companion Website contains a wealth of support material for students and teachers:

- **Chapter Review Questions** which can be used as a diagnostic tool or for revision. These are self-correcting and include multiple-choice, pattern-matching and fill-in the-gaps-style questions. Results can be emailed directly to the teacher or parents.
- **Quick Quizzes** for most chapters.
- **Destinations** — links to useful websites which relate directly to the chapter content.
- **Technology Applications** — activities that apply concepts covered in most chapters and are designed for students to work independently:

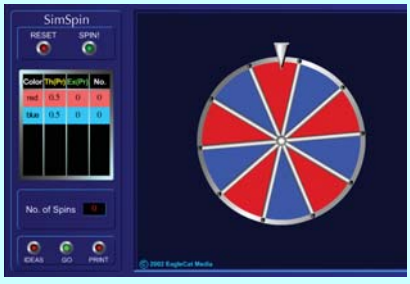
Activities and investigations using technology, such as Excel spreadsheets and The Geometer's Sketchpad.



Sample Drag and Drop

Drag and Drop interactives to improve mastery of basic skills.

Animations to develop key skills by manipulating visually stimulating and interactive demonstrations of key mathematical concepts.



Sample Animation

- **Teacher's Resource Centre** — provides a wealth of teacher support material and is password protected:
  - — Coursebook corrections
  - — Topic Review Tests and answers
  - — Foundation and Challenge Worksheets and answers

Teacher's resource

This material is provided as both a printout and as an electronic copy on CD:

- Electronic copy of the complete Student Coursebook in PDF format
- Teaching Program, including treatment of learning outcomes, in both PDF and editable Microsoft Word formats
- Practice Tests and Answers
- Foundation and Challenge Worksheets and answers
- Answers to some of the Technology Application Activities and Investigations

Most of this material is also available in the Teacher's Resource Centre of the Companion Website.

Using this Book for Teaching MYP for the IB

- **Holistic Learning**
- **Intercultural Awareness**
- **Communication**

These elements of the MYP Mathematics course are integrated throughout the text. Links are made possible between subjects, and different methods of communicating solutions to problems through investigations allow students to explore their own ideas.

The Areas of Interaction

- **Approaches to Learning**
- **Community and Service**
- **Health and Social Education**
- **Environment**
- **Homo Faber**

Areas of Interaction covered are outlined at the start of each chapter, allowing teachers to develop links between subjects and formulate their own Interdisciplinary Units with additional assistance in the Teacher's Resource.

Addressing the Objectives

Assessment grids are provided for Investigations throughout the text to not only help teachers assess criteria B, C and D of the MYP, but also to assist students in addressing the criteria. The assessment grids should be modified to suit the student where necessary.

A Knowledge and Understanding
This criterion is addressed in the Diagnostic Tests and Revision Assignments that accompany each chapter. Teachers can also use the worksheets from the CD to add to material for this criterion.

B Investigating Patterns
It is possible to address this criterion using the Working Mathematically sections accompanying each chapter, and also using the Investigations throughout the text.

C Communication
This can be assessed using the Investigations throughout the book.

D Reflection in Mathematics
This can be assessed using the Investigations throughout the book.

Fulfilling the Framework for Mathematics

The content of the text covers the five broad areas required to fulfil the Framework:

- **Number**
- **Algebra**
- **Geometry**
- **Statistics**
- **Discrete Mathematics**

Although the material in the text is not exhaustive, it covers the required areas in sufficient depth. Teachers can use the text as a resource to build on as they develop their own scheme of work within their school.

Metric Equivalents

Length
1 m = 1000 mm
= 100 cm
= 10 dm
1 cm = 10 mm
1 km = 1000 m
Area
1 m^2 = 10 000 cm^2
1 ha = 10 000 m^2
1 km^2 = 100 ha
Mass
1 kg = 1000 g
1 t = 1000 kg
1 g = 1000 mg
Volume
1 m^3 = 1 000 000 cm^3
= 1000 dm^3
1 L = 1000 mL
1 kL = 1000 L
1 m^3 = 1 kL
1 cm^3 = 1 mL
1000 cm^3 = 1 L
Time
1 min = 60 s
1 h = 60 min
1 day = 24 h
1 year = 365 days
1 leap year = 366 days

Months of the year

30 days each has September,
April, June and November.
All the rest have 31, except February alone,
Which has 28 days clear and 29 each leap year.

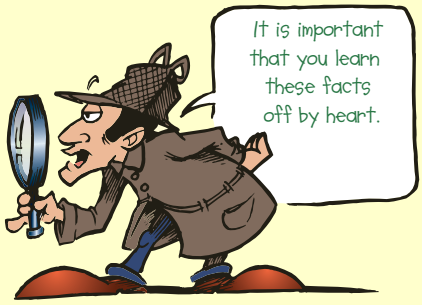
Seasons

Northern Hemisphere

Summer:	June, July, August
Autumn/Fall:	September, October, November
Winter:	December, January, February
Spring:	March, April, May

Southern Hemisphere

Summer:	December, January, February
Autumn:	March, April, May
Winter:	June, July, August
Spring:	September, October, November



The Language of Mathematics

You should regularly test your knowledge by identifying the items on each card.

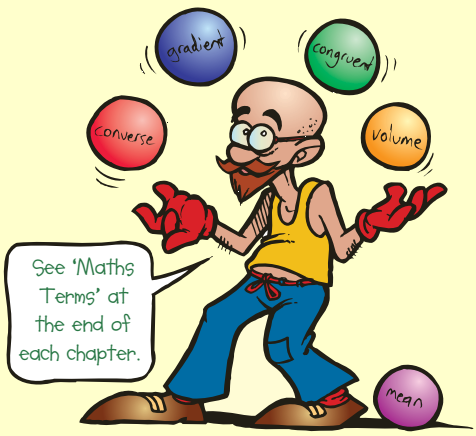
id

ID Card 1 (Metric Units)			
1 m	2 dm	3 cm	4 mm
5 km	6 m^2	7 cm^2	8 km^2
9 ha	10 m^3	11 cm^3	12 s
13 min	14 h	15 m/s	16 km/h
17 g	18 mg	19 kg	20 t
21 L	22 mL	23 kL	24 °C

See page 427 for answers.

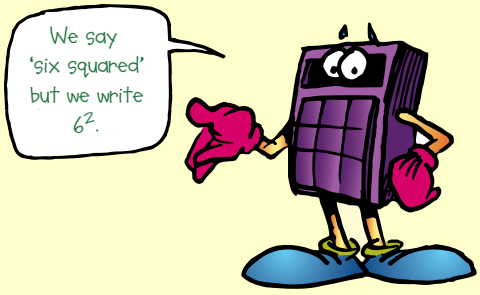
ID Card 2 (Symbols)			
1 =	2 $\doteq$ or $\approx$	3 $\neq$	4 $<$
5 $\leqslant$	6 $\not<$	7 $>$	8 $\geqslant$
9 4^2	10 4^3	11 $\sqrt{2}$	12 $\sqrt[3]{2}$
13 $\llcorner$	14 $\parallel$	15 $\equiv$	16 $\parallel\!\!\parallel$
17 %	18 $\therefore$	19 eg	20 ie
21 π	22 Σ	23 $\bar{x}$	24 P(E)

See page 427 for answers.



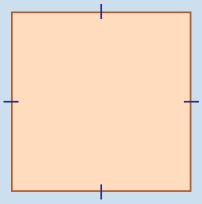
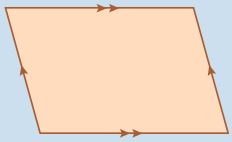
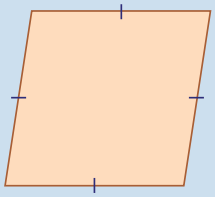
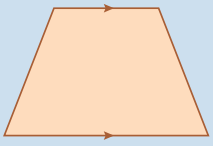
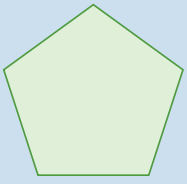
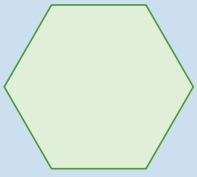
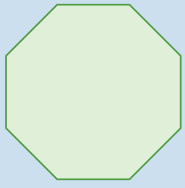
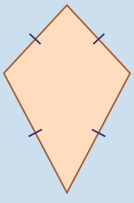
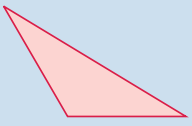
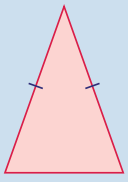
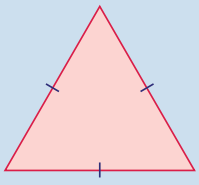
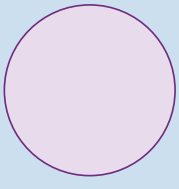
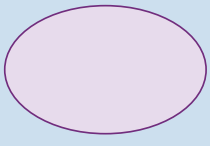
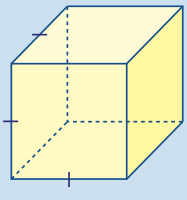
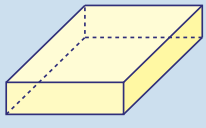
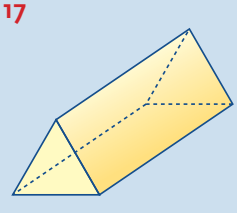
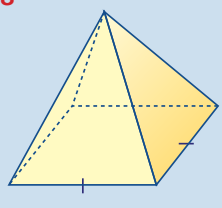
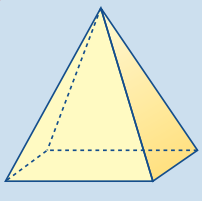
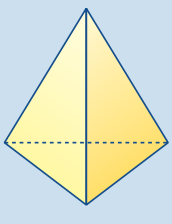
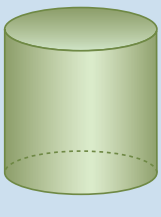
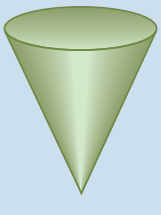
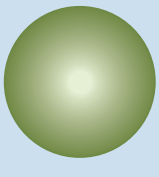
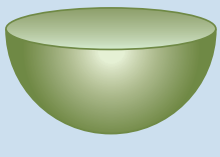
ID Card 3 (Language)			
1 6 minus 2	2 the sum of 6 and 2	3 divide 6 by 2	4 subtract 2 from 6
5 the quotient of 6 and 2	6 $2\overline{)6}$ the divisor is	7 $2\overline{)6}$ the dividend is	8 6 lots of 2
9 decrease 6 by 2	10 the product of 6 and 2	11 6 more than 2	12 2 less than 6
13 6 squared	14 the square root of 36	15 6 take away 2	16 multiply 6 by 2
17 average of 6 and 2	18 add 6 and 2	19 6 to the power of 2	20 6 less 2
21 the difference between 6 and 2	22 increase 6 by 2	23 share 6 between 2	24 the total of 6 and 2

See page 427 for answers.



id

ID Card 4 (Language)

1 	2 	3 	4 
5 	6 	7 	8 
9 	10  All sides different	11 	12 
13 	14 	15 	16 
17 	18 	19 	20 
21 	22 	23 	24 

See page 427 for answers.

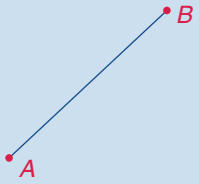
ID Card 5 (Language)

1

A

.....

2



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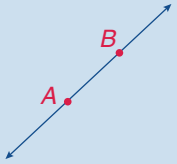
3



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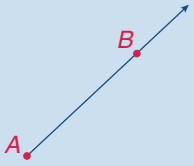
4



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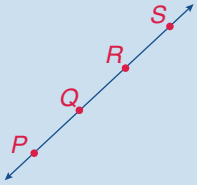
5

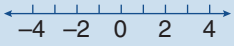


..... points

6

C is the .....

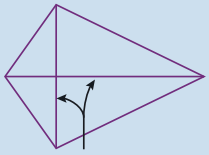
7



.....

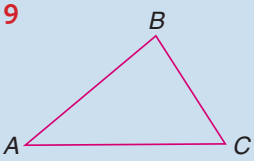
.....

8



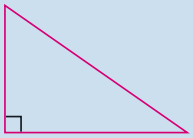
.....

9



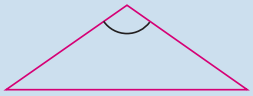
all angles less than 90°

10



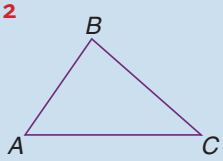
one angle 90°

11



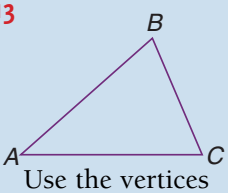
one angle greater than 90°

12



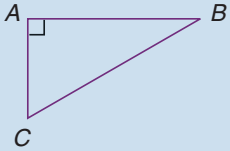
A, *B* and *C* are ..... of the triangle.

13



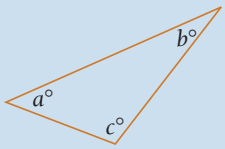
Use the vertices to name the Δ

14



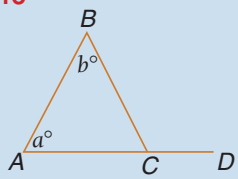
BC is the ..... of the right-angled Δ.

15



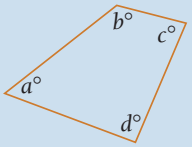
$a° + b° + c°$ = ...

16



$\angle BCD$ = ...

17

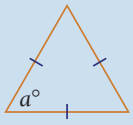


$a° + b° + c° + d°$ = ...

18

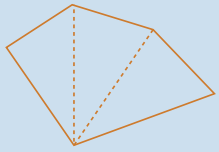
Which is true?
(a) $a° < b°$
(b) $a° = b°$
(c) $a° > b°$

19



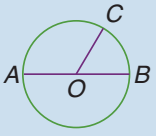
$a°$ = ...

20



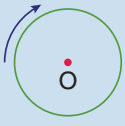
Angle sum = ...

21



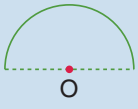
AB is a .....
OC is a .....

22



Name of distance around the circle.
.....

23



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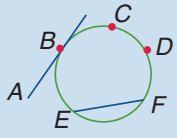
24



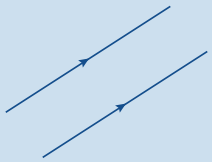
AB is a .....
CD is an .....
EF is a.....

See page 427 for answers.

id

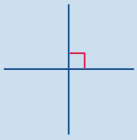
ID Card 6 (Language)

1



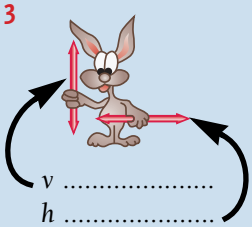
..... lines

2

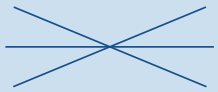


..... lines

3

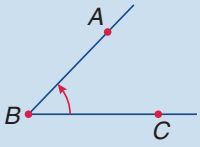


4



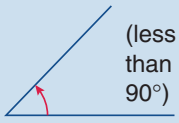
..... lines

5



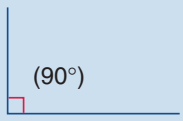
angle .....

6



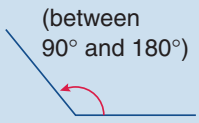
..... angle

7



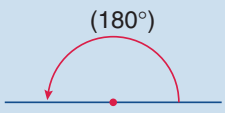
..... angle

8



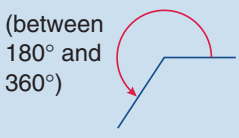
..... angle

9



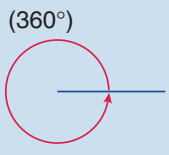
..... angle

10



..... angle

11



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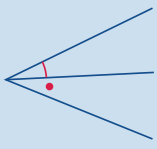
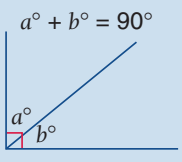
12



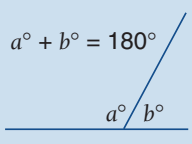
..... angles

13



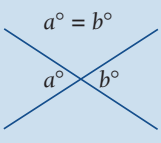
..... angles

14



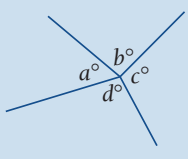
..... angles

15



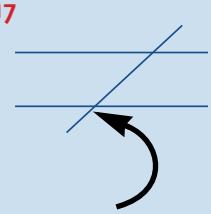
..... angles

16



a° + b° + c° + d° = ...

17



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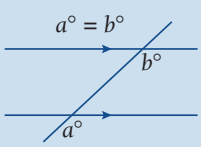
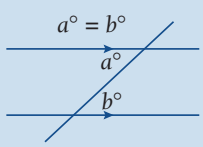
18



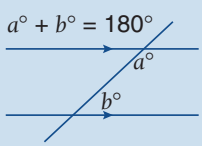
..... angles

19



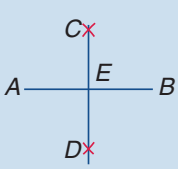
..... angles

20



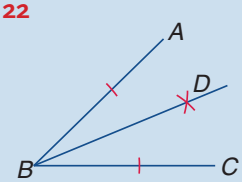
..... angles

21



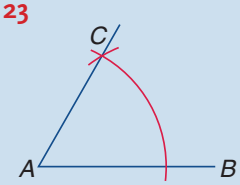
b..... an interval

22



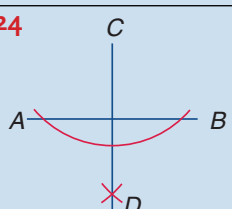
b..... an angle

23



∠*CAB* = ...

24



CD is *p*..... to *AB*.

See page 427 for answers.

ID Card 7 (Language)

1

AD

a..... *D*.....

2

BC

b..... *C*.....

3

am

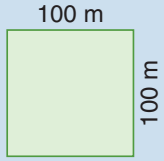
a..... *M*.....

4

pm

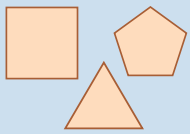
p..... *m*.....

5



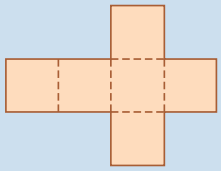
area is 1 .....

6



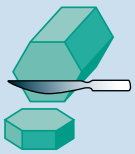
r..... shapes

7



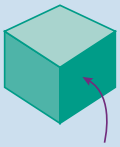
..... of a cube

8



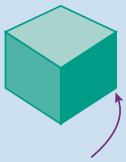
c.....-*s*.....

9



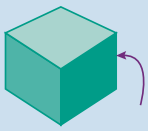
f.....

10



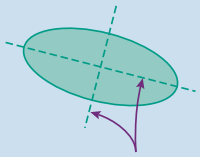
v.....

11



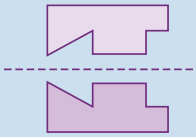
e.....

12



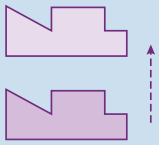
axes of .....

13



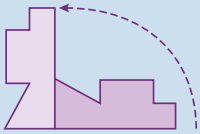
r.....

14



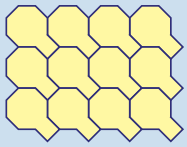
t.....

15



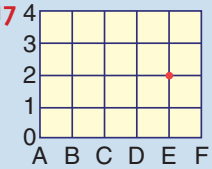
r.....

16



t.....

17



The *c*.....
of the dot are E2.

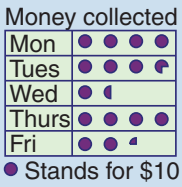
18

Cars sold

Mon	𝍸 I
Tues	𝍸 𝍸 𝍸 I
Wed	𝍸 𝍸 II
Thurs	𝍸 𝍸 𝍸
Fri	𝍸 𝍸 II

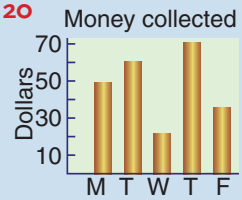
t.....

19



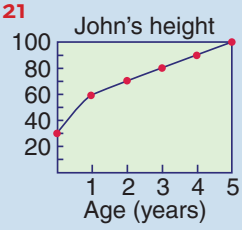
p..... graph

20



c..... graph

21

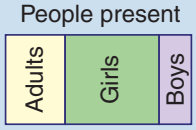


l..... graph

22

s..... graph

23



b..... graph

24

s..... *d*.....

See page 427 for answers.

Algebra Card

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	3	2·1	$\frac{1}{4}$	m	$\frac{2m}{3}$	$-3m$	$5m^2$	$-5x$	$\frac{x}{6}$	$-3x$	$-\frac{x}{2}$	$x+2$	$x-3$	$2x+1$	$3x-8$
2	-1	$-0{\cdot}4$	$\frac{1}{8}$	$-4m$	$\frac{m}{4}$	$2m$	$-2m^3$	$3x$	$-\frac{x}{3}$	$5x^2$	$\frac{x}{4}$	$x+7$	$x-6$	$4x+2$	$x-1$
3	5	0·8	$\frac{1}{3}$	$10m$	$-\frac{m}{4}$	$-5m$	$8m^5$	$10x$	$-\frac{2x}{7}$	$-8x$	$\frac{2x}{5}$	$x+5$	$x+5$	$6x+2$	$x-5$
4	-2	1·5	$\frac{1}{20}$	$-8m$	$-\frac{3m}{2}$	$7m$	$6m^2$	$-15x$	$\frac{x}{10}$	$-4x^4$	$-\frac{x}{5}$	$x+1$	$x-9$	$3x+3$	$2x+4$
5	-8	$-2{\cdot}5$	$\frac{3}{5}$	$2m$	$-\frac{m}{5}$	$10m$	m^2	$7x$	$\frac{2x}{3}$	$2x^3$	$\frac{x}{3}$	$x+8$	$x+2$	$3x+8$	$3x+1$
6	10	$-0{\cdot}7$	$\frac{2}{7}$	$-5m$	$-\frac{3m}{7}$	$-6m$	$-9m^3$	$9x$	$-\frac{2x}{5}$	x^2	$\frac{3x}{5}$	$x+4$	$x-7$	$3x+1$	$x+7$
7	-6	$-1{\cdot}2$	$\frac{3}{8}$	$8m$	$-\frac{m}{6}$	$9m$	$2m^6$	$-6x$	$\frac{5x}{6}$	$5x^2$	$\frac{2x}{3}$	$x+6$	$x-1$	$x+8$	$2x-5$
8	12	0·5	$\frac{9}{20}$	$20m$	$\frac{2m}{5}$	$-4m$	$-3m^3$	$-12x$	$\frac{3x}{4}$	$4x^3$	$-\frac{x}{7}$	$x+10$	$x-8$	$5x+2$	$x-10$
9	7	0·1	$\frac{3}{4}$	$5m$	$\frac{3m}{5}$	$-10m$	m^7	$5x$	$-\frac{3x}{7}$	$-3x^5$	$-\frac{3x}{7}$	$x+2$	$x+5$	$2x+4$	$2x-4$
10	-5	$-0{\cdot}6$	$\frac{7}{10}$	$-9m$	$-\frac{4m}{5}$	$-7m$	$-8m^4$	$-3x$	$-\frac{x}{6}$	$-7x^5$	$\frac{2x}{9}$	$x+1$	$x-7$	$5x+4$	$x+7$
11	-11	$-1{\cdot}8$	$\frac{1}{10}$	$-7m$	$\frac{m}{5}$	$-8m$	$-4m$	$-4x$	$\frac{x}{5}$	$-x^3$	$\frac{x}{3}$	$x+9$	$x+6$	$2x+7$	$x-6$
12	4	$-1{\cdot}4$	$\frac{2}{5}$	$3m$	$\frac{m}{3}$	$12m$	$7m^2$	$-7x$	$-\frac{3x}{4}$	x^{10}	$\frac{x}{6}$	$x+3$	$x-10$	$2x+3$	$2x+3$

How to use this card

If the instruction is 'column D + column F', then you add corresponding terms in columns D and F.

eg

1 $m + (-3m)$ **2** $(-4m) + 2m$ **3** $10m + (-5m)$

4 $(-8m) + 7m$ **5** $2m + 10m$ **6** $(-5m) + (-6m)$

7 $8m + 9m$ **8** $20m + (-4m)$ **9** $5m + (-10m)$

10 $(-9m) + (-7m)$ **11** $(-7m) + (-8m)$ **12** $3m + 12m$

1

Basic Number Skills

Chapter Contents

Learning Outcomes

Students will:

- Revise comparing, ordering and calculating with integers.
- Revise operating with fractions, decimals, percentages, ratios and rates.
- Round decimals to a specified number of significant figures, express recurring decimals in fraction form and convert rates from one set of units to another.

Areas of Interaction

Approaches to Learning, Homo Faber

1:01 | The Language of Mathematics

Much of the language met so far is reviewed in the identification cards (ID Cards) found on pages xiv to xix. These should be referred to throughout the text. Make sure that you can identify every term.

Exercise 1:01

1 Write down the symbol and its meaning for each of (1) to (24) of ID Card 1 (Metric Units) on page xiv.

2 Write down the symbol and its meaning for each of (1) to (24) of ID Card 2 (Symbols) on page xiv.

3 Use ID Card 3 (Language) on page xvii to write down a number sentence, giving the answer to each of (1) to (24).
For example, for '6 minus 2' write '$6 - 2 = 4$'.

4 Write true or false, without using a calculator.

a $344 \times 1{\cdot}5 \neq 246$ **b** $830 \div 0{\cdot}9 > 830$ **c** $830 \div 1{\cdot}1 < 830$

d $199 \times 0{\cdot}9 > 199$ **e** $699 \div 0{\cdot}99 > 699$ **f** $699 \div 1{\cdot}01 < 699$

g $1{\cdot}1^2 \not< 1{\cdot}1^3$ **h** $\pi \div 3{\cdot}14$ **i** $0{\cdot}1\% = 0{\cdot}001$

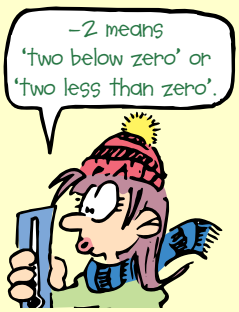
j $\sqrt{1{\cdot}1} < 1{\cdot}1$ **k** $\sqrt[3]{1{\cdot}2} > 1{\cdot}2$ **l** $0{\cdot}8^2 < 0{\cdot}8$

m $\sqrt{2} > \sqrt[3]{2}$ **n** $\sqrt{1} > \sqrt[3]{1}$ **o** $\sqrt{0{\cdot}8} < \sqrt[3]{0{\cdot}8}$

- Estimate the width of this bridge, in metres.

1:02 | Diagnostic Tests

Without obtaining help, complete the diagnostic tests below to determine which areas need attention. Errors made will indicate areas of weakness. Each weakness should be treated by referring to the section found on the Interactive Student CD. There you will find explanations and worked examples relating to these skills.
DO NOT USE A CALCULATOR!



1:02A Integers

					CD Appendix
1	a $-7+14$	b $2-15$	c $-2-8$		Set A
2	a $3--6$	b $12+(-5)$	c $6-(3-8)$		Set B
3	a -3×2	b -5×-6	c $7\times(-9)$		Set C
4	a $(-15)\div(-3)$	b $63\div -9$	c $\frac{-156}{-3}$		Set C
5	a $14-7\times 10$	b $(-3+4)\div 3$	c $(4-18)\div(-8+6)$		Set D

1:02B Fractions

					CD Appendix
1	Write these improper fractions as mixed numerals.				Set A
	a $\frac{7}{4}$	b $\frac{13}{3}$	c $\frac{141}{10}$		
2	Write these mixed numerals as improper fractions.				Set B
	a $2\frac{1}{2}$	b $5\frac{3}{10}$	c $3\frac{1}{7}$		
3	Simplify these fractions.				Set C
	a $\frac{16}{24}$	b $\frac{100}{650}$	c $\frac{240}{3600}$		
4	Complete the following to make equivalent fractions.				Set D
	a $\frac{3}{4}=\frac{\square}{28}$	b $\frac{17}{20}=\frac{\square}{100}$	c $\frac{3}{8}=\frac{\square}{1000}$		
Give the simplest answer for ...					
5	a $\frac{3}{8}+\frac{2}{8}$	b $\frac{9}{10}+\frac{3}{10}$	c $\frac{7}{9}+\frac{2}{9}$		Set E
6	a $\frac{9}{10}-\frac{7}{10}$	b $\frac{13}{14}-\frac{9}{14}$	c $\frac{37}{100}-\frac{11}{100}$		Set E
7	a $\frac{3}{4}+\frac{4}{5}$	b $\frac{3}{10}+\frac{2}{5}$	c $\frac{7}{100}+\frac{3}{40}$		Set F
8	a $\frac{7}{8}-\frac{3}{4}$	b $\frac{9}{10}-\frac{1}{4}$	c $\frac{5}{6}-\frac{3}{5}$		Set F
9	a $3\frac{1}{2}+4\frac{3}{5}$	b $6\frac{7}{10}+5\frac{3}{4}$	c $1\frac{5}{6}+\frac{7}{8}$		Set G
10	a $4\frac{1}{2}-1\frac{2}{9}$	b $10\frac{3}{4}-5\frac{1}{10}$	c $20\frac{3}{8}-\frac{1}{5}$		Set G
11	a $7\frac{1}{2}-\frac{7}{8}$	b $6\frac{3}{5}-2\frac{7}{10}$	c $3\frac{1}{2}-1\frac{5}{6}$		Set H
12	a $\frac{4}{5}\times\frac{3}{11}$	b $\frac{3}{10}\times\frac{7}{10}$	c $\frac{1}{10}\times\frac{3}{5}$		Set I
13	a $\frac{7}{8}\times\frac{3}{7}$	b $\frac{15}{38}\times\frac{19}{20}$	c $\frac{7}{10}\times\frac{5}{6}$		Set I
14	a $3\frac{1}{2}\times\frac{5}{7}$	b $1\frac{3}{10}\times 1\frac{4}{5}$	c $5\frac{1}{4}\times 2\frac{2}{3}$		Set J
15	a $4\times 3\frac{4}{5}$	b $2\frac{1}{4}\times 3$	c $5\times 6\frac{3}{8}$		Set J
16	a $\frac{8}{10}\div\frac{2}{10}$	b $\frac{9}{20}\div\frac{3}{20}$	c $\frac{7}{10}\div\frac{7}{10}$		Set K
17	a $\frac{3}{5}\div\frac{1}{2}$	b $\frac{8}{9}\div\frac{3}{4}$	c $\frac{5}{8}\div\frac{4}{7}$		Set K
18	a $1\frac{7}{8}\div\frac{3}{4}$	b $3\frac{4}{7}\div 2\frac{1}{2}$	c $3\frac{5}{8}\div 2\frac{9}{10}$		Set L
19	a $7\frac{1}{2}\div 3$	b $4\frac{9}{10}\div 7$	c $6\frac{7}{8}\div 5$		Set L
20	a $5\div\frac{1}{4}$	b $10\div\frac{1}{5}$	c $4\div\frac{1}{10}$		Set L

■ 3 of 4 equal parts

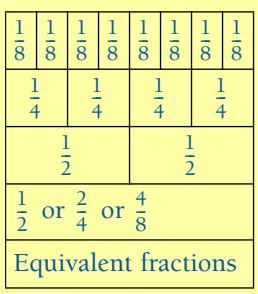
$\frac{3}{4}$ → Numerator

$\frac{3}{4}$ → Denominator

■ Fractions should always be expressed in lowest terms.

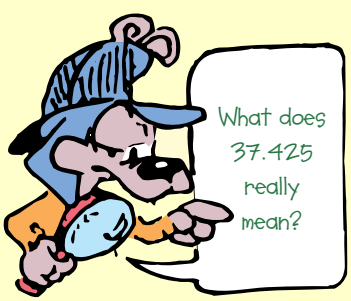
$\frac{4}{6}=\frac{2}{3}$

■



1:02C Decimals

				CD Appendix
1	Put in order, smallest to largest.			Set A
	a 0·505, 0·5, 0·55	b 8·4, 8·402, 8·41	c 1·01, 1·1, 1·011	
2	a $2·6 + 3·14$	b $18·6 + 3$	c $0·145 + 0·12$	Set B
3	a $12·83 - 1·2$	b $9 - 1·824$	c $4·02 - 0·005$	Set B
4	a $0·7 \times 6$	b $(0·3)^2$	c $0·02 \times 1·7$	Set C
5	a $3·142 \times 100$	b $0·04 \times 1000$	c $0·065 \times 10$	Set D
6	a $2·1 \times 10^4$	b $8·04 \times 10^6$	c $1·25 \times 10^2$	Set D
7	a $4·08 \div 2$	b $12·1 \div 5$	c $0·19 \div 4$	Set E
8	Write answers as repeating decimals.			
	a $2·5 \div 6$	b $5·32 \div 9$	c $28 \div 3$	Set F
9	a $24·35 \div 10$	b $6·7 \div 100$	c $0·7 \div 1000$	Set G
10	a $6·4 \div 0·2$	b $0·824 \div 0·08$	c $6·5 \div 0·05$	Set H
11	Convert these decimals to fractions.			
	a 0·5	b 0·18	c 9·105	Set I
12	Convert these fractions to decimals.			
	a $\frac{4}{5}$	b $\frac{3}{8}$	c $\frac{5}{6}$	Set J



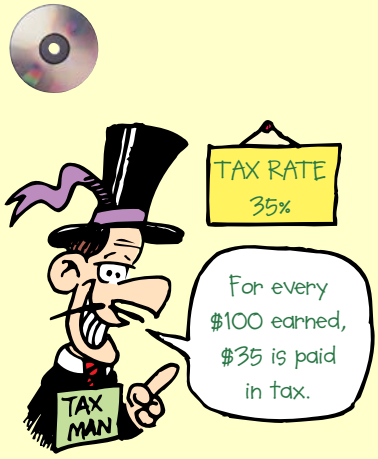
3 tens
7 units
4 tenths
2 hundredths
5 thousandths

37·425

10	1	·	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
3	7	·	4	2	5

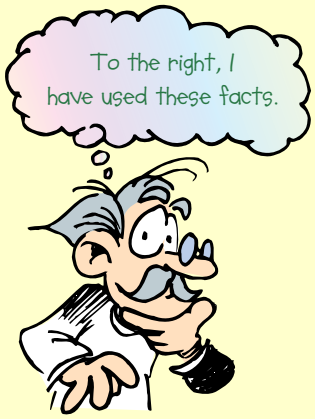
1:02D Percentages

		CD Appendix
1	Convert to fractions. **a** 18% **b** 7% **c** 224%	Set A
2	Convert to fractions. **a** 9·5% **b** $6\frac{1}{4}\%$ **c** 12·25%	Set A
3	Convert to percentages. **a** $\frac{11}{20}$ **b** $\frac{5}{6}$ **c** $1\frac{1}{4}$	Set B
4	Convert to decimals. **a** 9% **b** 16% **c** 110%	Set C
5	Convert to decimals. **a** 23·8% **b** $12\frac{1}{2}\%$ **c** $4\frac{2}{3}\%$	Set C
6	Convert to percentages. **a** 0·51 **b** 0·085 **c** 1·8	Set D
7	Find: **a** 35% of 600 m **b** 162% of $8	Set E
8	Find: **a** 7% of 84·3 m **b** $6\frac{1}{4}\%$ of 44 tonnes	Set E
9	**a** 7% of my spending money was spent on a watch band that cost $1.12. How much spending money did I have? **b** 30% of my weight is 18 kg. How much do I weigh?	Set F
10	**a** 5 kg of sugar, 8 kg of salt and 7 kg of flour were mixed accidentally. What is the percentage (by weight) of sugar in the mixture? **b** John scored 24 runs out of the team's total of 60 runs. What percentage of runs did John score?	Set G
11	**a** Increase $60 by 15%. **b** Decrease $8 by 35%.	Set H



1:03 | Conversion Facts You Should Know

Percentage	*Decimal*	*Fraction*
1%	0·01	$\frac{1}{100}$
5%	0·05	$\frac{1}{20}$
10%	0·1	$\frac{1}{10}$
$12\frac{1}{2}\%$	0·125	$\frac{1}{8}$
20%	0·2	$\frac{1}{5}$
25%	0·25	$\frac{1}{4}$
$33\frac{1}{3}\%$	$0\cdot\dot{3}$	$\frac{1}{3}$
50%	0·5	$\frac{1}{2}$
100%	1	1

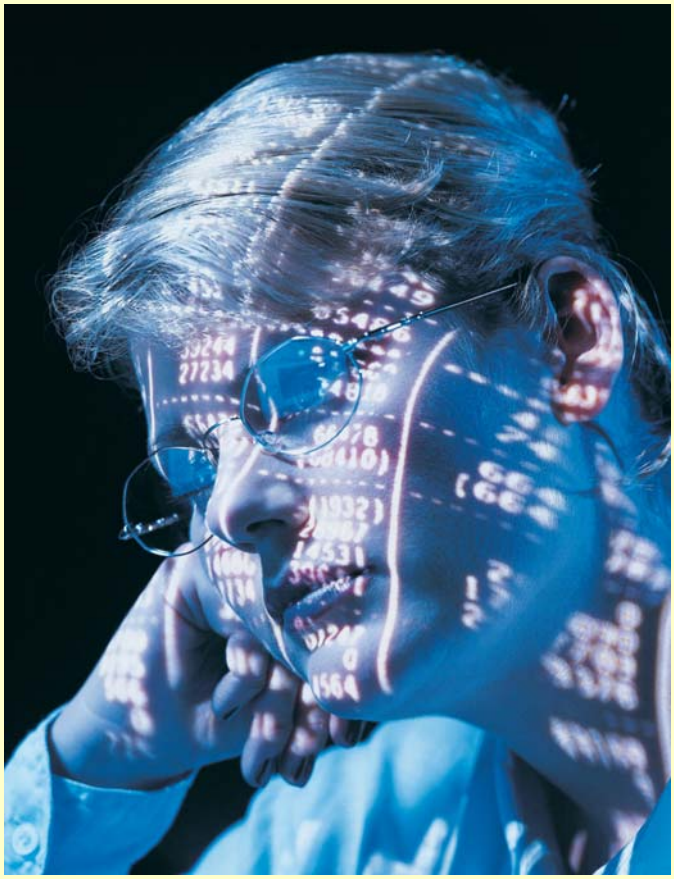


a $10\% = 0{\cdot}1 = \frac{1}{10}$
Multiply each by 6.
$60\% = 0{\cdot}6 = \frac{6}{10}$

b $5\% = 0{\cdot}05 = \frac{1}{20}$
Multiply each by 7.
$35\% = 0{\cdot}35 = \frac{7}{20}$

c $20\% = 0{\cdot}2 = \frac{1}{5}$
Add 1 or 100% to each.
$120\% = 1{\cdot}2 = 1\frac{1}{5}$

d $12\frac{1}{2}\% = 0{\cdot}125 = \frac{1}{8}$
Add 1 or 100% to each.
$112\frac{1}{2}\% = 1{\cdot}125 = 1\frac{1}{8}$



- How many fractions can you convert to decimals and percentages in your head?

Fun Spot 1:03 | What was the name of the General Secretary of the United Nations in 1978?

Work out the answer to each part and put the letter for that part in the box that is above the correct answer.

Write the basic numeral for:

A $-8 + 10$ **A** $-7 - 3$ **A** -6×4

A $6 - (3 - 4)$ **A** $(-5)^2$

Y Write $\frac{15}{4}$ as a mixed numeral.

M Change $1\frac{3}{4}$ to an improper fraction.

Write the simplest answer for:

I $\frac{44}{32}$ **I** $\frac{37}{100} - \frac{12}{100}$ **I** $\frac{3}{8} + \frac{1}{3}$

T $\frac{4}{5} - \frac{2}{3}$ **T** $\frac{7}{8} \times \frac{8}{7}$ **T** $(\frac{1}{3})^2$

T $4\frac{3}{8} + \frac{5}{8}$ **T** $2\frac{5}{8} - \frac{1}{2}$ **N** $\frac{1}{2} \div \frac{1}{8}$

N $0{\cdot}05 + 3$ **O** $0{\cdot}3 - 0{\cdot}02$ **O** $0{\cdot}3 \times 5$

E $(0{\cdot}3)^2$ **E** $3{\cdot}142 \times 100$ **E** $6{\cdot}12 \div 6$

E $20{\cdot}08 \div 10$ **C** $1{\cdot}8 \div 0{\cdot}2$

G $\frac{3}{4}$ of 60 kg **D** What fraction is 125 g of 1 kg?

H 5% of 80 kg **H** Write $\frac{2}{5}$ as a percentage.

H Write 0·75 as a fraction. **H** Increase 50 kg by 10%.

D 40% of my weight is 26 kg. How much do I weigh?

S Write $4 \div 9$ as a repeating (recurring) decimal.

S 10 cows, 26 horses and 4 goats are in a paddock. What is the percentage of animals that are horses?

S Increase \$5 by 20%.

S 600 kg is divided between Alan and Rhonda so that Alan gets $\frac{3}{5}$ of the amount. How much does Alan get?

$\frac{1}{9}$	$\frac{3}{4}$	1·02	0·4	7	$\frac{7}{4}$	0·09	2	\$6	$\frac{1}{4}$	1	$\frac{17}{24}$	65%	$\frac{2}{15}$	1·5	$\frac{1}{8}$	25	$3\frac{3}{4}$

55 kg	314·2	4 kg	−10	360 kg	4	0·28	5	9	40%	−24	3·05	45 kg	2·008	65 kg	$1\frac{3}{8}$	$2\frac{1}{8}$

1:04 | Rational Numbers

Integers, fractions, decimals and percentages (both positive and negative) are *rational numbers*.

eg $\frac{3}{4}$, 8, 52%, 0·186, $0.\dot{3}$, −1·5, −10

Exercise 1:04

1 From the list on the right, choose two equivalent numbers for:

a $2\frac{1}{2}$ **b** 130%

c 2·8 **d** $1\frac{1}{4}$

125%	114%	$2\frac{4}{5}$	28%	280%
$2\frac{1}{8}$	1·4	2·5	208%	13
1·25	1·3	$1\frac{3}{10}$	250%	25%

2 Write in order:

a 0·85, 0·805, 0·9, 1 **b** 87·5%, 100%, 104%, $12\frac{1}{4}$%

c $\frac{5}{8}$, $\frac{2}{7}$, $\frac{2}{3}$ and $\frac{6}{100}$ **d** $1\frac{3}{4}$, 150%, 1·65, 2

e 1·42, $\sqrt{2}$, 1·41, 140% **f** π, $3\frac{1}{4}$, 3·1, $\sqrt{12}$

3 Find the number halfway between:

a 6·8 and 6·9 **b** 12% and 20%

c 1 and $\frac{1}{5}$ **d** 6·3 and 6·4

4 Two unit fractions have a difference of $\frac{3}{8}$. What are they? (One half is one of the fractions.)

5 What are the next three numbers in the sequence:

a 0·2, 0·4, 0·8, 1·6, . . . **b** 2·4, 1·2, 0·6, 0·3, . . .

6 The average (ie mean) of five numbers is 5·8.

a What is the sum of these numbers?

b If four of the numbers are 5s, what is the other number?

7 What is meant by an interest rate of 8% p.a.?

8 An advertisement reads: '67% leased. Only one tenancy remaining for lease. Building ready October.' How many tenants would you expect in this building?

9 Using a diameter growth rate of 4·3 mm per year, find the number of years it will take for a tree with a diameter of 20 mm to reach a diameter of 50 mm.

10 At the South Pole the temperature dropped 15°C in two hours, from a temperature of −18°C. What was the temperature after that two hours?

11 Julius Caesar invaded Britain in 55 BC and again one year later. What was the date of the second invasion?

12 Chub was playing 'Five Hundred'.

a His score was −150 points. He gained 520 points. What is his new score?

b His score was 60 points. He lost 180 points. What is his new score?

c His score was −120 points. He lost 320 points. What is his new score?

13 What fraction would be displayed on a calculator as:

a 0·3333333 **b** 0·6666666

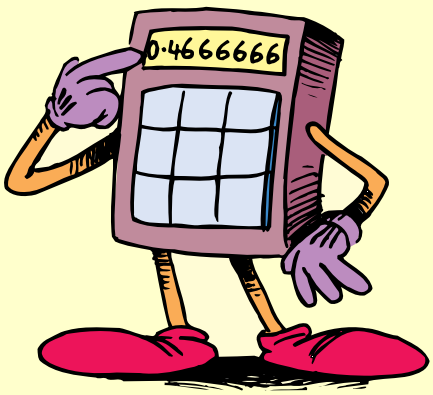
c 0·1111111 **d** 0·5555555

14 To change $\frac{7}{15}$ to a decimal approximation, push (7) (÷) (15) (=) on a calculator.

Use this method to write the following as decimals correct to five decimal places.

a $\frac{8}{9}$ **b** $\frac{2}{7}$ **c** $\frac{7}{13}$

d $\frac{20}{21}$ **e** $\frac{4}{11}$ **f** $\frac{5}{18}$



15 Since I started work my income has increased by 200%. When I started work my income was $21 500. How much do I earn now?

16 What information is needed to complete the following questions?

a If Mary scored 40 marks in a test, what was her percentage?

b In a test out of 120 Nandor made only 3 mistakes. What was his percentage?

c If 53% of cases of cancer occur after the age of 65, what is the chance per 10 000 of developing cancer after the age of 65?

- How many notes are there in an 'octave' on a piano. (There are more than eight.)
- What fraction are white notes?
- What fraction are black notes?

1:05 | Recurring Decimals

prep quiz

1:05

Write these fractions as decimals.

1 $\frac{1}{4}$ **2** $\frac{2}{5}$ **3** $\frac{1}{3}$ **4** $\frac{5}{6}$

$0{\cdot}63974974974\ldots$ is written as $0{\cdot}6\dot{3}9\dot{7}4$

Rewrite these recurring decimals using the 'dot' notation.

5 $0{\cdot}4444\ldots$ **6** $0{\cdot}631631631\ldots$

7 $0{\cdot}166666\ldots$ **8** $0{\cdot}72696969\ldots$

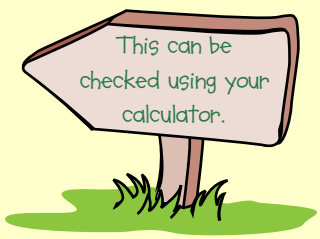
Rewrite these decimals in simplest fraction form.

9 $0{\cdot}75$ **10** $0{\cdot}875$

To write fractions in decimal form we simply divide the numerator (top) by the denominator (bottom). This may result in either a 'terminating' or 'recurring' decimal. For example:

For $\frac{3}{8}$: $8\overline{)3{\cdot}^{3}0^{6}0^{4}0}$ $0{\cdot}375$

For $\frac{1}{6}$: $6\overline{)1{\cdot}^{1}0^{4}0^{4}0^{4}0}$ $0{\cdot}1666\ldots$



To rewrite a terminating decimal as a fraction the process is easy. We simply put the numbers in the decimal over the correct power of 10, ie 10, 100, 1000, etc, and then simplify.

For example: $0{\cdot}375 = \frac{375}{1000}$ $(\div 125)$ $(\div 125)$

$= \frac{3}{8}$

■ Recurring decimals are sometimes called *repeating decimals*.

To rewrite a recurring decimal as a fraction is more difficult. Carefully examine the two examples given below and copy the method shown when doing the following exercise.

worked examples

Example 1

When each number in the decimal is repeated.

Write $0{\cdot}636363\ldots$ as a fraction

Let $x = 0{\cdot}6363\ldots$

Multiply by 100 because two digits are repeated.

Then $100x = 63{\cdot}6363\ldots$

Subtract the two lines.

So $100x - x = 63{\cdot}6363\ldots - 0{\cdot}6363\ldots$

ie $99x = 63$

$\therefore x = \frac{63}{99}$ $\div 9$ $\div 9$

Simplifying this fraction.

$\therefore x = \frac{7}{11}$

Example 2

When only some digits are repeated.

Write $0{\cdot}617777\ldots$ as a fraction

Let $x = 0{\cdot}61777\ldots$

Multiply by 100 to move the non-repeating digits to the left of the decimal point.

Then $100x = 61{\cdot}777\ldots$

Multiply by 1000 to move one set of the repeating digits to the left of the decimal point.

And $1000x = 617{\cdot}777$

Subtract the previous two lines.

So $1000x - 100x = 617{\cdot}777 - 61{\cdot}777$

ie $900x = 556$

$$\therefore\ x = \frac{\cancel{556}^{\div 4}}{\cancel{900}^{\div 4}}$$

Simplifying this fraction using your calculator.

$$\therefore\ x = \frac{139}{225}$$

This answer can be checked by performing $139 \div 225$ using your calculator.

Exercise 1:05

Foundation Worksheet 1:05

Recurring decimals

1 Write as decimals.

a $\frac{1}{5}$ **b** $\frac{17}{100}$

2 Write as fractions.

a $0{\cdot}6$ **b** $0{\cdot}95$

1 Write these fractions as terminating decimals.

a $\frac{3}{4}$ **b** $\frac{4}{5}$ **c** $\frac{5}{8}$ **d** $\frac{7}{10}$

e $\frac{7}{100}$ **f** $\frac{35}{20}$ **g** $\frac{4}{25}$ **h** $\frac{17}{50}$

i $\frac{19}{40}$ **j** $\frac{117}{125}$

2 Write these fractions as recurring decimals.

a $\frac{2}{3}$ **b** $\frac{5}{9}$ **c** $\frac{8}{9}$ **d** $\frac{2}{11}$ **e** $\frac{1}{7}$

f $\frac{1}{6}$ **g** $\frac{1}{15}$ **h** $\frac{7}{15}$ **i** $\frac{1}{24}$ **j** $\frac{17}{30}$

3 Write these terminating decimals as fractions.

a $0{\cdot}47$ **b** $0{\cdot}16$ **c** $0{\cdot}125$ **d** $0{\cdot}85$ **e** $0{\cdot}035$

4 By following Example 1, rewrite these recurring decimals as fractions.

a $0{\cdot}4444\ldots$ **b** $0{\cdot}575757\ldots$ **c** $0{\cdot}173173173\ldots$

d $0{\cdot}\dot{7}$ **e** $0{\cdot}\dot{3}\dot{6}$ **f** $0{\cdot}\dot{1}2\dot{3}\dot{4}$

5 Determine the value of $0{\cdot}\dot{3}$.

6 By following Example 2, rewrite these decimals as fractions.

a $0{\cdot}83333\ldots$ **b** $0{\cdot}6353535\ldots$ **c** $0{\cdot}197777\ldots$

d $0{\cdot}6\dot{4}$ **e** $0{\cdot}73\dot{6}$ **f** $0{\cdot}82\dot{4}\dot{9}$

g $0{\cdot}5\dot{1}2\dot{3}$ **h** $0{\cdot}5278$ **i** $0{\cdot}64\dot{7}3\dot{4}$

challenge

1:05

Challenge 1:05 | Try this with repeating decimals

Here is a clever shortcut method for writing a repeating decimal as a fraction.
Follow the steps carefully.

Example 1

1 $0\cdot\dot{2}\dot{6} = \frac{26-0}{99}$

$= \frac{26}{99}$

Step 1 (Numerator)
Subtract the digits before the repeating digits from all the digits.
Step 2 (Denominator)
Write down a 9 for each repeating digit and then a zero for each non-repeating digit in the decimal.
Step 3
Simplify the fraction if possible.

Example 2

2 $0\cdot32\dot{7} = \frac{327-32}{900}$

(two 0s, one 9)

$= \frac{295}{900}$

$= \frac{59}{180}$

Try converting these repeating decimals to fractions using this method.

1 $0\cdot\dot{7}$ **2** $0\cdot6\dot{7}$ **3** $0\cdot3\dot{1}\dot{2}$ **4** $0\cdot1\dot{6}$ **5** $0\cdot32\dot{1}\dot{7}$

fun spot

1:05

Fun Spot 1:05 | Speedy addition

Rachel discovered an interesting trick.

1 She asked her father to write down a 5-digit number.

2 Rachel then wrote a 5-digit number below her father's. She chose each digit of her number so that when she added it to the digit above, she got 9.

3 She then asked her father to write another 5-digit number.

4 She then repeated step 2.

5 She then asked her father to write one more 5-digit number.

6 She now challenged her father to a race in adding these 5 numbers.

7 Rachel wrote down the answer immediately and surprised her father. Look at the example to see how she did it.

8 She then asked her father to work out how she did it.

9 What should you do if the last number chosen ends with 00 or 01?

4	1	8	4	9
5	8	1	5	0
3	8	1	4	6
6	1	8	5	3
2	1	4	1	1
2	2	1	4	0	9

Put a 2 at the front.
These three digits are the same as in the last number.
Make this 2-digit number two less than the one above it.

1:06 | Simplifying Ratios

prep quiz 1:06

Simplify the fractions:

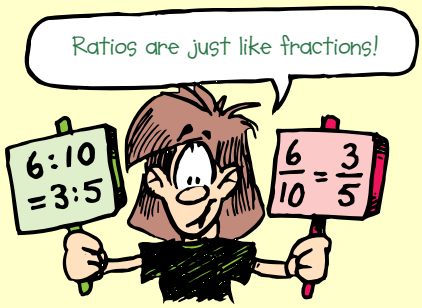
1 $\frac{50}{60}$ **2** $\frac{16}{20}$ **3** $\frac{72}{84}$ **4** $\frac{125}{625}$

What fraction is:

5 50c of $1? **6** 40c of 160c? **7** 8 kg of 10 kg?

8 100 cm of 150 cm? **9** 1 m of 150 cm? **10** $2 of $2.50?

■ A ratio is a comparison of like quantities
eg Comparing 3 km to 5 km we write:
3 to 5 or 3 : 5 or $\frac{3}{5}$.



worked examples

1 Jan's height is 1 metre while Diane's is 150 cm. Find the ratio of their heights.

2 $\frac{3}{5}$ of the class walk to school while $\frac{1}{4}$ ride bicycles. Find the ratio of those who walk to those who ride bicycles.

3 Express the ratio 11 to 4 in the form **a** X : 1 and **b** 1 : Y.

Solutions

1 Jan's height to Diane's height
= 1 m to 150 cm
= 100 cm to 150 cm
= 100 : 150
Divide both terms by 50.
= 2 : 3 or $\frac{2}{3}$

■ Each term is expressed in the same units, then units are left out.

■ We may simplify ratios by dividing or multiplying each term by the same number.

From this ratio we can see that Jan is $\frac{2}{3}$ as tall as Diane.

2 Those walking to those cycling
$= \frac{3}{5} : \frac{1}{4}$
Multiply both terms by 20.
$= \frac{3}{\cancel{5}_1} \times \frac{\cancel{20}^4}{1} : \frac{1}{\cancel{4}_1} \times \frac{\cancel{20}^5}{1}$
= 12 : 5

■ To remove fractions multiply each term by the lowest common denominator.

continued →→→

3 **a** 11 to 4
$= 11 : 4$
Divide both terms by 4.
$= \frac{11}{4} : 1$
$= 2\frac{3}{4} : 1$
This is in the form X : 1.

b 11 to 4
$= 11 : 4$
Divide both terms by 11.
$= 1 : \frac{4}{11}$
This is in the form 1 : Y.

Exercise 1:06

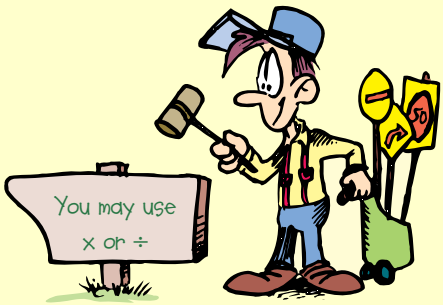
1 Express the first quantity as a fraction of the second each time.

a 7 men, 15 men	**b** 10 kg, 21 kg	**c** 3 cm, 4 cm
d \$5, \$50	**e** 8 m, 10 m	**f** 10 bags, 100 bags
g 75 g, 80 g	**h** 6 runs, 30 runs	**i** 25 goals, 120 goals

■ Simplify the fractions.

2 Simplify the ratios.

a 6 : 4	**b** 10 : 5
c 65 : 15	**d** 14 : 35
e 20 : 45	**f** 42 : 60
g 60 : 15	**h** 45 : 50
i 1000 : 5	**j** 1100 : 800
k 55 : 20	**l** 16 : 28
m 10 : 105	**n** 72 : 2
o 4 : 104	**p** $10 : \frac{1}{2}$
q $\frac{1}{2} : 2\frac{1}{2}$	**r** $2\frac{1}{2} : 2$
s $2\frac{3}{4} : 1$	**t** $2 : 3\frac{1}{4}$
u $6\frac{3}{4} : 3$	**v** $\frac{3}{4} : \frac{1}{2}$
w $\frac{11}{16} : \frac{1}{2}$	**x** $\frac{2}{3} : \frac{1}{2}$



3 In each, find the ratio of the first quantity to the second, giving your answers in simplest form.

a 7 men, 9 men	**b** 13 kg, 15 kg	**c** 7 cm, 8 cm
d \$8, \$12	**e** 16 m, 20 m	**f** 15 bags, 85 bags
g 90 g, 100 g	**h** 9 runs, 18 runs	**i** 50 goals, 400 goals
j 64 ha, 50 ha	**k** 25 m, 15 m	**l** 100 m^2, 40 m^2

4 Find the ratio of the first quantity to the second. Give answers in simplest form.

a \$1, 50c	**b** \$5, \$2.50	**c** \$1.20, \$6
d 1 m, 60 cm	**e** 25 cm, 2 m	**f** 100 m, 1 km
g 600 mL, 1 L	**h** 1 L, 600 mL	**i** 1 L, 250 mL
j 2 h, 40 min	**k** 50 min, 1 h	**l** 2 h 30 min, 5 h

■ Are units the SAME?

5 Express each ratio in the form X : 1.
a 2 : 5 **b** 3 : 4 **c** 7 : 10 **d** 5 : 3 **e** 20 : 9

6 Express each ratio in question **5** in the form 1 : Y.

7 **a** A recipe recommends the use of two parts sugar to one part flour and one part custard powder. What does this mean?

b A mix for fixing the ridge-capping on a roof is given as 1 part cement to 5 parts sand and a half part of lime. What does this mean?

c The ratio of a model to the real thing is called the scale factor. My model of an aeroplane is 40 cm long. If the real plane is 16 m long, what is the scale factor of my model?

d My father is 180 cm tall. If a photograph of him has a height of 9 cm, what is the scale of the photograph?

1:07 | Rates

prep quiz 1:07

If Wendy earns $16 per hour, how much would she earn in:

1 2 hours? **2** 3 hours? **3** 5 hours? **4** half an hour?

Complete:

5 1 kg = . . . g **6** 1 tonne = . . . kg **7** 1 hour = . . . min

8 1 cm = . . . mm **9** 1 m^2 = . . . cm^2 **10** 1·5 litres = . . . millilitres

A rate is a comparison of unlike quantities:
eg If I travel 180 km in 3 hours my average rate of speed is $\frac{180\text{ km}}{3\text{ h}}$ or 60 km/h or 60 km per h.

Usually we write down how many of the first quantity correspond to one of the second quantity, eg 60 kilometres per one hour, ie 60 km/h.

worked examples

1 84 km in 2 hours

Divide each term by 2.

= 42 km in 1 hour

= 42 km/h

or

84 km in 2 hours

$= \frac{84\text{ km}}{2\text{ h}}$

$= \frac{84}{2}\ \frac{\text{km}}{\text{h}}$

= 42 km/h

■ Units must be shown.

■ Example (1) is an average rate because, when you travel, your speed may vary from moment to moment.
Example (2) is a constant rate, because each kg will cost the same.

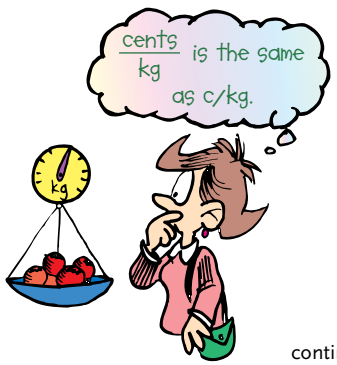
2 16 kg of tomatoes are sold for $10.
What is the cost per kilogram?

$\text{Cost} = \frac{\$10}{16\text{ kg}}$

$= \frac{1000}{16}\ \frac{\text{cents}}{\text{kg}}$

$= \frac{125}{2}\ \frac{\text{cents}}{\text{kg}}$

= 62·5 cents/kg



continued →→→

3 A plumber charges a householder \$24 per hour to fix the plumbing in a house. Find the cost if it takes him $4\frac{1}{2}$ hours.

Rate = \$24 per 1 hour

Multiply both terms by $4\frac{1}{2}$

$= \$24 \times 4\frac{1}{2}$ per $4\frac{1}{2}$ h

$\therefore$ Cost = \$108

4 Change 72 litres per hour into cm^3 per second.

$$72\text{ L per h} = \frac{72\text{ L}}{1\text{ h}}$$

$$= \frac{72\,000\text{ mL}}{60\text{ min}}$$

$$= \frac{72\,000\text{ cm}^3}{60 \times 60\text{ s}}$$

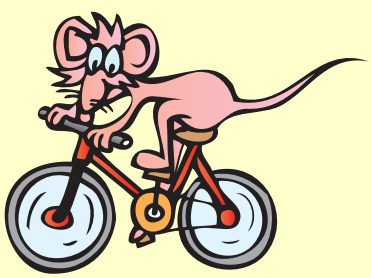
$\therefore$ 72 L/h = 20 cm^3/s

Exercise 1:07

1 Write each pair of quantities as a rate in its simplest form.

a 6 km, 2 h
b 10 kg, \$5
c 500c, 10 kg
d 100 mL, 100 cm^3
e 160 L, 4 h
f \$100, 5 h
g \$315, 7 days
h 70 km, 10 L
i 20 degrees, 5 min
j 7000 g, 100 cm
k 50 t, 2 blocks
l 60 km, $\frac{1}{2}$ h
m 88 runs, 8 wickets
n 18 children, 6 mothers
o 75 g, 10 cm^3

2 **a** I walk at 5 km/h. How far can I walk in 3 hours?
b Nails cost \$2.45 per kg. What is the cost of 20 kg?
c I can buy four exercise books for \$5. How many books can I buy for \$20?
d I earn \$8.45 per hour. How much am I paid for 12 hours work?
e On a cycling trip, Cherry cycles exactly 37.5 km per day. How far will she cycle in 6 days?
f The fuel value of milk is measured as 670 kilojoules per cup. What is the fuel value of 3 cups of milk?
g If the rate of exchange for one English pound is 1·60 American dollars, find the value of ten English pounds in American currency.
h The density of iron is 7·5 g/cm^3. What is the mass of 1000 cm^3 of iron? (Density is mass per unit of volume.)
i If light travels at 300 000 km/s, how far would it travel in one minute?
j If I am taxed 1·6c for every \$1 on the value of my \$50 000 block of land, how much must I pay?



3 Complete the equivalent rates.

a 1 km/min = . . . km/h
b 40 000 m/h = . . . km/h
c \$50/kg = . . . c/kg
d \$50/kg = . . . c/g
e 144 L/h = . . . mL/s
f 60 km/h = . . . m/s
g 7 km/L = . . . m/mL
h 25c/h = . . . \$/week
i 30 mm/s = . . . km/h
j 90 beats/min = . . . beats/s
k 800 kg/h = . . . t/day
l 3 t/h = . . . kg/min
m 10 jokes/min = . . . jokes/h
n 50c/m^2 = . . . \$/ha
o 1·05 cm^3/g = . . . cm^3/kg

1:08 | Significant Figures

No matter how accurate measuring instruments are, a quantity such as length cannot be measured exactly. Any measurement is only an approximation.

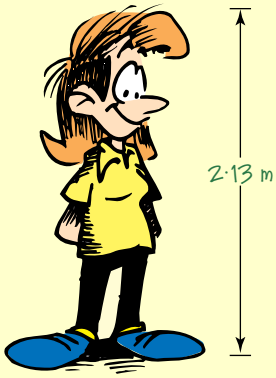
- A measurement is only useful when one can be confident of its validity. To make sure that a measurement is useful, each digit in the number should be significant.

  For example, if the height of a person, expressed in significant figures, is written as 2·13 m it is assumed that only the last figure may be in error.

  Clearly any uncertainty in the first or second figure would remove all significance from the last figure. (If we are not sure of the number of metres, it is pointless to worry about the number of centimetres.)
- It is assumed that in the measurement 2·13 m we have measured the number of metres, the number of tenths of a metre and the number of hundredths of a metre.

  Three of the figures have been measured so there are *three significant figures*.
- To calculate the number of significant figures in a measurement we use the rules below.

■ A significant figure is a number that we believe to be correct within some specific or implied limit of error.



RULES FOR DETERMINING SIGNIFICANT FIGURES

1. **Coming from the left, the first non-zero digit is the first significant figure.**
2. **All figures following the first significant figure are also significant, unless the number is a *whole* number ending in zeros.**
3. **Final zeros in a whole number may or may not be significant, eg 0·00120 has three significant figures, 8800 may have two, three or four.**

Putting this more simply:

1 Starting from the left, the first significant figure is the first non-zero digit.

eg 0·003 250 865 000 8·007

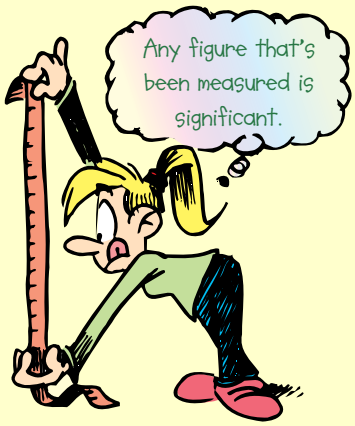
the first non-zero digit

2 Final zeros in a whole number may or may not be significant.

eg 56 000 000 73 210 18 000

Unless we are told, we cannot tell whether these zeros are significant.

3 All non-zero digits are significant.



4 Zeros at the end of a decimal are significant.

eg 3·0 213·123 0 0·000 010 0 0

These final zeros are significant.

Method for counting the number of significant figures

Every figure between the first and last significant figure is significant.

Locate the first and last significant figures, then count the significant figures, including all digits between the first and last significant figures.

worked examples

Example A

How many significant figures has:

1 316 000 000 (to nearest million)?
2 316 000 000 (to nearest thousand)?
3 42 007?
4 31·005 0?
5 0·000 130 50?

Solutions A

1 316 000 000 (to nearest million)
last significant figure
first significant figure
Number of significant figures = 3

2 316 000 000 (to nearest thousand)
last sig. fig. (thousands)
first significant figure
Number of significant figures = 6

3 42 007
last
first
Number of significant figures = 5

4 31·005 0
last
first
Number of significant figures = 6

5 0·000 130 50
last
first
Number of significant figures = 5

Example B

1 The distance of the earth from the sun is given as 152 000 000 km.
In this measurement it appears that the distance has been given to the nearest million kilometres. The zeros may or may not be significant but it seems that they are being used only to locate the decimal point. Hence the measurement has three significant figures.

2 A female athlete said she ran 5000 metres.
This is ambiguous. The zeros may or may not be significant. You would have to decide whether or not they were significant from the context of the statement.

Exercise 1:08

1 How many significant figures are there in each of the following numerals?

a 2·1	**b** 1·76	**c** 9·05	**d** 0·62	**e** 7·305	**f** 0·104
g 3·6	**h** 3·60	**i** 0·002	**j** 0·056	**k** 0·04	**l** 0·40
m 0·00471	**n** 3·040	**o** 0·5	**p** 304	**q** 7001	**r** 0·001 50
s 0·000 000 125		**t** 0·000 000 100		**u** 0·000 000 001	

2 **a** 2000 (to the nearest thousand) **b** 2000 (to the nearest hundred)
c 53 000 (to the nearest thousand) **d** 530 000 (to the nearest thousand)
e 25 000 (to the nearest ten) **f** 26 300 (to the nearest hundred)
g 26 000 (to the nearest hundred) **h** 8 176 530 (to the nearest ten)

3 A newspaper article reported that 20 000 people attended the 'Festival of Lights' concert. How accurate would you expect this number to be? (That is, how many significant figures would the number have?)

1:09 | Approximations

How many significant figures do the following numbers have?

1 3·605 **2** 0·06 **3** 0·1050

Write one more term for each number sequence.

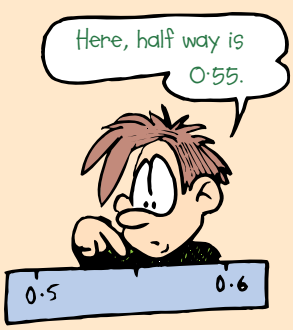
4 3·06, 3·07, . . . **5** 0·78, 0·79, . . . **6** 2·408, 2·409, . . .

7 Is 3·7 closer to 3 or 4?

8 Is 2·327 closer to 2·32 or 2·33?

9 What number is halfway between 3·5 and 3·6?

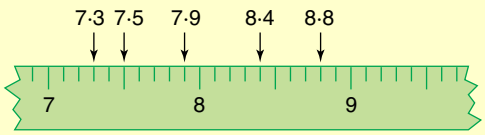
10 What number is halfway between 0·06 and 0·07?



prep quiz 1:09

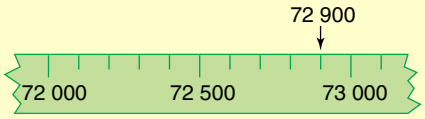
Discussion

- To round off a decimal to the nearest whole number, we write down the whole number closest to it.



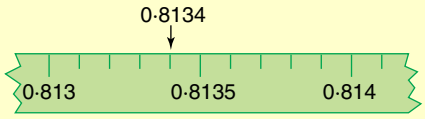
7·3 is closer to 7	7·9 is closer to 8	8·4 is closer to 8	8·8 is closer to 9
7·5 is exactly halfway between 7 and 8. In cases like this it is common to round up. We say 7·5 = 8 correct to the nearest whole number.			

- To round off 72 900 to the nearest thousand we write down the thousand closest to it.



72 900 is closer to 73 000 than to 72 000.
∴ 72 900 = 73 000 correct to the nearest thousand.

- To round off 0·813 4 to the nearest thousandth we write down the thousandth closest to it.



0·813 4 is closer to 0·813 than to 0·814.
∴ 0·813 4 = 0·813 correct to the nearest thousandth.

To round off (or approximate) a number correct to a given place we round up if the next figure is 5 or more, and round down if the next figure is less than 5.

worked examples

Example A

Round off:

1 56 700 000 to the nearest million

2 0·085 1 to the nearest hundredth

3 86·149 to one decimal place

4 0·666 15 correct to four decimal places

Solutions A

1 56 700 000 has a 6 in the millions column.
The number after the 6 is 5 or more (ie 7).
∴ 56 700 000 = 57 000 000 correct to the nearest million.

2 0·085 1 has an 8 in the hundredths column.
The number after the 8 is 5 or more (ie 5).
∴ 0·085 1 = 0·09 correct to the nearest hundredth.

3 86·149 has a 1 in the first decimal place.
The number after the 1 is less than 5 (ie 4).
∴ 86·149 = 86·1 correct to one decimal place.

4 0·666 15 has a 1 in the fourth decimal place.
The number after the 1 is 5 or more (ie 5).
∴ 0·666 15 = 0·666 2 correct to four decimal places.

To approximate correct to a certain number of significant figures, we write down the number that contains only the required number of significant figures and is closest in value to the given number.

Example B

Round off:

5 507 000 000 to 2 significant figures.

6 1·098 to 3 significant figures.

7 0·006 25 correct to 1 sig. fig.

8 0·080 25 correct to 3 sig. figs.

Solutions B

5 The 2nd significant figure is the 0 between the 5 and 7. The number after the zero is 5 or more (ie 7).
∴ 507 000 000 = 510 000 000 correct to 2 significant figures.

6 The 3rd significant figure is the 9. The number after the 9 is 8, so increase the 9 to 10. Put down the 0 and carry the 1.
∴ 1·098 = 1·10 correct to 3 significant figures.

7 The 1st significant figure is 6. The number after the 6 is less than 5 (ie 2).
∴ 0·006 25 = 0·006 correct to 1 sig. fig.

8 The 3rd significant figure is 2. The number after the 2 is 5 or more (ie 5).
∴ 0·080 25 = 0·080 3 correct to 3 sig. figs.

Exercise 1:09

Foundation Worksheet 1:09

Approximation

1 Write 7463·9 to the nearest:
a integer **b** ten **c** hundred

2 Write 6·4937 correct to:
a 3 dec. pl. **b** 2 dec. pl. **c** 1 dec. pl.

1 Round off these numbers to the nearest hundred.

a 7923	**b** 1099	**c** 67 314	**d** 853·461
e 609·99	**f** 350	**g** 74 932	**h** 7850

2 Round off these numbers to the nearest whole number.

a 9·3	**b** 79·5	**c** 45·1	**d** 2·7
e 2·314	**f** 17·81	**g** 236·502	**h** 99·5

3 Round off these numbers to the nearest hundredth.

a 243·128	**b** 79·664	**c** 91·351	**d** 9·807
e 0·3046	**f** 0·085 2	**g** 0·097	**h** 1·991

4 Round off these numbers correct to one decimal place.

a 6·70	**b** 8·45	**c** 2·119	**d** 6·092
e 0·05	**f** 0·035	**g** 29·88	**h** 9·99

5 Round off these numbers correct to 2 significant figures.

a 8170	**b** 3504	**c** 655	**d** 849
e 14 580	**f** 76 399	**g** 49 788	**h** 76 500

6 Round off the numbers in question 5 correct to 1 sig. fig.

7 Round these off to 3 sig. figs.

a 694·8	**b** 35·085	**c** 320·5	**d** 0·081 54
e 0·666 66	**f** 9·3333	**g** 10·085	**h** 9·095

8 To change $1\frac{7}{9}$ to a decimal, Gregory divided 16 by 9 using his calculator. Give the answer correct to:

a 1 dec. pl.	**b** 2 dec. pl.	**c** 3 dec. pl.
d 1 sig. fig.	**e** 2 sig. figs.	**f** 3 sig. figs.

■ When you 'round off' you are making an approximation.

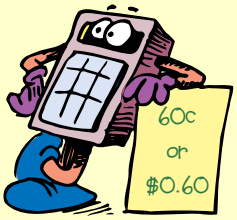
■ 'sig. figs.' is short for 'significant figures', 'dec. pl.' is short for 'decimal places'.

9 Diane cut 60 cm of blue ribbon into 11 equal parts to make a suit for her new baby. After dividing, she got the answer $5{\cdot}4\dot{5}$. Give the length of one part correct to:

a the nearest centimetre **b** the nearest millimetre
c 1 dec. pl. **d** 2 dec. pl. **e** 3 dec. pl.
f 1 sig. fig. **g** 2 sig. figs. **h** 3 sig. figs.

10 The following calculator display represents an answer in cents. 14059·705
Give this answer correct to:

a the nearest dollar **b** the nearest cent
c 1 dec. pl. **d** 2 dec. pl.
e 3 dec. pl. **f** 1 sig. fig.
g 2 sig. figs. **h** 3 sig. figs.



11 What level of accuracy do you think was used in each of these measurements and what would be the greatest error possible as a result of the approximation?

a The crowd size was 18 000.
b The nation's gross domestic product was \$62 000 000 000.

12 What approximation has been made in each of these measurements and what would be the greatest error possible?

a 6·4 cm
b 0·007 mg

13 A number is rounded to give 2·15. What could the number have been? What is the smallest the number could have been? Is it possible to write down the largest number that can be rounded to give 2·15?

14 An answer is given as 3 000 000 correct to 1 significant figure. What might the exact measure have been?

15 Seven people decide to share a bill of \$187.45 equally. How much should each person pay? What could be done with the remainder?

16 The area of a room is needed to order floor tiles. The room dimensions, 2·49 m by 4·31 m, were rounded off to 2 m by 4 m to calculate the area. What problems might arise?

17 A 10-digit calculator was used to change fractions into decimals. The truncating of the decimal produced an error. What error is present in the display after entering:

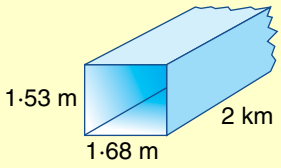
a $\frac{1}{3}$? **b** $\frac{2}{3}$? **c** $\frac{5}{9}$?

18 Find an approximation for $345{\cdot}45^2$ by first rounding off the 345·45 correct to:

a 1 sig. fig. **b** 1 dec. pl. **c** 2 sig. figs.

19 In question **18**, what is the difference between the answer to **a** and the real answer?

20 To find the volume of the tunnel drawn on the right, each measurement was rounded off correct to 1 significant figure before calculation. What error in volume occurred?



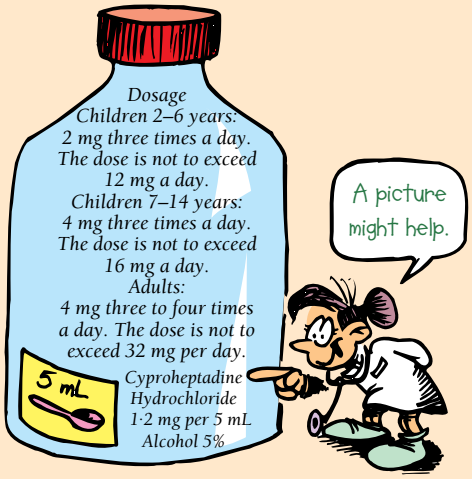
Note: Truncating or 'rounding' numbers before a calculation may produce unwanted errors or inaccuracy.

Reading mathematics 1:09 | Take your medicine!

Many deaths have occurred because people have misread or not understood directions on medicine bottles. Instructions are often difficult to read and require sophisticated measuring instruments.

Before beginning the investigations listed below, use the picture to answer these questions. The information shown was on the label of a 100 mL bottle of a certain medicine.

- Read the information carefully and answer the questions below.
  1. What is the dosage for an 8-year-old child and how often should it be taken?
  2. A 5-year-old girl has a dosage of 2 mg. What is the usual number of times she should take the dose? What is the maximum number of doses she may take?
  3. What is the youngest age for which the adult dose is recommended?
  4. In a 100-mL bottle, how many millilitres of alcohol are present?
  5. How many milligrams of the chemical cyproheptadine hydrochloride would be in a 100-mL bottle of this medicine?



A Investigate the labelling on medicine bottles and other medicinal preparations (eg pet worming tablets, etc).

B Suggest ways in which directions could be given that would make them easier to understand.

C Redesign the label in the picture so that it reflects your answer to part B.

D Present your findings in the form of a written report.

1:10 | Estimation

prep quiz

1:10

1 Write 216 to the nearest hundred.

2 Write 17·68 to the nearest ten.

3 $15{\cdot}61 \times 10$

4 $15{\cdot}61 \times 100$

5 Is $\frac{0{\cdot}716}{3{\cdot}5}$ less than or greater than 1?

6 If $3 < 4$ and $5{\cdot}3 < 7{\cdot}8$, what sign (< or >) can we put in the box?
$3 \times 5{\cdot}3 \square 4 \times 7{\cdot}8$

7 *True* or *false*?
$21{\cdot}68 \times 0{\cdot}716 < 21{\cdot}68 \times 1$

8 Which is the best approximation for $0{\cdot}316 \times 0{\cdot}81$?
a 2·5 **b** 0·25 **c** 0·025

9 $\frac{7{\cdot}6}{0{\cdot}25} > 7{\cdot}6$, *true* or *false*?

10 $\frac{7{\cdot}6}{8{\cdot}3} > 1$, *true* or *false*?

Like all machines, calculators only operate correctly if they are used correctly. Even when doing simple calculations it is still possible to press the wrong button. So it is essential that you learn how to estimate the size of the answer before the calculation is even started.

An estimate is a valuable means of checking whether your calculator work gives a sensible answer. If your estimate and the actual answer are not similar, then it tells you that a mistake has been made either in your estimate or your calculation.

The following examples will show you how to estimate the size of an answer.

worked examples

Estimate the size of each of the following calculations.

1 $14{\cdot}61 - 7{\cdot}15 + 3{\cdot}2$

2 $7{\cdot}56 \times 5{\cdot}173$

3 $0{\cdot}0253 \div 0{\cdot}45$

4 $\frac{21{\cdot}73 \times 0{\cdot}815}{7{\cdot}3}$

5 $\frac{\sqrt{86}}{2{\cdot}8 \times 16{\cdot}18}$

Both numbers are multiplied by 100 to simplify the question.

Solutions

1 $14{\cdot}61 - 7{\cdot}15 + 3{\cdot}2$
$\approx 15 - 7 + 3$
≈ 11

2 $7{\cdot}56 \times 5{\cdot}173$
$\approx 8 \times 5$
≈ 40

3 $0{\cdot}0253 \div 0{\cdot}45$
$= 2{\cdot}53 \div 45$
$\approx 3 \div 45$
$\approx \frac{1}{15}$ or 0·07

≈ or ≒ means 'is approximately equal to'.

4 $\frac{21{\cdot}73 \times 0{\cdot}815}{7{\cdot}3}$
$\fallingdotseq \frac{21 \times 1}{7}$
$\fallingdotseq 3$

5 $\frac{\sqrt{86}}{2{\cdot}8 \times 16{\cdot}18}$
$\fallingdotseq \frac{{}^{3}\cancel{9}}{{}_{1}\cancel{3} \times 16}$
$\fallingdotseq \frac{3}{16}$
$\fallingdotseq \frac{1}{5}$ or 0·2

These hints may be useful.

- When estimating, look for numbers that are easy to work with, eg 1, 10, 100.
- Remember it's an estimate. When you approximate a number you don't have to take the nearest whole number.
- Try thinking of decimals as fractions. It often helps.
- eg $7{\cdot}6 \times 0{\cdot}518 \approx 8 \times \frac{1}{2}$ or 4
- When dealing with estimates involving fraction bars, look for numbers that nearly cancel out.
- eg $\dfrac{{}^{2}\cancel{17{\cdot}68} \times 5{\cdot}8}{\cancel{8{\cdot}9}_{1}} \approx \dfrac{2 \times 6}{1} = 12$

Check that the answer makes sense.

Exercise 1:10

Foundation Worksheet 1:10

Estimation

1 Estimate:
a 4·9 + 10·2 **b** 6·15 × 9·89

2 Find an approximate answer to:
a 16·1 ÷ 7·9 **b** $(7{\cdot}1)^2 \div 9{\cdot}9$

1 Estimate the answers to the following calculations.

a $7{\cdot}9 + 0{\cdot}81 + 13{\cdot}56$ **b** $42{\cdot}56 - 15{\cdot}81 + 9{\cdot}2$
c $5{\cdot}6 \times (7{\cdot}2 + 5{\cdot}9)$ **d** $14{\cdot}31 \times 8{\cdot}97$
e $73{\cdot}95 \div 14{\cdot}2$ **f** $0{\cdot}73 \times 0{\cdot}05 \div 4{\cdot}53$
g $0{\cdot}916 \times 0{\cdot}032 \times 18{\cdot}34$ **h** $(15{\cdot}6 + 6{\cdot}82) \times 5{\cdot}31$
i $15{\cdot}6 + 6{\cdot}82 \times 5{\cdot}31$ **j** $(14{\cdot}56 + 3{\cdot}075) \div (0{\cdot}561 \times 20{\cdot}52)$

2 Estimate the answers for each of the following (giving your answer as an integer, ie a whole number).

a $\dfrac{7{\cdot}3 \times 9{\cdot}8}{15{\cdot}6 \times 3{\cdot}2}$ **b** $\dfrac{212 \times 71{\cdot}5}{15{\cdot}8 \times 0{\cdot}89}$ **c** $\dfrac{3{\cdot}6 \times 9{\cdot}7}{15{\cdot}8}$ **d** $\dfrac{7{\cdot}82 \times 5{\cdot}64}{9{\cdot}8 + 2{\cdot}96}$

e $\dfrac{19{\cdot}6 - 5{\cdot}8}{3{\cdot}6 \times 1{\cdot}72}$

f $\dfrac{205{\cdot}3 - 76{\cdot}8}{41{\cdot}16 + 13{\cdot}7}$

g $6{\cdot}28 \times \sqrt{\dfrac{9{\cdot}78}{0{\cdot}53}}$

h $\dfrac{76{\cdot}5(3{\cdot}7 + 15{\cdot}6)}{2{\cdot}9 \times 15{\cdot}8}$

i $9{\cdot}6 \times 0{\cdot}75 + \dfrac{10{\cdot}6 \times (3{\cdot}5)^2}{2}$

Note:

- The fraction bar acts a little like grouping symbols. You work out the numerator and denominator separately.
- In $\sqrt{41{\cdot}6 + 39{\cdot}5}$ you must work out the addition first. The square root sign also acts like grouping symbols.

3 When estimating the size of a measurement, both the number and the unit must be considered. In each case, choose the most likely answer by estimation.

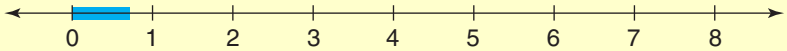
a The weight of the newborn baby was:
i 350 g **ii** 7·8 kg **iii** 3·1 kg **iv** 50 pounds

b The length of a mature blue whale is about:
i 27 m **ii** 3 km **iii** 32 cm **iv** 98 m

c 12% discount on a television set marked $2300 is:
i $86.60 **ii** $866 **iii** $276 **iv** $27.60

d I just borrowed more than $80 000 from the bank. Next year the interest on the loan is:
i $873 **ii** $9400 **iii** $185.60 **iv** $21 140

4



Use the number line above to estimate each of the following as a whole number.

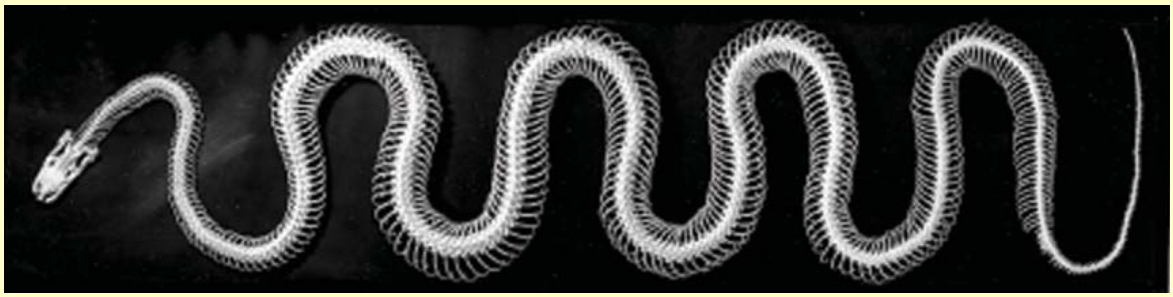
a four lots of 0·75
b 0·75 multiplied by 7
c $0{\cdot}75 \times 3$
d $0{\cdot}75 \times 5$
e how many 0·75s make 8?
f how many 0·75s in 1?
g $6 \div 0{\cdot}75$
h $8 \div 0{\cdot}75$

5 **a** A pile of paper is 3·2 cm thick. If there are 300 sheets in the pile, estimate the thickness of one sheet of paper.

b Peter estimated that there were 80 people sitting in an area of 50 m^2 at the 'Festival of Lights' service. He estimated that about 2000 m^2 of area was similarly occupied by the crowd. To the nearest 100, what is his estimate of the crowd size?

c Would $8{\cdot}6 \times 84{\cdot}4$ be between 8×80 and 9×90? Explain why.

6 Two measurements were rounded off correct to two significant figures and then multiplied to estimate an area. The working was: 92 m × 0·81 m = 74·52 m^2. Between which two measurements would the real area lie? How many of the figures in this estimate are useful, given the possible spread of the area?



• Estimate the number of vertebrae in this python skeleton. (The numerous vertebrae, or spinal-column segments, give the backbone great flexibility.)

Mathematical terms 1

approximate

- To replace a number with a less accurate one, often to make it simpler.
  eg 3·94 m might be approximated to 4 m.

decimal place

- The position of a numeral after the decimal point, each position being $\frac{1}{10}$ of the one before it.
  eg the number 0·639 has three decimal places.
  $0{\cdot}639 = \frac{6}{10} + \frac{3}{100} + \frac{9}{1000}$

estimate

- To calculate roughly (*v*).
- A good guess or the result of calculating roughly (*n*).

fraction

- One or more parts of a whole expressed in the form $\frac{a}{b}$ where '*a*' and '*b*' are integers and $b \neq 0$.
  eg $\frac{7}{8}$ ← numerator / denominator

integer

- A whole number which may be positive, negative or zero.
  eg 7, −23, 0.

percentage

- Fraction which has a denominator of 100, written using the symbol %.
  eg $\frac{27}{100} = 27\%$

rational number

- A number which can be expressed in fraction form. This includes integers, percentages, terminating and recurring decimals.
  eg $\frac{4}{7}$, $-2\frac{1}{9}$, 7, 16%, −0·69, $4{\cdot}6\dot{3}\dot{2}$

recurring decimal (repeating)

- A decimal for which the digits set up a repeating pattern.
  eg 0·737373 . . . or 0·69444 . . .
  These numbers can be written as:
  $0{\cdot}\dot{7}\dot{3}$ $0{\cdot}69\dot{4}$

significant figure

- A number that we believe to be correct within some limit of error.
- To round off a number to a number of significant figures is to specify the accuracy required from a calculation.
  eg 16·483 to 3 sig. figs. = 16·5
  0·00475 to 1 sig. fig. = 0·005

terminating decimal

- A decimal number which has a limited number of decimal places.
  eg 0·6, 0·475, 0·0069

- The mean earth–moon distance is 380 000 km, to the nearest thousand kilometres. How many significant figures is this?

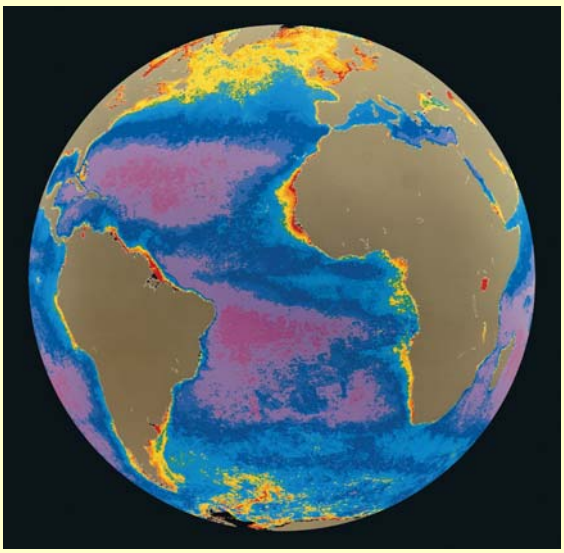
diagnostic test

1

Diagnostic Test 1: | Basic Number Skills

- These questions reflect some of the important skills introduced in this chapter. Other skills have been covered in diagnostic tests within the chapter or previously in Book 2.
- Errors made in this test will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
1 Write these fractions as recurring decimals. **a** $\frac{2}{3}$ **b** $\frac{1}{6}$ **c** $\frac{7}{9}$ **d** $\frac{4}{15}$	1:05
2 Write these recurring decimals in fraction form. **a** 0·5555 . . . **b** $0\cdot\dot{3}\dot{7}$ **c** 0·5777 . . . **d** $0\cdot6\dot{8}\dot{9}$	1:05
3 How many significant figures has: **a** 316 000 000 (to the nearest thousand) **b** 42 007 **c** 0·00013050 **d** 6200	1:08
4 Round off: **a** 56 700 000 to the nearest million **b** 0·66615 correct to 4 dec. pl. **c** 7·983 correct to 1 dec. pl. **d** 4·673 m correct to nearest cm	1:09
5 Round off: **a** 0·00625 to 1 sig. fig. **b** 507 000 000 to 2 sig. figs. **c** 0·06075 to 2 sig. figs. **d** 93 784 231 to 3 sig. figs.	1:09
6 Estimate the size of each calculation to the nearest whole number. **a** $8{\cdot}73 \times 4{\cdot}132$ **b** $0{\cdot}0394 \div 0{\cdot}00812$ **c** $\frac{25{\cdot}4 \times 7{\cdot}98}{9{\cdot}93}$ **d** $\sqrt{65{\cdot}4} \times (4{\cdot}97)^2$	1:10



- The mean radius of the earth is 6370 km, correct to three significant figures. This means it has been measured to the nearest how many kilometres?

Chapter 1 | Revision Assignment

assignment 1A

1 Simplify:
- a $15 - 3 \times 2$
- b $24 \div 4 + 5 \times 2$
- c $-6 \times (-5) \div (-10)$

2 Give the simplest answer to:
- a $\frac{7}{10} + \frac{1}{10}$
- b $\frac{5}{6} - \frac{2}{3}$
- c $\frac{2}{5} \times \frac{1}{4}$
- d $\frac{5}{9} \div \frac{2}{3}$
- e $2\frac{3}{4} + 3\frac{1}{5}$
- f $6\frac{1}{3} - 4\frac{3}{5}$
- g $3\frac{1}{2} \times 2\frac{2}{3}$
- h $5\frac{1}{4} \div \frac{7}{8}$

3 Simplify:
- a $9{\cdot}2 - 4{\cdot}73$
- b $6{\cdot}2 \times 0{\cdot}7$
- c $24{\cdot}3 \div 0{\cdot}6$
- d $(0{\cdot}3)^2$

4 Write these decimals as fractions:
- a $0{\cdot}35$
- b $0{\cdot}875$
- c $0{\cdot}\dot{2}\dot{3}$
- d $0{\cdot}3\dot{4}\dot{7}$

5 a Find $7\frac{1}{2}\%$ of \$350.
- b What percentage is \$65 of \$325?

6 Simplify these ratios:
- a 25 : 45
- b \$6 : \$18
- c 50 cm : 2 m
- d 15 min : $1\frac{1}{2}$ h

7 How many significant figures are there in each of these numerals?
- a 5·27
- b 0·00604
- c 6·90
- d 93 000 (to nearest thousand)

8 Round off these numbers as indicated.
- a 6·4472 to 1 dec. pl.
- b 6·91673 to nearest hundredth
- c 47 643 908 to nearest thousand
- d 647·542 to nearest whole number
- e 6·3942 to 2 sig. figs.
- f 0·005817 to 3 sig. figs.
- g 47 649 503 to 3 sig. figs.
- h 0·704906 to 2 sig. figs.

9 A number with three decimal places was rounded to 3·7.
- a What is the smallest the number could have been?
- b What is the largest the number could have been?

10 Estimate each of these calculations to the nearest integer.
- a $4{\cdot}2 + 15{\cdot}9 - 9{\cdot}9 + 6{\cdot}1$
- b $(15{\cdot}84 + 9{\cdot}21) \div (12{\cdot}97 - 7{\cdot}86)$
- c $\frac{8{\cdot}3 \times 6{\cdot}8}{3{\cdot}9 \times 7{\cdot}2}$
- d $(7{\cdot}1)^2 \times \sqrt{15{\cdot}98}$

Technology Applications Activities — Coldest temperatures

1 Converting fractions to decimals
2 Simplifying ratios
3 Rates
4 Significant figures

Technology Applications Drag and Drops

Chapter Review Questions

- What sort of estimates would organisers of large fun runs have to make?

Chapter 1 | Working Mathematically

1 Use ID Card 1 on page xiv to give the mathematical term for:

a 9 b 10 c 15 d 16
e 18 f 19 g 20 h 22
i 23 j 24

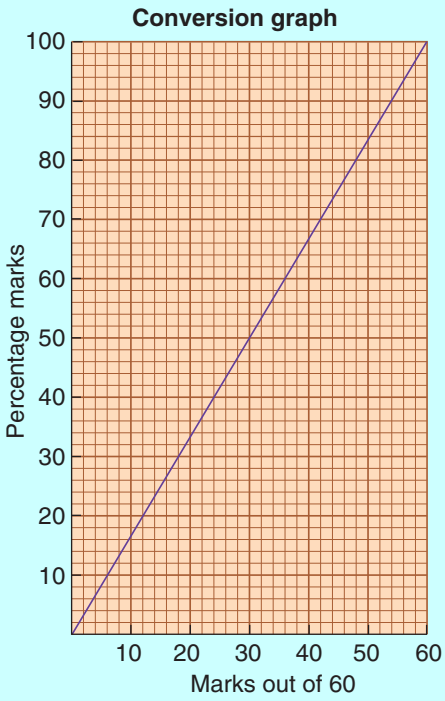
2 Use ID Card 2 on page xvi to give the mathematical term for:

a 2 b 5 c 10 d 11
e 14 f 15 g 16 h 18
i 19 j 23

3 Twelve schools participate in a knockout competition. How many matches must be played before the winner is decided?

4 How many different counting numbers less than one thousand could be formed if only the digits 5, 6 and 7 could be used? (These could be used more than once in a number.)

5 This graph converts marks out of 60 into percentages and vice versa.



a Use the graph to convert the following marks out of 60 into percentages. (Give your answer to the nearest whole per cent.)

i 30 ii 40
iii 12 iv 16

b Use the graph to convert the following percentage marks to marks out of 60. (Give your answer to the nearest integer.)

i 80% ii 90%
iii 56% iv 38%

c Two students score 30 and 40 marks out of 60. What is the difference in their percentage marks?

d Tony scores 80% and Mark scores 60% in the same test. If the test was marked out of 60, by how many marks did Tony outscore Mark?

e Is there a point on the graph corresponding to a horizontal reading of 66? Why or why not?

6 a How much fuel was in the tank at 9:00 a.m.?

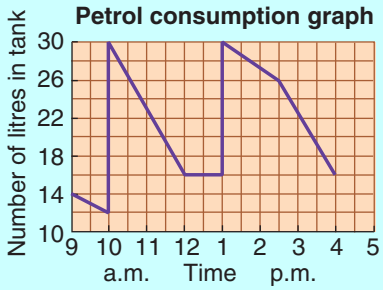
b How much fuel was used from 9:00 a.m. to 10:00 a.m.?

c When did the driver fill the car?

d How much fuel was used from 12 noon to 1:00 p.m.? What does this tell us about the car?

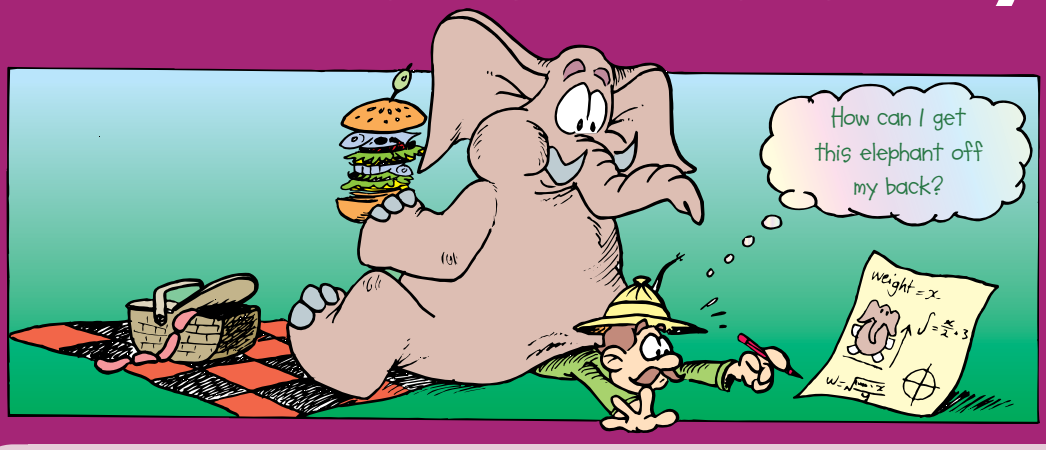
e How much fuel was used altogether?

f How much more fuel was in the tank at 4 p.m. than at 9 a.m.?



2

Working Mathematically



Chapter Contents

Learning Outcomes

Students will:

- Operate with fractions, decimals, ratios and rates.
- Use formulae in calculating area and perimeter of circles and figures composed of triangles and rectangles.
- Calculate the surface area of rectangular and triangular prisms and the volume of right prisms and cylinders.
- Select and use appropriate problem-solving strategies including selecting and organising key information.
- Use appropriate mathematical language and algebraic, statistical and other notations.
- Communicate using oral or graphic form as appropriate.
- Use mathematical arguments to reach and justify conclusions.
- Link ideas and make connections and generalise using existing knowledge and understanding.

Areas of Interaction

Approaches to Learning, Environment

2:01 | Solving Routine Problems

In mathematics the learning of new skills and concepts is usually followed by the use of that newly acquired knowledge in the solving of problems.

These problems are generally routine in nature as the mathematical knowledge and skills needed are fairly obvious. The problem may still be hard to do but at least what the problem is about is clear.

Hence, problems on percentages or measurement or geometry, for instance, are routine in that we know what mathematical knowledge we are trying to use.

No matter what type of problem we are trying to solve, the following steps are important.

Steps for solving problems
Step 1 Read the question carefully.
Step 2 Decide what you are asked to find.
Step 3 Look for information that might be helpful.
Step 4 Decide on the method you will use.
Step 5 Set out your solution clearly.
Step 6 Make sure that your answer makes sense.

The problems in the following exercises are based on stage 4 content.

2:01A | Rates

worked examples

Example 1

A 20 kg box of tomatoes is sold for $13.

a What is the cost per kilogram?

b How much should 12 kg of these tomatoes cost?

• 20 kg for $13. How much per kilogram?

Solution 1

a 20 kg cost $13

1 kg cost $\$\frac{13}{20}$

$\therefore$ Cost is $0.65/kg

b 1 kg cost $0.65

12 kg cost $12 \times \$0.65$

$\therefore$ 12 kg cost $7.80

Example 2

Wendy earns $16 per hour.

a How much would she earn in $7\frac{1}{2}$ h?

b How many hours would she have to work to earn $200?

Solution 2

a Amount earned $= 7\frac{1}{2} \times \$16$

$= \$120$

b Number of hours $= \$200 \div \$16$

$= 12\frac{1}{2}$

Exercise 2:01A

1 **a** Apples cost $4.69/kg.
- **i** How much will 20 kg cost?
- **ii** How many kilograms can be bought for $30? Answer correct to 1 decimal place.

b I earn $9.60 per hour.
- **i** How much will I earn for 35 hours work?
- **ii** How many hours would I need to work to earn $2400?

c Sharon bought 1000 manila folders for €40.
- **i** Express this as a rate in Euros/folder.
- **ii** How much would she pay for 700 folders at this rate?

d The council rates on my block of land are calculated at the rate of 0.83 cents for every $1 on the value of my land. If my land is valued at $145 000, how much must I pay to the nearest dollar?

e If the exchange rate for one Euro is 0·68 British pounds:
- **i** How many British pounds would I get for 500 Euros?
- **ii** How many Euros would I get for 500 British pounds?

Answer to the nearest integer.

- €1 = £0.68

f A block of iron has a volume of 13 500 cm^3. If the density of iron is 7·5 g/cm^3 what is the mass of the block in kilograms?

g A car is travelling at an average speed of 80 km/h. At this rate find:
- **i** how far it will travel in $6\frac{3}{4}$ h
- **ii** how long it would take to travel 980 km

2 On a trip my car usually averages 8·4 L per 100 km. When carrying a heavy load it averages only 11·2 L per 100 km.

a What is the difference in petrol consumption between these two conditions?

b What is the difference between the distance travelled per litre of petrol used for the two conditions?

3 While on holiday in Hawaii, Ming How from Singapore bought a surfboard for US$580. If the exchange rate for the Singapore dollar is 0.63 cents (US), what was the cost of the surfboard in Singaporean currency? (Answer correct to the nearest 5 cents.)

4 Sandy's pulse rate was calculated to be 84 beats/min, calculated over 5 minutes. During this time, Sandy took 60 breaths. Calculate Sandy's rate of heartbeats/breath during this time.

5 Toni's average reaction time when driving was measured to be 0·8 seconds. How far would her car travel in this time if its speed is 60 km/h? (Answer correct to the nearest metre.)

2:01B | Ratio

worked examples

1 The ratio of John's weight to Andrew's is 5 : 4. If Andrew's weight is 56 kg, how much does John weigh?

2 The ratio of profit to cost price on sales at a store was 2 : 3. Find the cost price for an item if the store's profit is $600.

Solutions

1 John's weight : Andrew's weight is
5 lots : 4 lots.
As Andrew's weight is 56 kg,
4 lots = 56 kg.
∴ 1 lot = 56 ÷ 4 kg
= 14 kg
Here John's weight is *5 lots*.
∴ 5 lots = 14 kg × 5
= 70 kg
John's weight is 70 kg.

2 Profit : Cost price is
2 lots : 3 lots.
As the profit is $600,
2 lots = $600.
∴ 1 lot = $600 ÷ 2
= $300
Here the cost price is *3 lots*.
∴ 3 lots = $300 × 3
= $900
The cost price is $900.

Exercise 2:01B

1 **a** The ratio of Jamie's weight to Alan's is 1 : 7. If Jamie's weight is 9 kg, what is Alan's weight?

b The ratio of time working to time playing is 5 : 1. If seven hours were spent playing, how much time was spent working?

c The ratio of balls caught to balls dropped was 9 : 2. If the number of balls dropped was 16, how many balls were caught?

d The ratio of the number of bananas to the number of apples used in a fruit punch is 2 : 5. Find the number of apples used if 6 bananas were used.

2 **a** In church on Sunday, the ratio of women to men was 5 : 4. How many men were present if 45 women were there?

b At the Entertainment Centre the ratio of basketball players to spectators was 3 : 500. How many spectators were there if there were 30 players?

c The ratio of Rhonda's savings to Alan's was 7 : 4. If Rhonda had saved $8708, how much had Alan saved?

d The ratio of zebra finches to Gouldian finches in our aviaries is 13 : 2. If we have 234 zebra finches, how many Gouldian finches do we have?

3 **a** Two streets on a map are 2·8 cm apart. If the scale of the map is 1:20 000, what is the distance between the streets?

b Our town hall is 1·8 km from the church. How far apart are they on the map referred to in part **a**?

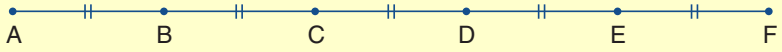
4 **a** On a holiday the ratio of the cost of accommodation to the cost of food was 3 : 7. Find the cost of accommodation if the food cost $420.

b For every $3 in tax that I pay, I get to keep $7. How much money do I keep if I pay $8400 tax?

c From Sharon's home, the ratio of the distance to the swimming pool to the distance to the tennis court is 3 : 8. How far is it to the pool if the court is 4 km away?

d Peter is trying to fill a bottle with water, but for every 5 mL of water he puts in the bottle, he wastes another 4 mL. How much water will he waste in filling a 600 mL bottle?

5



These points are equally spaced along the interval AF.

a Find the ratio of AC to CF.

b Find the ratio of AD to CF.

c If CF = 108 cm, find the length of AC.

2:01C | Dividing a Quantity in a Given Ratio

worked examples

1 $9000 is divided between Tom and James in the ratio 4 : 5. How much does each receive?

2 The sizes of the angles of a triangle are in the ratio 2 : 3 : 5. Find the size of each angle.

Solutions

1 There are 9 parts; 4 for Tom and 5 for James.

$\therefore$ Tom receives $\frac{4}{9}$ of $9000

$= \$ \frac{4}{\cancel{9}_{1}} \times \frac{\cancel{9000}^{1000}}{1}$

$= \$4000$

James receives $\frac{5}{9}$ of $9000

$= \$ \frac{5}{\cancel{9}_{1}} \times \frac{\cancel{9000}^{1000}}{1}$

$= \$5000$

(*Note:* The sum of the parts is equal to the whole $9000.)

2 There are 10 parts; 2, 3 and 5.

The angle sum of a triangle is 180°.

$\text{First angle} = \frac{2}{10}$ of 180°

$= \frac{2}{\cancel{10}_{1}} \times \frac{\cancel{180}^{18}}{1}$

$= 36°$

$\text{Second angle} = \frac{3}{\cancel{10}_{1}} \times \frac{\cancel{180}^{18}}{1}$

$= 54°$

$\text{Third angle} = \frac{5}{10}$ of 180°

$= \frac{1}{2}$ of 180°

$= 90°$

(*Note:* 36° + 54° + 90° = 180°)

Exercise 2:01C

1 **a** $10 000 is divided between two charities, A and B, in the ratio 3 : 7. How much does each charity receive?

b During the basketball season Geoff and Dennis scored 105 points. If the ratio of their points was 2 : 3, find how many points each took.

c A cordial is made by mixing syrup and water in the ratio 3 : 8. If 5·5 L of the mixture was made, how much syrup was used?

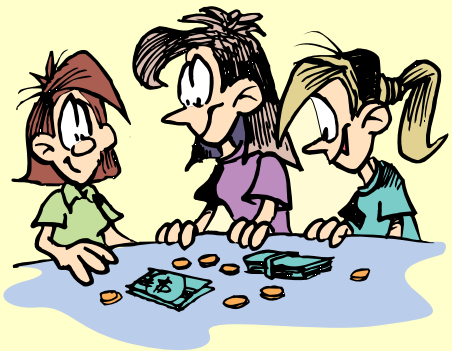
d Divide $770 between Mike and Joy in the ratio 3 : 4.

e A chocolate bar has 40 pieces. If Mary and Sue shared the pieces in the ratio 3 : 5, how many pieces did each get?

f John is paid €5 an hour while his younger brother is paid €3 an hour. They worked together for several hours and earned a total of €56. How much did each earn? How long did they work?

g Defence spending for the Army and Navy is allocated in the ratio of 9 : 4. If an additional $179 400 000 must be allocated, how much would each receive?

2 **a** $200 is to be divided between Rachel, Alana and Naomi in the ratio 3 : 5 : 2. How much does each receive?



b A band of three robbers divided their spoils so that the leader took 3 shares while the others took one share each. When caught they were given punishment in the same ratio as they shared the money. In total the punishment was 15 years imprisonment. How many years was the leader given?

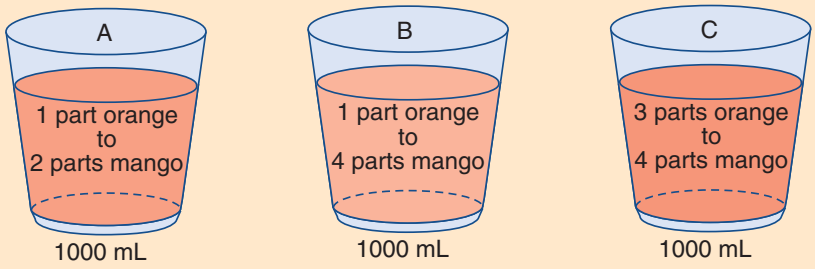
c An author wrote three novels which had their number of pages in the ratio 9 : 10 : 11. If the total number of pages is 900, how long is each novel?

d For every $100 I earn, the Taxation Department gets $38, living expenses account for $60 and I save $2. How much do I save in a year if my salary is $27 000 per year?

e The lengths of the three sides of a triangle are in the ratio 4 : 5 : 3. Find the length of each side if the perimeter of the triangle is 1·08 metres.

- How many litres of petrol need to be mixed with 80 mL of this oil?

Challenge 2:01 | Mixing drinks



- In each container above there is a mixture of orange and mango fruit juice.
- If the contents of containers A and B are mixed, would the ratio of orange to mango be the same as in container C? Explain why or why not.

Questions

1 Find to the nearest millilitre, the amount of orange juice in:
a A **b** B **c** C

2 Find to the nearest millilitre, the amount of mango juice in:
a A **b** B **c** C

3 What is the total amount of orange juice (to the nearest mL) in:
a A and B? **b** B and C?

4 What is the total amount of mango juice (to the nearest mL) in:
a A and B? **b** B and C?

5 What is the percentage of orange juice (to the nearest per cent) in the mixture of:
a A and B? **b** B and C?

6 Use your answers to question 5 to find the ratio of orange to mango juice in the mixture of:
a A and B? **b** B and C?

- Ratios are used to record the 'secret recipes' of fruit juice mixes. By varying the numbers in the ratio different tastes are created.
- Discuss the flavour of the fruit juices made from carrot, apple and celery juice in the following ratios.

Juice	Carrot	:	Apple	:	Celery
A	3	:	2	:	1
B	2	:	3	:	1
C	1	:	4	:	3
D	8	:	5	:	2

Questions

1 Which juice do you think would have the strongest carrot taste?

2 In which juice would you have the strongest apple taste?

2:01D | Percentages

worked examples

Example 1

Larry earns a commission of $3\frac{1}{2}\%$.

How much will he earn on a sale of \$70 000.

Solution 1

$$\text{Commission} = 3\tfrac{1}{2}\% \text{ of } \$70\,000$$
$$= \frac{3{\cdot}5}{100} \times \$70\,000$$
$$= \$2450$$

Example 2

On New Year's Eve 8000 drivers were given random breath tests. Of these, 160 were found to be over the limit of 0·05. What percentage of the drivers tested were over the limit?

- What percentage is 160 of 8000?

Solution 2

160 drivers as a % of 8000

$$= \frac{160}{8000} \times 100\%$$
$$= 2\%$$

Example 3

My salary was increased in February by 12%. This meant I would get an extra \$110.40 per week. What is my new weekly salary?

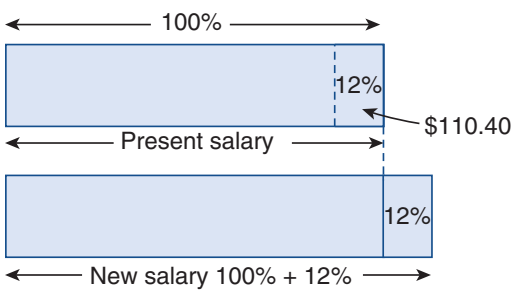
Solution 3

12% of present salary = \$110.40

∴ 1% of present salary = \$110.40 ÷ 12
= \$9.20

∴ 100% of present salary = \$9.20 × 100
= \$920

∴ New salary (112%) = \$920 + \$110.40
= \$1030.40



Exercise 2:01D

1 **a** A theatre holds 1200 people when full. If it is 80% full, how many people are present?

b Larry has a trade discount of $15\frac{1}{2}\%$ on the selling price of all the articles at a hardware store. How much will he pay for a saw priced $275?

c A farmer lost 65% of his cattle in a prolonged drought. If he originally had 2300 cattle, how many does he have now?

d My car depreciated in value by 12% in 1 year. If its value was $15 800 what is its value now?

e Carol's present wage is $47 000 p.a. What will her wage be if it is increased by 4·5%?

2 **a** Wendy earns $220 interest on an investment of $4000. What is the interest as a percentage of her investment?

b Cliff scores 486 baskets in basketball from 573 attempts. What is his success rate as a percentage?

c Alan bought some land for $165 000 and later sold it for $210 000. Find:

i his profit

ii his profit as a percentage of the selling price

d In a town with a population of 30 689 there are 3829 people in the 10–19 age group. Express the number in this age group as a percentage of the total population to the nearest whole percent.

e Packets of food are checked for weight. A machine records the following:

Overweight:	643
Underweight:	490
Correct weight:	24 608

What percentage of the packets weighed were underweight? Answer to the nearest whole percent.

• 486 baskets from 573 attempts. What is the shooting %?

3 **a** A discount of 15% is given on a TV set. This reduces the price by $194.85. What was the original price?

b A town's population increased by 422 people in a year. If this represented an increase in the population of $2\frac{1}{2}\%$, what was the town's population at the start of the year?

c A property agent earns a commission of 3% on sales. If his commission for the year was $43 500, what was the value of his sales?

d Max's shares rose in value by 18%. If this increased their value by $14 040, what is their present value?

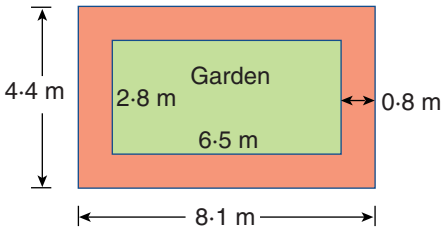
e A married couple spend 78% of their monthly income on major expenses. If this amounts to $5616, what is their monthly income?

worked examples

Example 1

A path 0·8 m wide is to be put around a rectangular garden which is 6·5 m long and 2·8 m wide. What is the area of the path?

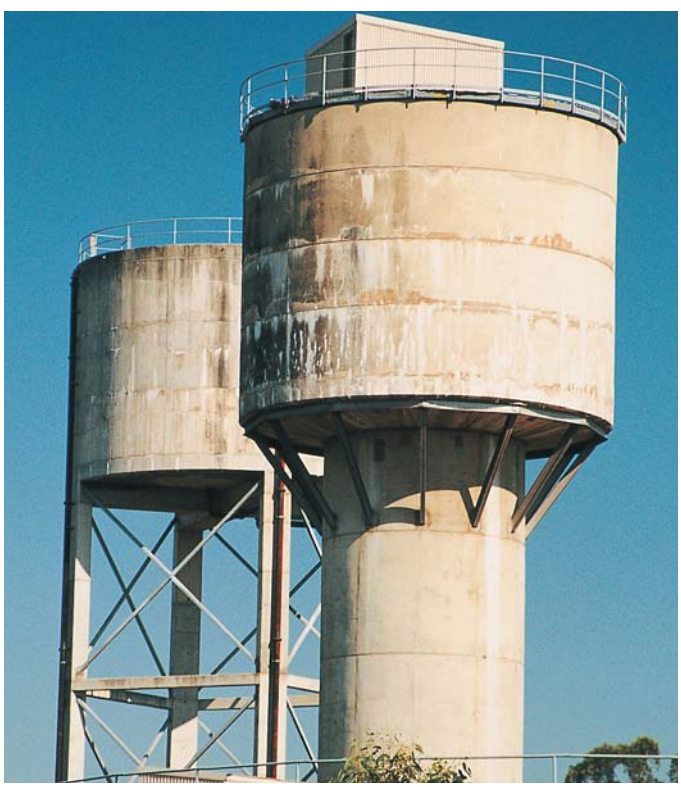
Solution 1



Area = $(8{\cdot}1 \times 4{\cdot}4) - (6{\cdot}5 \times 2{\cdot}8)$
$= 17{\cdot}44\ \text{m}^2$

Example 2

A cylindrical tank has a diameter of 8 metres. How high must the tank be if it has to have a volume of 400 m^3?



Solution 2

$V = \pi r^2 h$

$400 = (\pi \times 4^2) \times h$

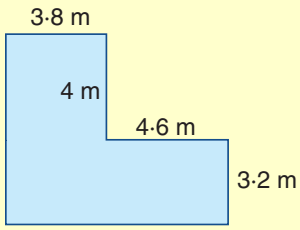
$$h = \frac{400}{(\pi \times 4^2)}$$

$= 7{\cdot}96$ m (correct to 2 decimal places)

Exercise 2:01E

1 **a** Adam's trailer is 2·4 m long, 1·2 m wide and 0·4 m high. What is the maximum volume of soil it can carry?

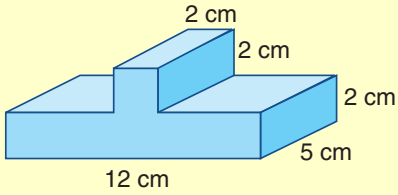
b A lawn is shaped as shown in the diagram. The lawn is to be covered to a depth of 4 cm with soil. What volume of soil is required?



c A water tank is cylindrical in shape. It is 9·8 m high and 12·4 m in diameter. Calculate its capacity if $1\ m^3 = 1$ kL. (Give your answer to the nearest 100 L.)

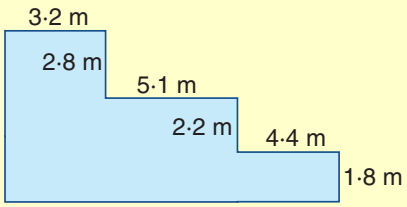
d A cylindrical cup has a diameter of 7 cm. How high must it be if it is to hold 250 mL? ($1\ cm^3 = 1$ mL)

e Calculate the volume of the steel solid shown and calculate its mass if $1\ cm^3$ of steel weighs 7·8 g.

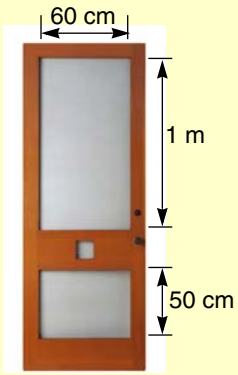


2 **a** The triangular gable shown in the photograph is to be painted. It has a base of 12.6 m and a perpendicular height of 1·8 m. Calculate the area to be painted and the cost at $\$30/m^2$.

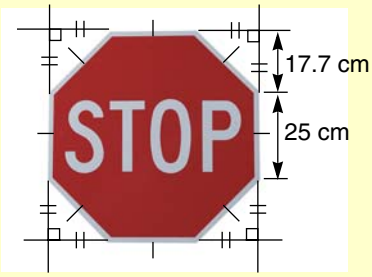
b The area shown below is to be tiled. Calculate the area and the cost of the tiles at $\$45/m^2$.



c Calculate the cost of replacing the glass in the door shown if glass costs $\$38/m^2$.

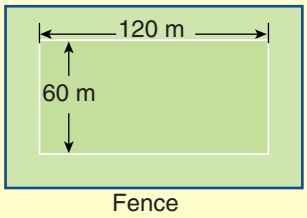


d Calculate the area of metal in the 'STOP' sign.



e A room has a rectangular prism shape. It is 7·6 m long, 6·4 m wide and 2·8 m high. Calculate the area of the four walls.

3 **a** A football field is 120 m long and 60 m wide. A fence is to be placed around the field 3 m back from the field. Calculate the length of fencing needed.

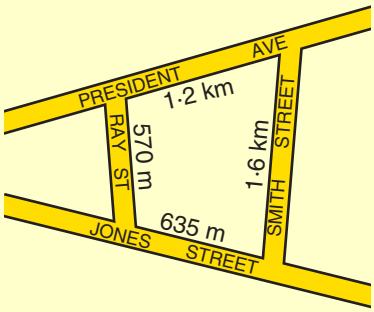


b A rectangular area is fenced off using 200 m of rope. If the rectangle is to be 65 m long, how wide will it be?

c The diagram shows the course followed by an athlete.

i If she runs the block bounded by the streets given, how far does she run?

ii How many complete laps would she need to run around this course if she wished to travel at least 20 km?



d A trundle wheel is to have a circumference of 1 metre. What must its diameter be, to the nearest millimetre?

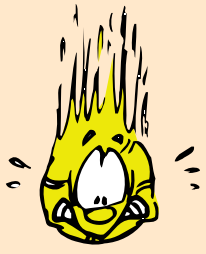
e The diameter of a car wheel is 40 cm. How many complete revolutions will this wheel need to make to travel 1 km?

fun spot
2:01

Fun Spot 2:01 | Bouncing ball

A ball is known to rebound to half the height that it drops. If the ball is dropped from a height of 80 m, how far will it have travelled by the time it hits the ground:

a for the 1st time? **b** for the 2nd time?
c for the 3rd time? **d** for the 4th time?
e for the 5th time? **f** for the 6th time?

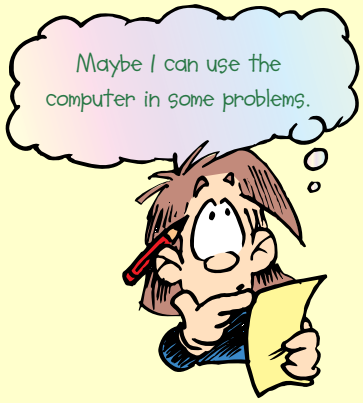


2:02 | Solving Non-routine Problems

Often in mathematics (as well as in real life) we get a problem which is unlike any we have seen before. We need to reflect on what we already know and see how our existing knowledge can be used. Sometimes the problem will need us to develop new skills, or we may need to look at the problem in a different way.

Applying strategies is one of the processes involved in Working Mathematically.

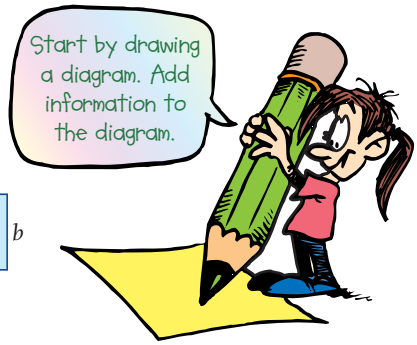
Some useful strategies for problem solving are:
- Eliminating possibilities
- Working backwards
- Acting it out
- Looking for patterns
- Solving a simpler problem
- Trial and error
- Making a drawing, diagram or model
- Using algebra
- Using technology



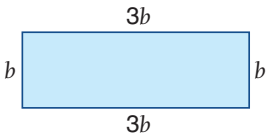
worked examples

Example 1

A rectangle has a perimeter of 44 m. If its length is three times as long as its breadth, find its dimensions.



Solution 1



Let the breadth be b.
The length then must be $3b$.
Now the perimeter is 44 m.

$\therefore 3b + b + 3b + b = 44$

$8b = 44$

$\therefore b = 5{\cdot}5$

$\therefore$ The breadth is 5·5 m and the length is 16·5 m.

(*Note*: This problem could also be solved using 'trial and error'.)

continued ➜➜➜

Example 2

Screwdrivers come in four different sizes. The cost of these four sizes are \$8.90, \$7.80, \$5.40 and \$4.80. I bought seven screwdrivers. Which of the following amounts was the total cost?

a \$48.40 **b** \$34 **c** \$50.65 **d** \$61.40

Solution 2

Try to eliminate possibilities.

- \$50.65 cannot be the answer as the costs of all screwdrivers are multiples of 10 cents. It's impossible to get the 65 cents.
- Look at the maximum cost.
  7 at \$8.90 = \$62.30
- \$61.40 is less than \$62.30 but close to it.
  6 at \$8.90 and 1 at \$7.80 = \$61.20
  Since all other combinations would cost less than \$61.20, \$61.40 cannot be the answer.
- Look at the minimum cost.
  7 at \$4.80 = \$33.60
- \$34 is not much more than \$33.60.
  6 at \$4.80 and 1 at \$5.40 = \$34.20
  Since all other combinations would cost more than \$34.20, \$34 is not the answer.
- The only possibility remaining is \$48.40, so it is the answer.

Note: We could continue to try different combinations of prices but we were told that one of the possibilities was correct and we have eliminated the other three.

Note: $(2 \times \$8.90) + (2 \times \$7.80) + (1 \times \$5.40) + (2 \times \$4.80) = \$48.40$.

Exercise 2:02

I Use problem solving strategies to solve these problems.

a Vickie is expecting a baby. She must pick two *given names* (in order) for her child from the names she is considering. Girls' names being considered are Rachel, Jessye, Faith and Kate. Boys' names are Jason, Brent and Grant. She is also considering the name Sandy for both a girl or a boy. How many ways of naming the child are being considered?

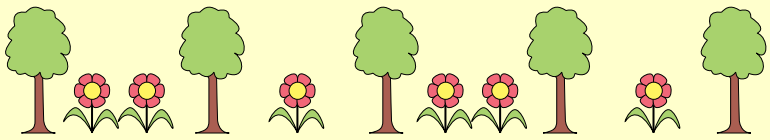
b If tyres sell for \$96.70, \$113.50, \$125.90 and \$143.30, which of the following amounts could be the cost of 5 tyres?
A \$483.30 **B** \$592.90 **C** \$610 **D** \$717.50

c Luke has 13 finches now but yesterday he lost $\frac{3}{4}$ of his finches when part of the roof blew off his aviary. The day before that he had given 6 zebra finches to his cousin and two days before that he had purchased two pairs of Gouldian finches. How many finches did Luke have before he bought the Gouldians?

d One 'move' involves turning three coins over. What is the least number of 'moves' needed to change these five 'tails' to five 'heads'?

e The difference of two numbers plus their sum is equal to 1. Write down one of these two numbers.

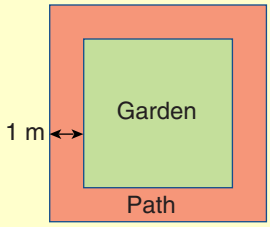
f 865 trees were planted in a row. Between the first and second trees 2 flowers were planted, between the second and third trees 1 flower, between the third and fourth 2 flowers, and so on to the end of the row. How many flowers were planted?



g We supply textbooks that have a mass six times as great as our summary books. Five textbooks and ten summary books have a mass of 8 kg altogether. What is the mass of one textbook?

h What is the greatest number of points in which four circles can intersect?

2 **a** A square garden is surrounded by a path of width 1 m. What is the area of the garden if the outside perimeter of the path is 34 m? Report on the process you used to obtain a solution.



b A rectangle has a perimeter of 60 cm. If its length is four times as long as its breadth, find its dimensions. How could you check your answer?

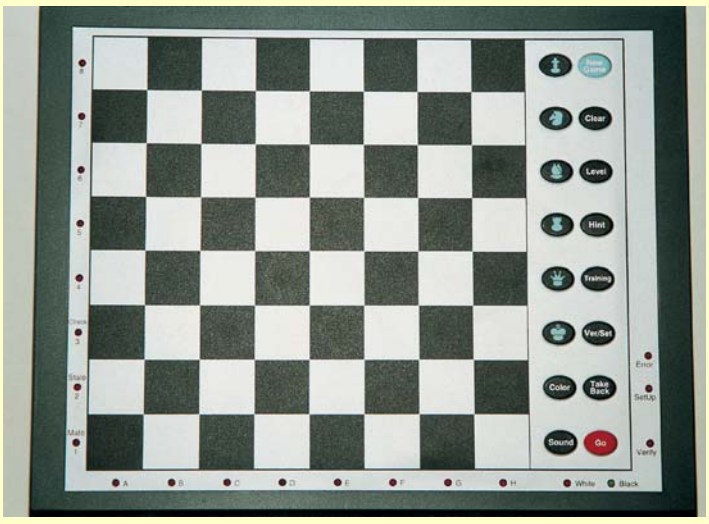
c In my moneybox I have $2.95. The 37 coins are all either 5-cent or 10-cent coins. How many 5-cent coins are in my moneybox? Report on the process you used to obtain a solution. How could you check your answer?

■ Compare and contrast your solution with those of other students.

3 For the chessboard on the right, how many squares are there of side length:

a 8 units? **b** 7 units?
c 6 units? **d** 5 units?
e 4 units? **f** 3 units?
g 2 units? **h** 1 unit?

Write the total number of squares as the sum of eight square numbers.



4 **a** How many pieces of paper would you have if you cut a strip of paper:

i 1 time? **ii** 2 times?
iii 3 times? **iv** 4 times?
v 15 times? **vi** 186 times?

How is the number of pieces related to the number of cuts?

b If the strip of paper above is folded at the centre and the result folded at the centre again, 3 creases are made. On the second fold, 2 *additional creases* are made. If this folding process were to continue how many *additional creases* would be made on the:

i 3rd fold? **ii** 4th fold? **iii** 5th fold?
iv 6th fold? **v** 7th fold? **vi** 15th fold?

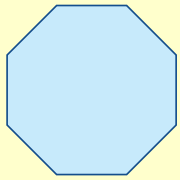
How is the number of additional folds related to the fold number?

5 **a** Which two numbers have a sum of 79 and a product of 1288?
b Which two numbers have a difference of 11 and a product of 5226?
c Which two numbers have a sum of 1200 and a difference of 68?

6 In our school's House basketball competition each of the six teams must play the other five on two occasions. How many games must be played?

7 Plastic figures are needed for the doors on the 5th floor of a large hotel. If all of the numbers from 500 to 550 are needed, how many of each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 will be needed?

8 How many diagonals does an octagon have?



9 A pentomino is formed by joining 5 squares together so that each square is joined to another square along an edge. How many different pentominoes are there? (Note: They are the same if one can be turned into the other by turning it upside down.)

10 I own 3 ties, 5 shirts and 2 belts. How many different ways could I wear a tie, shirt and belt?

- How many times in a day are the hands of the clock at right angles?

Fun Spot 2:02 | What nationality is Santa Claus?

Work out the answer to each part and put the letter for that part in the box that is above the correct answer.

H Find:
$3\frac{1}{2}\%$ of 1450

Simplify:

H $-8-(-16)$

I $(-4)^2$

L $64 \div (4-8)$

N $-1-1-1$

O $1{\cdot}25 \times 100$

O $30 \times 0{\cdot}2$

P 7% of 300

R 125% of 10

S $\frac{3}{4}$ of (-60)

T $\frac{4}{5} \times 1\frac{1}{4}$

−3	125	12·5	1	50·75	21	6	−16	16	−45	8

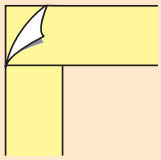
Investigation 2:02 | Line marking

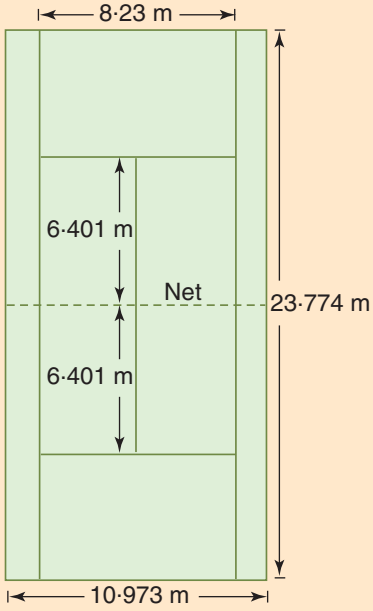
Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Special adhesive tape is used to mark the lines on the tennis court.

Use the dimensions given to calculate the length of tape required.

Assume the tape overlaps at each point of intersection.





Assessment Grid for Investigation 2:02 | Line marking

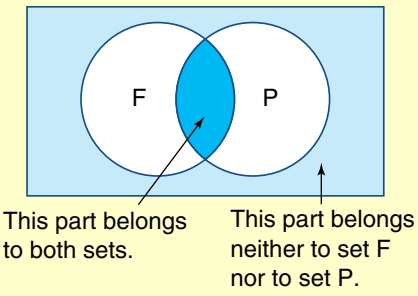
The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	The approach is very haphazard and disorganised.	1	
			2	
	b	An organised approach has been attempted with some success.	3	
			4	
	c	An organised approach has been used and any short cuts are explained.	5	
			6	
	d	A successful organised approach has been used, short cuts have been fully investigated and their effectiveness explained.	7	
			8	
Criterion C Communication	a	Little or no working out is shown.	1	
			2	
	b	Working out is shown and there is some explanation of the steps undertaken.	3	
			4	
	c	The steps undertaken are well communicated and the work is well structured.	5	
			6	
Criterion D Reflection in Mathematics	a	The student has attempted to explain the methods used and to evaluate the reliability of the answers.	1	
			2	
	b	Any short cuts taken to evaluate answers are explained and the answers have been checked for reasonableness.	3	
			4	
	c	Clear and concise justifications are given for methods used and accuracy of results; and possible alternatives or improvements have been investigated.	5	
			6	

2:03 | Using Venn Diagrams (Extension)

Drawing Venn diagrams is a useful problem solving strategy.

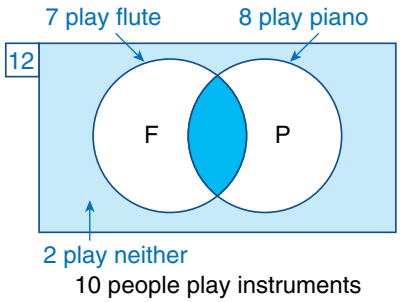
- A rectangle is used to stand for the larger group out of which smaller sets are considered.
- Circles are used to represent those smaller sets.
- If the smaller sets have members in common, the circles will overlap.



worked examples

Example 1

Of 12 students, 7 could play the flute and 8 could play the piano. If 2 students could play neither instrument how many students could play both instruments?



Solution 1

A Venn diagram has been drawn on the right.

- As 2 students play neither instrument, 10 students must be placed inside the circles.
- The coloured section is in both circles so, if we add the numbers playing each instrument, the students playing both are *counted twice*.

(number in F + number in P) − number in overlap = number inside the circles.

$(7 + 8) - n = 10$ (because 10 people play instruments)

$\therefore n = 5$

Therefore 5 students play both instruments.

Example 2

Of 30 students, 20 could play the piano, 5 the flute and 6 the violin. 3 violinists could also play the piano but only one of these 3 could also play the flute. 5 students could play both the piano and the flute. None of the students could play any other type of musical instrument. How many students played:

a the piano and the flute but not the violin?
b only the piano?
c only the flute?
d only the violin?
e no instrument at all?

Solution 2

Figure 1

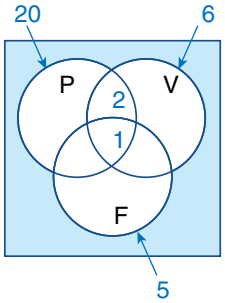


Figure 2

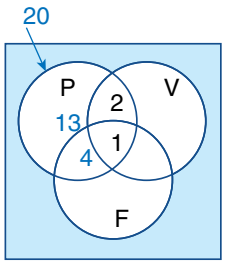
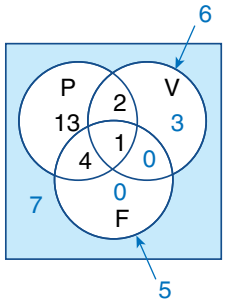


Figure 3



We need to put the information in the question onto the Venn diagram. As we enter this, we can deduce other information.

Figure 1: Of the 3 violinists who also play piano, one can play the flute. So only one person can play all three instruments.

Figure 2: 5 students play both piano and flute, so 4 would play only the piano and flute. Since there are 20 piano players 13 must play the piano alone.

Figure 3: As there are only 5 flute players the other two sections of that circle will be zero and the remaining section of the violin circle will have 3 members (to make the total of violin players 6). Within the circles there are 23 students, so 7 must be outside. These 7 play no instrument.

The answers can now be read from figure 3.

a 4 **b** 13 **c** 0 **d** 3 **e** 7

challenge

2:03

Challenge 2:03 | Venn diagrams

1 Of 30 people in our class, 9 can catch left-handed, 24 can catch right-handed and 6 can catch with both their left hands and their right hands.

a How many can catch only with the left hand?
b How many can catch only with the right hand?
c How many cannot catch with one hand?

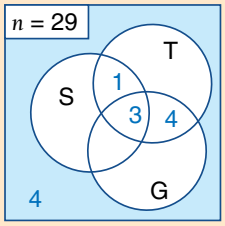
2 The *Herald* was delivered to 75 homes and the *Telegraph-Mirror* was delivered to 68 homes. If these papers were delivered to 110 homes:

a how many homes received both papers?
b how many homes received only the *Herald*?
c how many homes received only the *Telegraph-Mirror*?

3 Of the 30 students in 9M, 13 love surfing, 15 love hiking and 7 love neither.

a How many students love both surfing and hiking?
b How many love surfing but not hiking?
c How many love hiking but not surfing?

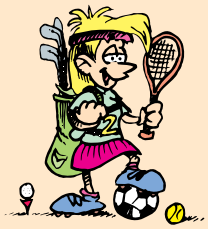
4 The Venn diagram on the right shows three intersecting sets: those who play soccer (S), those who play tennis (T) and those who play golf (G). A number in one part of the diagram shows the number of people in that part.



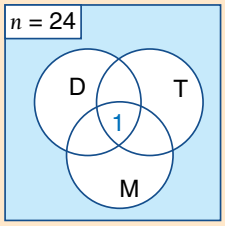
- Our class has 29 students and all but 4 students play at least one of these sports.
- 3 students play all three sports, one plays both soccer and tennis but not golf, 4 play tennis and golf but not soccer, a total of 5 play soccer and golf, a total of 16 play tennis and altogether 11 play soccer.

How many play:

a soccer and golf but not tennis?
b only tennis? **c** only soccer? **d** only golf?
e a sport other than golf? (Consider only these sports.)

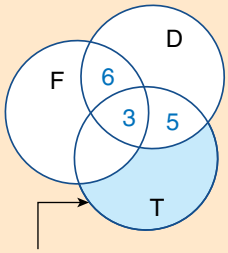


5 When 24 adults were asked which of Dubbo, Maitland and Terrigal they had visited, the following information was obtained.



- One person had visited all three places, 3 had visited Maitland and Dubbo but not Terrigal, 2 had visited Dubbo and Terrigal but not Maitland, 4 had visited Maitland, 16 had visited Dubbo and 3 of the adults had visited none of these places.
  a How many people had visited only Dubbo?
  b How many people had visited only Maitland?
  c How many people had visited only Terrigal?
  d How many people had visited Terrigal?
  e Are those who have visited Maitland a part of those who have visited Dubbo?

6 This Venn diagram shows three intersecting sets: those friends of Alan that can drive a car (D), those who play tennis (T) and those who are females (F).



Play tennis but are not female and do not drive (male non-driving tennis players)

- Alan has 40 friends.
- 5 friends fit into none of the three categories, 3 friends fit all three categories, 6 who do not play tennis are females who drive, 5 are male tennis players who drive, 8 are females who play tennis, a total of 20 drive and altogether 17 are female.

By drawing the Venn diagram and placing numbers in specific sections, find how many friends are:

a male drivers who don't play tennis.
b non-driving females who play tennis.
c females who neither drive nor play tennis.
d male tennis players who don't drive.
e tennis players.
f females who drive.
g either female or drivers.
h either drivers or tennis players.
i male.

- Hi! I'm a female driver who doesn't play tennis. Does Alan have any more friends like me?

fun spot

2:03

Fun Spot 2:03 | What kind of breakfast takes an hour to finish?

Work out the answer to each part and put the letter for that part in the box that is above the correct answer.

10 g of salt is dissolved in 1 litre of water. How much salt would there be in:

I 500 mL **L** 2.5 L **M** 1.8 L
R 200 mL **S** 50 mL **A** 250 mL

3 litres of water drips steadily from a tap in 5 hours. How much water would be wasted in:

A one hour? **T** 7 hours? **A** 4 hours?
Y 30 minutes? **D** 10 minutes? **E** 7 minutes?

E 4 litres of mercury was shared between two schools in the ratio 3 : 7. What was the smaller share?
I Change 180 km/h into m/s.

2·5 g	100 mL	600 mL	300 mL	4·2 L	50 m/s	18 g	70 mL	0·5 g	1·2 L	2 g	5 g	2·4 L	25 g

investigation

2:03

Investigation 2:03 | Comparing speeds

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

1. Find the Olympic record for:
   - **a** the men's 100 m, 200 m, 400 m, 800 m and marathon running
   - **b** the women's 100 m, 200 m, 400 m, 800 m and marathon running
2. For either men's or women's records, find the average speed for each distance in m/s and km/h. (Give answers correct to 3 significant figures.)
3. Report on your findings. What conclusions can you draw?

Assessment Grid for Investigation 2:03 | Comparing speeds

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	An organised approach has not been used to find answers or to explain them.	1	
			2	
	b	An organised approach has been attempted and some effort has been made to calculate the average speeds.	3	
			4	
	c	An organised approach has been described and conclusions regarding the speeds have been drawn.	5	
			6	
	d	A systematic approach has been adopted to successfully find the speeds, and correct, relevant conclusions and justifications have been made.	7	
			8	
Criterion C Communication	a	No working out is shown and no explanations are given.	1	
			2	
	b	Working out is shown and there is some interpretation of the results.	3	
			4	
	c	Working out is clear and easy to follow and results have been interpreted successfully.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt has been made to explain how the calculations were performed and to check the results.	1	
			2	
	b	The calculations and methods are briefly described and checked for reliability.	3	
			4	
	c	Methods are explained fully with possible improvements and there is a full interpretation of the results and their significance.	5	
			6	

mathematical terms 2

Mathematical terms 2

rate

- A comparison of unlike quantities, usually expressed by writing how many of the first quantity corresponds to one of the second.
  eg 6000 mL in 10 minutes
  = 600 mL per 1 minute
  = 600 mL/min

ratio

- A comparison of numbers in a definite order.

percentage

- A fraction with a denominator of 100.
  eg $7\% = \frac{7}{100}$

perimeter

- The length of the boundary of a figure.

area

- The amount of space inside a 2-D shape.

volume

- The amount of space inside a 3-D shape.

- A jogger jogs at a rate of 5 min/km.

1 How long will it take to jog 10 km?

2 How far could she jog in an hour?

Chapter 2 | Revision Assignment

assignment 2A

1 **a** A motor mechanic charges $56/h for his labour. How much should he charge for a job that takes 75 minutes?

b Australia scored 265 runs off 50 overs in a one-day cricket match against New Zealand. Calculate the average run-rate per over. What should New Zealand's score be after 15 overs if they score at the same rate?

c Light travels at a speed of 300 000 km/s. How long will it take light to travel from the sun to the earth if the distance between the two is 150 000 000 km. (Answer in minutes and seconds.)

d A nurse has to set the drip rate for an IV drip for a patient. The drip has to empty a 500 mL flask in 8 hours. Work out the correct drip rate in drops/minute if it is known that 1 mL = 15 drops.

2 **a** The ratio of two angles α and θ is 7 : 4. If the size of θ is 92°, what is the size of α?

b An interval is divided in the ratio of 11 : 4. If the larger part is 143 mm long, what is the length of the smaller part? What was the length of the original interval?

c The ratio of the area of a triangle to the area of a rhombus is 6 : 11. Find the area of the rhombus if the area of the triangle is 126 cm^2.

d The ratio of the volume of a service station's underground petrol tank to the volume of the tank on a petrol tanker is 7 : 4. Find the volume of the service station's underground tank if the volume of the tank on a petrol tanker is 15·4 m^3.

3 **a** A type of solder is made by mixing lead and tin in the ratio 2 : 3. How much lead is needed to make 300 g of solder?

b Concrete is made by mixing sand, gravel and cement in the ratio 5 : 8 : 2 by volume. How many cubic metres of cement will be needed to make 4 m^3 of concrete. (Answer correct to 1 decimal place.)

c Heather shared a kilogram pack of grass seed between two plots in proportion to their areas. One plot had an area of 20 m^2, the other 30 m^2. How much seed was used for each plot?

d Two friends invested in a business in the ratio 3:2. The profit is to be distributed in proportion to their investments. What percentage of the profits does each receive?

4 Using the ingredients below, Heather was able to make 40 vanilla slices. Make a list of the ingredients she would need to make 100 vanilla slices.

VANILLA SLICE

500 g of packaged puff pastry	1·25 litres milk
1 cup sugar	60 g butter
$\frac{3}{4}$ cup cornflour	2 egg-yolks
$\frac{1}{2}$ cup custard powder	2 teaspoons vanilla

5 a A discount of 5% was given on a computer priced at $2750. How much was paid for the computer?

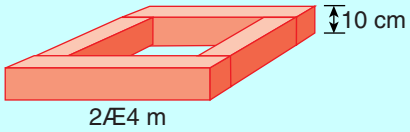
b A saleswoman gave a discount of $120 on a computer with a selling price of $1890. What was the discount as a percentage of the selling price?

c A discount of 6% on the cost of a new computer saved me $168.

i What was the original price of the computer?

ii How much did I pay?

6 Four identical garden beds (as shown in the diagram) are to be made from treated pine sleepers. The sleepers are rectangular prisms 2·4 m long, 20 cm wide and 10 cm high.



a How many sleepers will be needed to build the four beds?

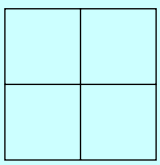
b What volume of soil is needed to fill the beds?

7 My salary has changed twice this year. At the start of the year it was €48 000 p.a. In February my salary increased by 12%, but in November my salary was decreased by 12%. What was my salary at the end of the year?

8 Six teams enter a round-robin competition where each team plays every other team. How many games will there be in the competition?

9 A section of fence is 12 m long. How far apart will the posts be if there are:

a 6 posts? b 9 posts?

10 

Four non-zero numbers are placed in these four squares to make two two-digit numbers across, and two two-digit numbers down. If the sum of these four two-digit numbers is 237, what are they?

Sharing the prize

Technology Applications Activities

Chapter Review Questions

- A cheetah can run at 80 km/h. At this rate, how long would it take a cheetah to run 100 km?

assignment 2B

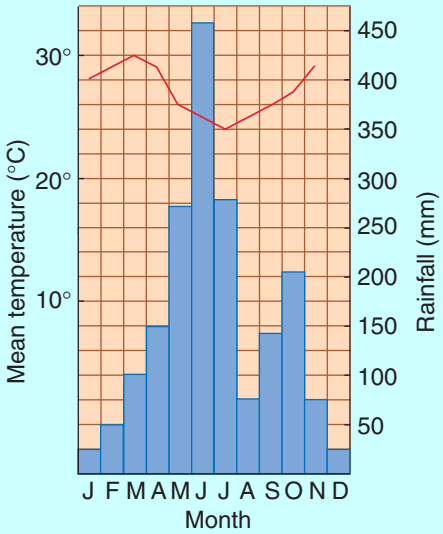
1 Use ID Card 4 on page xvi to give the number or numbers of:
   a the solids or 3-D shapes
   b the prisms
   c the pyramids
   d the 3-D shapes with curved surfaces
   e a triangular prism

2 Use ID Card 1 on page xiv to give the number or numbers of:
   a the units that measure capacity
   b the units that use 'kilo' in their name
   c the units that measure time
   d the unit for a hectare

3 400 m of fencing encloses a square paddock of area one hectare. If 800 m of fencing were used to enclose a square paddock, how many hectares would be enclosed?

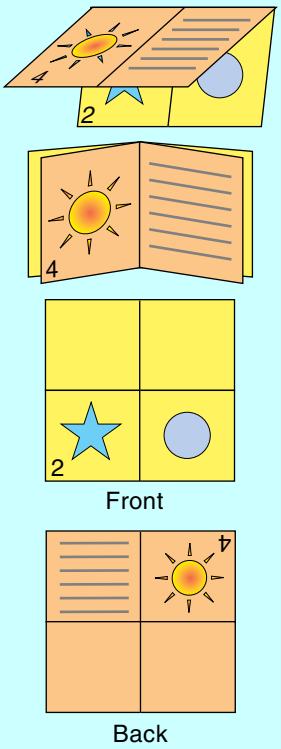
4 Mia is nine years older than Sandra. Alan is twice as old as Mia and Peter is three times as old as Sandra. If the sum of the ages of Alan and Peter is 88, how old is:
   a Sandra? b Mia?
   c Alan? d Peter?

5 This graph is a combination line and column graph often used in geography.



There are two vertical axes. The left-hand temperature axis refers to the line graph. The right-hand rainfall axis refers to the column graph.
   a In which month was the highest rainfall recorded?
   b Which month had the lowest mean temperature? What was this temperature?
   c How many months had a rainfall greater than 200 mm?
   d The scale on the rainfall axis is 1 cm : 75 mm. Use this fact to find the rainfall for September.

6 One large sheet of paper was ruled up and folded. It was then cut along the fold shown in the diagram on the top to form an 8-page booklet. On the diagrams to the right, put the page number on each quarter (as has been done for 2 and 4).



3

Ratio and Proportion

Chapter Contents

Learning Outcomes

Students will:

- Operate with ratios, percentages and rational numbers.
- Calculate percentage and proportional change.
- Apply proportional change to scale and other problems.

Areas of Interaction

Approaches to Learning, Homo Faber, Environment

3:01 | Ratio Review

A **ratio** is a comparison of two like quantities.

For example, a juice concentrate may ask for 1 part concentrate to be mixed with 3 parts water. This means that the ratio of concentrate to water = 1 to 3

$$= 1:3 \text{ or } \frac{1}{3}$$

So if you used 4 litres of concentrate, you would need to use $4 \times 3 = 12$ litres of water.

worked examples

1 Fred's class has 12 boys and 8 girls.
   - **a** What is the ratio of boys to girls in Fred's class in simplified form?
   - **b** What is the ratio of boys to the total number of students in Fred's class?
   - **c** What is the ratio of girls to the total number of students in Fred's class?

2 In Fred's class, what fraction of the class are:
   - **a** boys?
   - **b** girls?

Solutions

1 **a** 12:8 = 3:2 (You divide each part of the ratio by the common factor. In this case 4.)

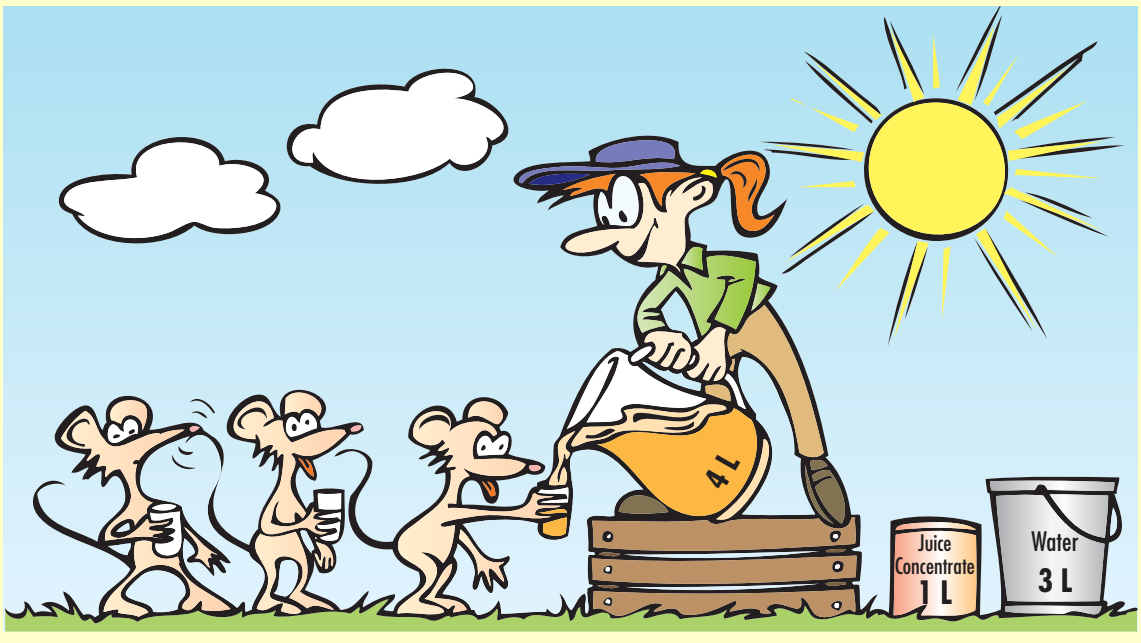
b 12:20 = 3:5 (You divide each part of the ratio by the common factor. In this case 4.)

c 8:20 = 2:5 (You divide each part of the ratio by the common factor. In this case 4.)

2 **a** $\frac{12}{20} = \frac{3}{5}$

b $\frac{8}{20} = \frac{2}{5}$

How could these simplified answers be obtained from the answers in example 1?



worked examples

1 Aurelien's class has 15 boys and 9 girls.
   a What is the ratio of boys to girls in Aurelien's class in simplified form?
   b Uta's class has boys to girls in the same ratio as Aurelien's class, but there are 6 girls in Uta's class. How many boys are in Uta's class?
   c Some girls moved into Aurelien's class and the ratio of boys to girls in his class is now 5:4. How many more girls are there in Aurelien's class?

2 To get the right shade of green for his painting, Theo mixes yellow and blue paint together so that $\frac{3}{7}$ of the total is yellow.
   a What is the ratio of yellow paint to blue paint in the final mixture?
   b If he used 1200 mL of blue paint, how much yellow paint did he use?

3 In a farmyard, the ratio of chickens to ducks to geese is 5:4:3.
   a If there are 20 ducks, how many chickens are there?
   b How many chickens, ducks and geese are there altogether?
   c A neighbour has these animals in the same ratio, but has 108 of them altogether. How many of each animal does the neighbour have?

Solutions

1 a $15:9 = 5:3$ (You divide each part of the raio by the common factor. In this case 3.)

b $\frac{boys}{girls} = \frac{5}{3}$

$\therefore \frac{x}{6} = \frac{5}{3}$ (Always put the unknown quantity in the numerator.)

$\therefore x = \frac{5 \times 6}{3}$

$\therefore x = 10$

To check: boys:girls = 10:6

= 3:2 (by dividing by 2)

c There is still the same number of boys so $\frac{x}{15} = \frac{4}{5}$ $\left(\frac{girls}{boys}\right)$

$\therefore x = \frac{4 \times 15}{5}$

$\therefore x = 12$

To check: boys:girls = 15:12

= 5:4

$\therefore 12 - 9 = 3$, so there must be 3 extra girls

2 a If $\frac{3}{7}$ is yellow then $\frac{4}{7}$ is blue

$\therefore$ ratio of yellow to blue $= \frac{3}{7} : \frac{4}{7}$ (Multiply by the common denominator: 7.)

$= \frac{3 \times 7}{4} : \frac{4 \times 7}{7}$

$= 4:3$

b $\frac{y}{1200} = \frac{4}{3}$ where y = the amount of yellow paint

$y = \frac{4 \times 1200}{3}$

$y = 1600$ mL To check: 1600:1200 = 4:3 (by dividing by 400)

3 a $\frac{chickens}{ducks} = \frac{5}{4}$

$\frac{x}{20} = \frac{5}{4}$

$x = \frac{5 \times 20}{4}$

$x = 25$

b $\frac{total}{ducks} = \frac{5+4+3}{4}$

$\frac{x}{20} = \frac{12}{4}$

$x = \frac{12 \times 20}{4}$

$x = 60$

c Let c = number of chickens, d = number of ducks and g = number of geese

$\frac{c}{total} = \frac{5}{12}$ $\frac{d}{total} = \frac{4}{12}$ $\frac{g}{total} = \frac{3}{12}$

$\frac{c}{108} = \frac{5}{12}$ $\frac{d}{108} = \frac{4}{12}$ $\frac{g}{108} = \frac{3}{12}$

$c = \frac{5 \times 108}{12}$ $d = \frac{4 \times 108}{12}$ $g = \frac{3 \times 108}{12}$

$c = 45$ $d = 36$ $g = 45$

To check: 45 + 36 + 27 = 108

And 45:36:27 = 5:4:3 (by dividing by 9)

Note: this is also known as dividing 108 in the ration 5:4:3.

Exercise 3:01

1 Lina says she earns three times as much money as Laura.

a What is the ratio of:

i Lina's income to Laura's income?

ii Laura's income to Lina's income?

b Do we know how much money either of them earns?

2 A birthday party has 18 boys and 22 girls.

a Write in simplified form the ratio of:

i boys to girls.

ii boys to the total number of people at the party.

iii girls to the total number of people at the party.

b What fraction of the people at the party are:

i boys?

ii girls?

3 Jeong An has $30, Jamie has $24 and Tam has $18.

a Write in simplified form the ratio of:

i Jeong An's money to Jamie's money.

ii Jeong An's money to Tam's money.

iii Jamie's money to Tam's money.

iv Jeong An's money to Jamie's money to Tam's money.

b If they put all their money together, what fraction of the total would be:

i Jeong An's?

ii Jamie's?

iii Tam's?

4 Masaki and Ryosuke both pitch for the same baseball club. In one season, Masaki pitched 27 innings and Ryosuke pitched 36 innings.

a Write in simplified form the ratio of:

i innings pitched by Ryosuke to innings pitched by Masaki.

ii innings pitched by Masaki to the total pitched by both players.

b What fraction of the innings pitched by these players was pitched by:

i Masaki?

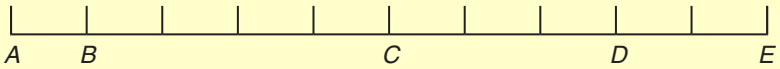
ii Ryosuke?

5 In a box of chocolates the ratio of hard centres to soft centres is 3:5.
If there are 30 hard centre chocolates:

a how many soft centres are there?

b how many chocolates are in the box?

6 The line below is divided into ten equal parts



a Find the following ratios:

i $\frac{AB}{AE}$ **ii** $\frac{BC}{AE}$ **iii** $\frac{AE}{DE}$ **iv** $\frac{DE}{AC}$

b What fraction of the total line is BC?

7 To get a new paint colour called 'Grape', red and blue paint are mixed together.

a If $\frac{3}{7}$ of Grape is blue, what fraction is red?

b What is the ratio of blue paint to red paint in Grape?

8 In a group of 18 people waiting for a bus, the ratio of men to women is 2:1.

a If there are six women, how many men are there?

b What fraction of the people waiting for the bus are women?

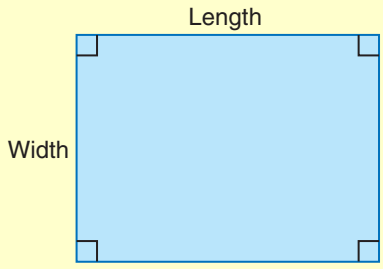
c Complete this sentence by filling in the missing word:
There are _____ as many men as women waiting for the bus.

9 Marie has $4, Ai has $6 and Wan Yu has $8. They decide to put all their money together and buy sweets to share in their home rooms. They find they can buy 1.8 kg of sweets.

i What is the ratio of Marie's money to Ai's money to Wan Yu's money in simplified form?

ii If they share the sweets in the same ratio, how much does each receive?

10 A rectangle is such that its length and width are in the ratio 3:4.
If its perimeter is 28 cm:
- **i** what are its dimensions?
- **ii** what is its area?



11 €24 000 is divided between three people in the ratio 1:3:4. How much does each receive?

12 The wingspan of a Boeing 747-400 is 64.4 metres.
- **a** If a scale model is built to the scale 1:20, what would the wingspan of the model be in centimetres?
- **b** If the wingspan of another model is $53\frac{2}{3}$ cm, find the scale of the model in the form 1:x.

13 In a car park, there are only sedans, station sedans and motorcycles. The ratio of sedans to station sedans is 7:6 and the ratio of station sedans to motorcycles is 3:2.
- **a** What is the ratio of sedans to station sedans to motorcycles?
- **b** If there are 204 vehicles in the car park altogether, how many of each type is there?

14 In her money box, Trudy has a total of 324 coins. The ratio of 1 dollar coins to 50-cent pieces to 20-cent pieces is 5:6:7.
- **a** How many of each coin does she have?
- **b** How much money does she have?

15 In his fish pond, Graham has goldfish and clownfish in the ratio 4:7. If he adds 6 more goldfish, he has the same number of each type of fish. How many fish did he have in the first place?

investigation 3:01

Investigation 3:01 | Ratios and Rational Numbers

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

- Perform the calculations in the first column and fill in the table as shown.
- Show all your working out clearly below the table.
- Describe any patterns in the table that can help you complete the table with less working out.
- Make up three similar problems of your own, using any short cuts you have discovered, to complete the last five lines in the table.
- Try to give a reason why these short cuts work.

		Write each amount as a simplified fraction of the original.	*Write the small amount as a simplified fraction of the big amount.*	*What is the difference between the two amounts?*	*Write the difference as a simplified fraction of the large amount.*
1	Divide \$64 in the ratio 5:3.				
2	Divide 108 g in the ratio 4:5.				
3	Divide 56 sweets in the ratio 3:4.				
4	Divide 72 goldfish in the ratio 2:1.				
5	Divide 81 jelly beans in the ratio 7:2.				
6	Divide £360 in the ratio 3:2.				
7	Divide 690 marbles in the ratio 11:12.				
8	Divide 115 lollipops in the ratio 10:13.				
9	Divide €24 in the ratio 1:3.				
10	Divide 84 toy cars in the ratio 3:1.				
11					
12					
13					

Assessment Grid for Investigation 3:01 | Ratio and Rational Numbers

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	A systematic approach has not been used. The student has had difficulty performing calculations.	1	
			2	
	b	A systematic approach has been attempted. The student has attempted to describe any patterns in the table.	3	
			4	
	c	An organised approach has been described and conclusions regarding the patterns have been drawn.	5	
			6	
	d	Further problems have been investigated and solved to fully explain the patterns demonstrated.	7	
			8	
Criterion C Communication	a	Little working out is shown and presentation is poor.	1	
			2	
	b	Working out is shown. Presentation is good with some structure to the work.	3	
			4	
	c	Communication is clear. Correct notation and terminology have been used.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt has been made to explain the steps undertaken and to check the results.	1	
			2	
	b	The majority of the processes are explained and any anomalies on the results have been checked.	3	
			4	
	c	Alternative methods for calculations have successfully been applied and these short cut answers have been checked and explained.	5	
			6	

3:02 | Increasing and Decreasing by a Ratio and a Percentage

worked examples

1 Increase \$42 in the ratio 3:4.

2 The distance between 2 towns on a map is 10 cm. If the scale on the map is 1:10 000, what is the actual distance between the towns?

3 All the marked prices in a shop are reduced by 15%. What would be the new price of an article originally marked at £80?

4 Over the past year, Casey's height has increased by 10%. If he was 150 cm tall at the start of the year, how tall is he now?

Solutions

1 $\frac{x}{42} = \frac{4}{3}$

$x = \frac{4 \times 42}{3}$ In this case the amount we are looking for corresponds to the big part of the ratio.

$x = \$64$

2 We must increase 10 cm in the ratio 1:10 000

$\frac{x}{10} = \frac{10\,000}{1}$

$x = \frac{10\,000 \times 10}{1}$

$x = 100\,000$ cm

$= 100$ km

3 The new price will be 85% of the old price (100% − 15%)

$\text{New price} = \frac{85}{100} \times 80$

$= 0.85 \times 80$

$= £68$

Note that $\frac{68}{80} = \frac{17}{25} = \frac{85}{100}$

4 Casey's new height is 110% of his height at the start of the year (100% + 10%)

$\text{New height} = \frac{110}{100} \times 150$

$= 1.10 \times 150$

$= 165$ cm

Note that $\frac{165}{150} = \frac{11}{10} = \frac{110}{100}$

So increasing and decreasing by a percentage is the same as increasing or decreasing in a ratio.

Exercise 3:02

1 **a** Increase €56 in the ratio 7:8.
b Decrease €56 in the ratio 7:8.
c Increase 48 m in the ratio 3:4.
d Decrease 48 m in the ratio 3:4.
e Increase $96 in the ratio 1:5.

2 A model of a building is made to a scale of 1:200. If the real building is 30 m high, how high is the model?

3 Due to a hurricane, the production of bananas at Bron's banana plantation has decreased in the ratio 5:4. If Bron produced 900 tonnes of bananas last year:
a what will this year's production be?
b what fraction of last year's production is this?

4 The number of people to visit the city museum this year has increased in the ratio 6:5 on last year's numbers. If 30 000 visited last year:
a how many visited this year?
b what is the percentage increase on last year's numbers?
c what percentage of last year's numbers visited this year?

5 Derek's weekly wage of $550 is increased by 20%.
a What is his new pay?
b What is the ratio of his new pay to his old pay?

6 The membership of our local football club has increased by 8% on last year. If there were 250 members last year:
a how many members are there this year?
b what is the ratio of the increase in members?

7 The number of tourists entering Canada in 2004 was 3 600 000. There was a 7.5% increase in 2005.
How many tourists entered Canada by air in 2005?

8 A photocopier is producing copies that are reduced by 25%. If a picture 10 cm long and 8 cm wide is copied:
a by what fraction is the picture reduced?
b what is the length of the new copy?
c what is the ratio of the new length to the old length?

9 From 1995 to 2005 the world's population grew by approximately 12%.
a If the population of the world was 5600 million in 1995, what would was it in 2005? Answer correct to 2 significant figures.
b What is the ratio of the population in 1996 to the population in 2005?
c By what ratio did the population increase?

10 Calvin sold a motorcycle which originally cost him $3600. He made a loss of 15% when he sold it.

- **a** What was the selling price?
- **b** By what ratio was the value decreased?
- **c** What fraction of the cost price was the selling price?

11 In a school bus, the ratio of girls to boys is 7:5. At the next stop some girls get off so that the ratio of girls to boys is now 6:5. If there are now 24 girls on the bus, how many got off at the stop?

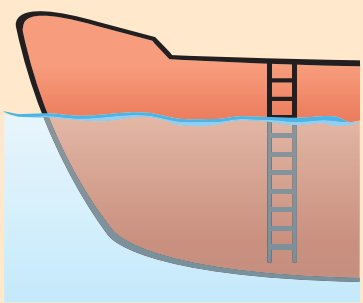
12

- **a** At Dan's discount sale, find the sale price of an item marked at £40.
- **b** At the sale, Bron buys a gravy boat for £30. What would the marked price have been?

fun spot 3:02

Fun Spot 3:02 | All Aboard!

A ship has a ladder over the side into the water with 10 rungs (steps). The rungs on the ladder are 30 cm apart. There are 2 rungs above the water and one rung right on the surface of the water. If the tide goes out 1.5 metres, how many rungs are now above the water?



3:03 | Proportional Change

If all parts of an object or shape are increased by a particular ratio or fraction then the original object or shape and the new object or shape are said to be **in proportion**.

worked examples

1 For two rectangles to be in proportion, all their sides must be in the same ratio. So if one rectangle measures 3 cm wide and 5 cm long, and the other is 9 cm wide, how long must the other one be?

2 Six people in a lifeboat need 12 litres of water per day to survive. If all the people drink the same amount, how much water would 8 people need?

Solutions

1 The two widths are 3 cm and 5 cm, so the ratio of the sides is 3:5.

If the length of the second ratio is l, then

$$\frac{l}{9} = \frac{5}{3}$$

$$\therefore l = \frac{5 \times 9}{3}$$

In this case the amount we are looking for corresponds to the big part of the ratio.

$$\therefore l = 15 \text{ cm}$$

Since the ratio of the widths is 3:5 and the ratio of the lengths is 9:15 = 3:5, the rectangles are *in proportion*

2 The water needs to be in the same ratio as the people.

The ratio of the people is 6:8. So if x is the amount of water needed, then

$$\frac{x}{12} = \frac{8}{6}$$

$$\therefore x = \frac{8 \times 12}{6}$$

$$\therefore x = 16 \text{ litres}$$

Since the ratio of the people is 6:8 = 3:4 and the ratio of the water is 12:16 = 3:4, the amount of water is *in proportion* to the number of people

Exercise 3:03

1 To make enough juice for 4 people, 300 mL of concentrate needs to be mixed with water. How much concentrate is needed to make enough juice for 9 people?

2 Six people can paint 9 metres of fencing in an hour. If everyone paints the same amount, how many metres can be painted by ten people in the same time?

3 For a particular building job, Kang needed 600 m^3 of concrete. He found that the best mix for the job required was to make concrete from 80 m^3 of cement, 200 m^3 of sand and 320 m^3 of aggregate (stone).

a What is the ratio of cement to sand to aggregate needed?

b How much of each would he need if his next job required 900 m^3 of concrete in the same proportion?

4 A recipe for apple cake needs 10 mL of baking powder, 70 mL of butter, 190 mL of sugar and 420 mL of flour. This is enough to make a cake for 8 people. How much of each ingredient would be needed to make a cake for 20 people?

5 Three people can dig a trench 10 m long in a day. If everyone digs the same amount, how long a trench could 5 people dig in the same time?

6 A special mix of oil and petrol is used for lawn mowers. This mix requires 20 mL of oil to be mixed with 1 litre of petrol. If Jerry has 45 mL of oil:

a how much petrol will he need to mix with the oil?

b how much mix will he have altogether?

7 Elaine has a photograph she wants to resize. The original is 360 pixels wide and 480 pixels long. She wants to reduce it so that it is 240 pixels wide. If the photograph is to remain in proportion, what must be the new length?

8 In a train carriage there are 20 men and 28 women. At a stop, 6 men get off and 11 men get on, 5 women off and some women get on. If the number of men and women in the carriage after the stop is in the same proportion as before the stop, how many women got on at the stop?

9 Jeremy and Martin both have scale models of the Boeing 787 Dreamliner. Jeremy's model is built on the scale 1:200 while Martin's is built on the scale 1:300. Since the models are built to scale, the models and the real aircraft are all in proportion.

a Which model is bigger, Jeremy's or Martin's?

If Jeremy's model has a wingspan of 30 cm:

b what is the wingspan of the real aircraft?

c what is the wingspan of Martin's model?

d what is the ratio of Jeremy's model to Martin's model?

10 For Christmas, the McAdam and Hazel families always buy nuts. The McAdam family likes macadamia nuts so their packet of mixed nuts has macadamias to hazelnuts to almonds to walnuts in the ratio 5:4:3:2. The Hazel family prefers hazelnuts so their packet of mixed nuts contains hazelnuts to macadamias to almonds to walnuts in the ratio 5:4:3:2.

a What fraction of the McAdam's packet is macadamia nuts?

If the McAdams eat 20 grams of macadamia nuts, the amount of macadamias left is the same as the amount of hazelnuts.

b What fraction of the packet did they eat?

c How much does the whole packet weigh?

d If the Hazel family's packet has the same amount of macadamias as the McAdam's family packet, what is the weight of the Hazel family's packet?

Mathematical terms 3

Ratio
- A comparison of two like quantities.

Percentage
- A way of writing a fraction out of 100.
  For example, $17\% = \frac{17}{100}$

Percentage increase
- The percentage by which an amount has increased.

Percentage decrease
- The percentage by which an amount has decreased.

Cost price
- The amount paid for an item before reselling.

Selling price
- The price paid when an item is purchased.

Proportion
- When all aspects of length of an object or an amount are kept in the same ratio.

Diagnostic Test 3: | Ratio and Proportion

- Each section of the test has similar items that test a certain type of question.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
1 Divide \$480 in the ratio: **a** 5:7 **b** 1:2 **c** 11:5	3:01
2 The ratio of men to women is 4:5. **a** How many men are there if there are 20 women? **b** How many women are there if there are 20 men? **c** How many men and women are there if there are 45 people altogether?	3:01
3 In a town, the distance between the town hall and the cinema is 2.4 km. If a map of the town is drawn to a scale of 1:20 000, what is the corresponding distance on the map?	3:01
4 In a packet of sweets, the ratio of lemon flavoured sweets to strawberry flavoured to orange flavoured is 3:5:7. **a** What fraction of the packet is strawberry flavoured? **b** If there are 18 lemon flavoured sweets, how many orange flavoured sweets are there?	3:01
5 Ja Young sold a car for 18% less than what it cost her. If the car cost her (Korean Won) ₩5 000 000: **a** for what percentage of the cost price did she sell the car? **b** how much did she sell the car for? **c** what is the ratio of the cost price to the selling price?	3:02

	Section
6 Reduce 360 g in the ratio: **a** 5:9 **b** 4:3 **c** 11:12	3:02
7 After the mid-term break, Patrice's class grew by 15%. If there were 20 students in his class before the break: **a** by how many students did the class grow? **b** how many students are in the class after the break? **c** express the number of students in the class after the break as a percentage of how many there were before the break.	3:02
8 Eddie has a 1:200 scale model of the Titanic. If the real Titanic was 268 metres long: **a** how long is Eddie's model? **b** what fraction of the real ship is the model? **c** what percentage of the real ship is the model?	3:03
9 A photograph is projected on a wall so that the original, which is 4 cm long and 3 cm high, is in proportion to the original, and is now 1 m long. **a** Express the scale of the projection in the form 1:n. **b** How high is the projection? **c** What fraction of the projection is the original photograph?	3:03
10 The number of pieces of pizza eaten by teenagers is in proportion to how many are present. If a group of 6 teenagers at a movie night can eat 21 pieces of pizza, how many will 10 teenagers eat if they all eat the same amount?	3:03

Chapter 3 | Revision Assignment

assignment 3A

1 In Ella's class there are 24 students, 10 of which are girls.
Write the following ratios in simplest form:
   a girls to total students
   b boys to girls
   c boys to total students.

2 Vinh has $15, Huy has $20 and My has $25.
   a What fraction of the total money has:
      i Vinh?
      ii Huy?
      iii My?
   b What is the ratio of Vinh's money to Huy's money to My's money in simplest form?

3 a There are red and green apples in a box. If $\frac{3}{5}$ of the apples in the box are red:
      i what fraction is green?
      ii what is the ratio of red apples to green apples?
   b If there are 30 green apples in the box above, how many apples are there altogether?

4 In her garden, Diana has red and yellow roses in the ratio 5:3.
   a What fraction of her roses are:
      i red?
      ii yellow?
   b If Diana has 10 more red roses than yellow ones, how many roses does she have altogether?

5 In a recent pay rise, Sean's pay increased in the ratio 5:7. If his pay was $700 per week before the increase:
   a what is his pay after the increase?
   b what is the increase in pay?
   c express the increase in pay as a fraction of Sean's original pay.

6 After three weeks of football training, Rick's weight decreased by 5%.
If he weighed 60 kg before training, what did he weigh after three weeks of training?

7 Brenda sold a set of golf clubs for €385. If she sold them at a loss of 70%:
   a what was the original price of the clubs?
   b what fraction of the original price did she lose?

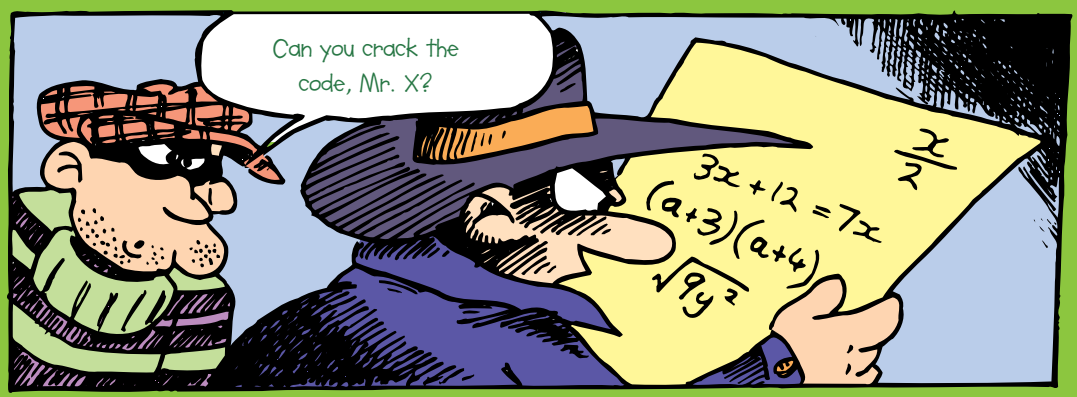
8 Laura has a photograph that is 150 mm wide and 210 mm long. She wants to have it enlarged so that it is 210 mm wide. How long will the enlargement be?

9 In a life raft it is calculated that 8 people can survive for 10 days on the food available. How long can 5 people last on the same food?

10 The scale on a map is 1:150 000. If on the map two towns are connected by a road 20 cm long, how long is the real road in kilometres?

4

Algebraic Expressions



Chapter Contents

Learning Outcomes

Students will:

- Use the algebraic symbol system to simplify, expand and factorise simple algebraic expressions.
- Simplify, expand and factorise algebraic expressions involving fractions and negative and fractional indices.

Areas of Interaction

Approaches to Learning, Homo Faber

4:01 | Addition and Subtraction in Algebra

prep quiz 4:01

Answer the following:

1 $12 + 7$ **2** $8 - 12$ **3** $-4 + 5$ **4** $-3 - 8$ **5** $6 - (-9)$

Complete the following:

6 $5x + 2x = (\ldots + \ldots)x$ **7** $9y - 4y = (\ldots - \ldots)y$ **8** $a \times b = b \times \ldots$

Simplify the following:

9 $2^2 + 2^3$ **10** $a + a$

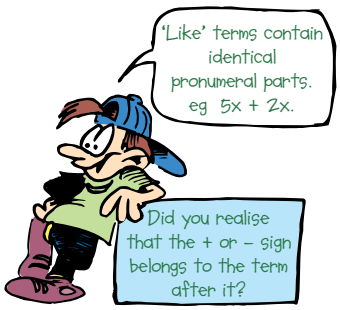
worked examples

1 $5a + 2b - 3a + b = 5a - 3a + 2b + b$
$= 2a + 3b$

2 $5p^2 + 2p - 3p^2 = 5p^2 - 3p^2 + 2p$
$= 2p^2 + 2p$
(Note: p^2 and p are *not* like terms.)

3 $6ab - 4ba = 6ab - 4ab$
$= 2ab$

4 $6a - 2x + 5 + x - 2a - 7 = 6a - 2a - 2x + x + 5 - 7$
$= 4a - x - 2$



Only *like* terms can be added or subtracted.

Exercise 4:01

Foundation Worksheet 4:01
Collecting like terms
1 Simplify: a 8 − 10
2 Complete the following:
a $3x + 4x = (\ldots + \ldots)x$
3 Simplify: a $9a - 2a$

1 Simplify the following expressions.

a $3x + 2x$	b $8a + 5a$	c $10p + 21p$
d $7a - 4a$	e $9b - 3b$	f $11q - q$
g $3p + 5p - 6p$	h $4x + 2x + x$	i $10x - 9x + 3x$
j $3x^2 + 9x^2$	k $8a^2 - 5a^2$	l $y^2 + 3y^2 - 2y^2$
m $8y - 10y$	n $x - 3x$	o $2n - 4n + 5n$
p $14ab + 2ab$	q $6pq - 5pq$	r $10x^2y - 12x^2y$

2 Collect the like terms to simplify these expressions.

a $2a + b + 3a$	b $8x - 9 + 5x$	c $10 + 3a + 7$
d $5p + 4q - 6p$	e $3y + 2x - x$	f $m + 3n - 2n$
g $6q - 5q + 3p$	h $8h - 10h + k$	i $5m + n + 2m + 3n$
j $2a + 3b + a + 2b$	k $5t + 2u - u + 4t$	l $9q + 4p - 10q + p$
m $2a + p - a + 3p$	n $a + m - a + m$	o $8 + 2x - 5x - 7$
p $8y - 1 - 8y - 1$	q $x^2 + 2x + 2x^2 - x$	r $p^2 + 4p + 3p^2 + p$
s $3q^2 + 8q - 4q - q^2$	t $y^2 + y + y^2 - y$	u $7 - p^2 + p - 5$
v $2a + a^2 + 7 + a$	w $8x - 7 - 7x - 3x^2$	x $5ab - 7 + 3ba - 9$

4:02 | Multiplication in Algebra

prep quiz 4:02

Answer the following:

1 -6×4 **2** -3×-2 **3** 7×-1 **4** $(3)^2$ **5** $(-2)^2$

Write in a simple form:

6 $7 \times a$ **7** $x \times y$ **8** $a \times a$

Simplify:

9 $2x + 2x + 2x$ **10** $3 \times 2x$

worked examples

1 $8 \times 7m = 8 \times 7 \times m$
$= 56 \times m$
$= 56m$

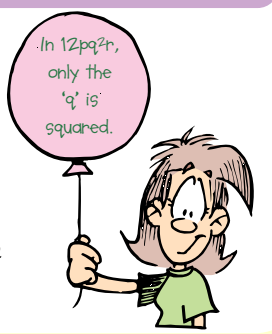
2 $5a \times 6b = 5 \times 6 \times a \times b$
$= 30ab$

3 $10x \times x = 10x \times 1x$
$= 10xx$
$= 10x^2$

4 $3pq \times 4qr = 12pqqr$
$= 12pq^2r$

5 $-4n \times 2y = -4 \times 2 \times n \times y$
$= -8ny$

6 $-7x \times -3xy^2 = -7 \times -3 \times x \times x \times y^2$
$= 21x^2y^2$



Multiply numbers first and then pronumerals.

Exercise 4:02

Foundation Worksheet 4:02
Multiplying pronumerals
1 Simplify: a -3×4
2 Write in simplified form:
a $6 \times a$
3 Simplify: a $5 \times 3a$

1 Simplify these products.

a $5 \times 2x$ b $7 \times 3p$ c $8y \times 3$
d $9 \times a$ e $b \times 5$ f $9p \times 12$
g $x \times y$ h $3x \times 2b$ i $8x \times 3y$
j $3m \times 10n$ k $x \times 17y$ l $10a \times 5x$
m $\frac{1}{2} \times 4x$ n $10y \times \frac{1}{2}x$ o $\frac{1}{4}a \times 12$

2 Write these products in their simplest form.

a $x \times x$ b $2y \times y$ c $a \times 3a$ d $4m \times 2m$
e $7p \times 2p$ f $8b \times 10b$ g $11t \times 11t$ h $10x \times 3x$
i $2a \times ab$ j $xy \times yz$ k $ab \times ac$ l $3pq \times 2p$
m $4mn \times 5np$ n $2ab \times 4ab$ o $xy \times 3yz$ p $4mn \times \frac{1}{2}m$
q $(-4x) \times 3y$ r $-2x \times -3x$ s $5a^2 \times (-7)$ t $9b \times a^2$
u $(-4a) \times (-7m)$ v $(-5p) \times 2pg$ w $(-ab) \times (-bc)$ x $14ab \times (-\frac{1}{2}ab)$

3 Simplify:

a $7 \times 2a \times 3$ b $5x \times 2y \times 3z$ c $2x \times 5 \times x$ d $ab \times bc \times ac$
e $x \times 2y \times 3z$ f $3a \times 5 \times 4a$ g $m \times n \times 7$ h $2k \times 3l \times k$
i $(-2) \times 7x \times (-5)$ j $10a \times (-3a) \times 4$ k $9x \times (-4y) \times (-5)$ l $\frac{1}{4}m \times 4n \times (-p)$

4:03 | Division in Algebra

prep quiz 4:03

Answer the following:

1 $72 \div 8$ **2** $(-12) \div 6$ **3** $20 \div (-5)$ **4** $(-24) \div (-6)$

Reduce these fractions to their simplest form:

5 $\frac{6}{2}$ **6** $\frac{10}{4}$ **7** $\frac{24}{10}$ **8** $\frac{100}{35}$

Simplify:

9 $5 \div 5$ **10** $x \div x$

worked examples

1 $6a \div 2 = \frac{{}^{3}\cancel{6}a}{\cancel{2}_{1}}$ (Divide top and bottom by 2.)

$= 3a$

2 $15xy \div 3x = \frac{{}^{5}\cancel{15}xy}{{}_{1}\cancel{3}x}$ (Divide top and bottom by 3 and x.)

$= 5y$

3 $12ac \div 8ab = \frac{{}^{3}\cancel{12}ac}{{}_{2}\cancel{8}ab}$

$= \frac{3c}{2b}$

4 $-6x \div 18xy = \frac{-{}^{1}\cancel{6}x}{{}_{3}\cancel{18}xy}$

$= -\frac{1}{3y}$

■ $8 \div 2$ and $\frac{8}{2}$ are different ways of writing the same thing.

Express the division sum as a fraction and reduce it to its lowest terms.

Exercise 4:03

Foundation Worksheet 4:03
Algebraic division
1 Reduce these fractions to their lowest terms: **a** $\frac{6a}{12}$
2 Simplify: **a** $a \div a$
3 Simplify: **a** $10x \div 5$

1 Reduce these fractions to their lowest terms.

a $\frac{3x}{6}$ **b** $\frac{12b}{30}$ **c** $\frac{8a}{8}$ **d** $\frac{5a}{10a}$

e $\frac{7x}{35x}$ **f** $\frac{12m}{4m}$ **g** $\frac{10ab}{5b}$ **h** $\frac{24a}{6b}$ **i** $\frac{7}{7t}$

2 Simplify these divisions.

a $12x \div 4$ **b** $16y \div 8$ **c** $5p \div 5$ **d** $15m \div 5$

e $2ab \div 2$ **f** $20xy \div 5$ **g** $9x^2 \div 3$ **h** $14x \div 7$

i $12a \div 3a$ **j** $8x \div 4x$ **k** $5x \div 5x$ **l** $36y \div 9y$

m $15m \div 10n$ **n** $18x \div 4x$ **o** $32a \div 12b$ **p** $5 \div 20x$

q $24xy \div 6x$ **r** $48ab \div 16a$ **s** $27x \div 21xy$ **t** $12a \div 10ba$

u $2 \div 18k$ **v** $a \div 3a$ **w** $5mn \div 10n$ **x** $45ab \div 20bc$

3 Simplify:

a $8p \div (-2)$ **b** $(-9x) \div (-3)$ **c** $(-20p) \div 4$ **d** $(-24x) \div (-12x)$

e $(-25) \div 15a$ **f** $(-xy) \div yz$ **g** $6ab \div 96$ **h** $(-54xy) \div (-9xy)$

i $14a \div (-a)$ **j** $20ab \div 12ba$ **k** $(-15x) \div 5xy$ **l** $-28mnp \div (-7mp)$

Challenge worksheet 4:03 Algebraic simplifications

4:04 | Simplifying Algebraic Fractions

4:04A Addition and subtraction

worked examples

1 $\frac{2x}{5} + \frac{x}{5} = \frac{2x + x}{5}$

$= \frac{3x}{5}$

2 $\frac{7m}{3} - \frac{5m}{3} = \frac{7m - 5m}{3}$

$= \frac{2m}{3}$

■ If the denominators are the same, simply add or subtract the numerators.

3 $\frac{3a}{5} + \frac{2a}{4} = \frac{3a \times 4}{5 \times 4} + \frac{2a \times 5}{4 \times 5}$

$= \frac{12a}{20} + \frac{10a}{20}$

$= \frac{22a}{20}$

$= \frac{11a}{10}$

■ Cancel if possible.

4 $\frac{2n}{5} - \frac{n}{10} = \frac{2n \times 2}{5 \times 2} - \frac{n}{10}$

$= \frac{4n}{10} - \frac{n}{10}$

$= \frac{3n}{10}$

■ If the denominators are different, rewrite each fraction with a common denominator, then add or subtract the numerators.

Always look for the lowest common denominator!

Exercise 4:04A

Foundation Worksheet 4:04A

Simplifying algebraic fractions

1 Simplify: a $\frac{1}{7} + \frac{3}{7}$

2 Simplify: a $\frac{7}{10} - \frac{3}{10}$

3 Simplify: a $\frac{x}{5} + \frac{2x}{5}$

Simplify each addition or subtraction.

■ Remember that:

- $\frac{x}{3}$ is the same as $\frac{1}{3}x$ and that $\frac{5m}{6}$ is the same as $\frac{5}{6}m$.
- $5y = \frac{5y}{1}$

1 a $\frac{x}{3} + \frac{x}{3}$ b $\frac{3m}{7} + \frac{2m}{7}$ c $\frac{5n}{6} + \frac{2n}{6}$

d $\frac{3x}{5} - \frac{2x}{5}$ e $\frac{7t}{8} - \frac{2t}{8}$ f $\frac{4m}{7} - \frac{2m}{7}$

2 a $\frac{7p}{10} + \frac{p}{10}$ b $\frac{3t}{8} + \frac{t}{8}$ c $\frac{5y}{6} - \frac{y}{6}$

d $\frac{9x}{10} - \frac{3x}{10}$ e $\frac{7x}{12} + \frac{7x}{12}$ f $\frac{5m}{16} + \frac{9m}{16}$

3 a $\frac{x}{3} + \frac{x}{2}$ b $\frac{2a}{3} + \frac{a}{4}$ c $\frac{m}{2} + \frac{2m}{5}$ d $\frac{2t}{3} + \frac{t}{2}$ e $\frac{2x}{5} + \frac{2x}{3}$

f $\frac{a}{2} - \frac{a}{3}$ g $\frac{3w}{5} - \frac{w}{3}$ h $\frac{3n}{2} - \frac{2n}{5}$ i $\frac{2m}{3} - \frac{m}{2}$ j $\frac{3a}{4} - \frac{3a}{5}$

4 a $\frac{2a}{3} + \frac{a}{6}$ b $\frac{3y}{2} + \frac{5y}{6}$ c $\frac{2m}{5} + \frac{m}{10}$ d $\frac{4m}{3} - \frac{5m}{6}$ e $\frac{3t}{5} - \frac{3t}{10}$

f $\frac{m}{6} + \frac{m}{9}$ g $\frac{3a}{4} + \frac{a}{6}$ h $\frac{7x}{10} + \frac{x}{4}$ i $\frac{3n}{8} + \frac{5n}{6}$ j $3y - \frac{3y}{8}$

5 a $\frac{5}{6}m - \frac{1}{4}m$ b $\frac{7}{12}y - \frac{3}{8}y$ c $\frac{3}{4}x - \frac{3}{10}x$ d $5y + \frac{5}{4}y$ e $2m - \frac{4}{5}m$

4:04B Multiplication and division

worked examples

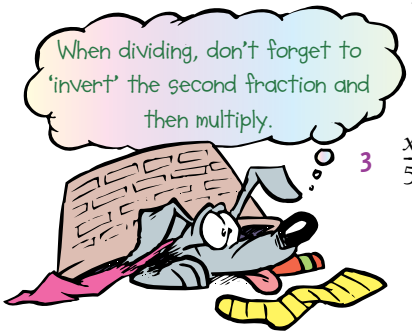
■ Cancel any common factors, then multiply the 'tops' and then multiply the 'bottoms'.

1 $\frac{a}{2} \times \frac{b}{3} = \frac{a \times b}{2 \times 3}$

$= \frac{ab}{6}$

2 $\frac{2x}{3} \times \frac{6}{5x} = \frac{2x^{1}}{\cancel{3}_{1}} \times \frac{\cancel{6}^{2}}{5\cancel{x}_{1}}$

$= \frac{2 \times 2}{1 \times 5} = \frac{4}{5}$



3 $\frac{x}{5} \div \frac{x}{3} = \frac{\cancel{x}^{1}}{5} \times \frac{3}{\cancel{x}_{1}}$

$= \frac{1 \times 3}{5 \times 1}$

$= \frac{3}{5}$

4 $\frac{ab}{4} \div \frac{b}{2} = \frac{ab^{1}}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{b}_{1}}$

$= \frac{a \times 1}{2 \times 1}$

$= \frac{a}{2}$

Exercise 4:04B

Foundation Worksheet 4:04B
Simplifying algebraic fractions
1 Simplify: a $\frac{1}{3} \times \frac{4}{5}$
2 Simplify: a $\frac{a}{2} \times \frac{b}{5}$
3 Simplify: a $\frac{a}{2} \div \frac{a}{8}$

Simplify these products.

1 a $\frac{x}{2} \times \frac{y}{3}$ b $\frac{a}{5} \times \frac{b}{6}$ c $\frac{m}{3} \times \frac{n}{4}$
d $\frac{a}{3} \times \frac{2}{x}$ e $\frac{5}{m} \times \frac{n}{3}$ f $\frac{2p}{3} \times \frac{5}{q}$

2 a $\frac{3x}{2} \times \frac{5y}{4}$ b $\frac{4m}{3} \times \frac{2n}{3}$ c $\frac{7x}{2} \times \frac{y}{3}$ d $\frac{2a}{5} \times \frac{3b}{5}$ e $\frac{3x}{2} \times \frac{5x}{4}$

3 a $\frac{2m}{3} \times \frac{m}{4}$ b $\frac{3x}{2} \times \frac{y}{6}$ c $\frac{a}{10} \times \frac{5}{6}$ d $\frac{3x}{2} \times \frac{4y}{3}$ e $\frac{5t}{2} \times \frac{3t}{2}$

4 a $\frac{2x}{3} \times \frac{9}{x}$ b $\frac{m}{6} \times \frac{3}{2m}$ c $\frac{a}{b} \times \frac{b}{a}$ d $\frac{2x}{y} \times \frac{3y}{x}$ e $\frac{ab}{5} \times \frac{a}{b}$

5 a $10 \times \frac{y}{5}$ b $4 \times \frac{n}{6}$ c $\frac{5a}{2} \times 6$ d $6 \times \frac{2y}{3}$ e $\frac{3a}{8} \times 4$

Simplify these divisions.

6 a $\frac{m}{3} \div \frac{m}{2}$ b $\frac{x}{4} \div \frac{x}{3}$ c $\frac{p}{6} \div \frac{p}{5}$ d $\frac{t}{2} \div \frac{t}{5}$ e $m \div \frac{m}{5}$
f $\frac{a}{4} \div 2$ g $\frac{8x}{5} \div 4$ h $\frac{m}{2} \div \frac{t}{5}$ i $2m \div \frac{n}{5}$ j $\frac{2x}{5} \div 5y$

7 a $\frac{2a}{3} \div \frac{a}{5}$ b $\frac{3a}{4} \div \frac{b}{2}$ c $\frac{3p}{4} \div \frac{p}{2}$ d $\frac{4a}{5} \div \frac{2x}{5}$ e $\frac{5a}{3} \div 3a$

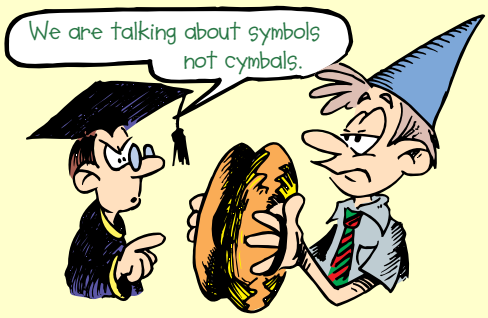
8 a $\frac{5}{x} \div \frac{x}{2}$ b $\frac{a}{3} \div \frac{2}{a}$ c $\frac{5m}{4} \div \frac{3}{2}$ d $\frac{ab}{3} \div \frac{ac}{2}$ e $\frac{2a}{b} \div \frac{3a}{c}$

Challenge worksheet 4:04 Algebraic fractions

4:05 | Grouping Symbols

The two most commonly used grouping symbols are:

parentheses ( )
brackets [ ]



prep quiz 4:05

Simplify:

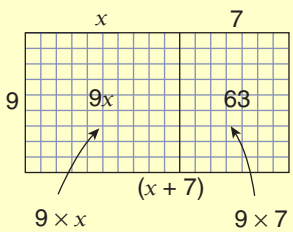
1 $5 \times 2a$ **2** $a \times 2a$ **3** $2x \times 5x$
4 -4×-5 **5** $-3 \times 2x$ **6** $-x \times x$
7 $7p \times (-2p)$

Complete:

8 $5 \times 6 + 3 \times 6 = \square \times 6$ **9** $7 \times 2 - 4 \times 2 = \square \times 2$
10 $3(8 + 5) = 3 \times \square + 3 \times \square$

Removing grouping symbols can be demonstrated using the area of a rectangle:

Area $= 9(x + 7)$
$= 9x + 63$



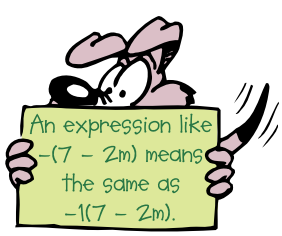
$a(b + c) = ab + ac$ or $a(b - c) = ab - ac$

To expand an expression, such as $a(b + c)$, each term inside the grouping symbols is multiplied by the term outside.

worked examples

1 $p(p + 3) = p \times p + p \times 3$
$= p^2 + 3p$

2 $3a(5 - 2a) = 3a \times 5 - 3a \times 2a$
$= 15a - 6a^2$



3 $-5(3x + 4) = (-5) \times 3x + (-5) \times 4$
$= -15x - 20$

4 $-(7 - 2m) = (-1) \times 7 - (-1) \times 2m$
$= -7 + 2m$

Exercise 4:05

Foundation Worksheet 4:05
Grouping symbols
1 Simplify: **a** $3 \times 4y$
2 Complete the following:
a $3(2m + 5) = 3 \times 2m + 3 \times \ldots$
3 Remove the grouping symbols.
a $3(2a + 5)$

1 Expand:

a $2(x + 3)$	**b** $3(a + 5)$	**c** $5(x - 1)$
d $10(a + 1)$	**e** $3(x - 2)$	**f** $8(y + 4)$
g $2(5a + 1)$	**h** $3(2x - 5)$	**i** $9(4x + 7)$
j $5(1 + x)$	**k** $4(3 - a)$	**l** $2(5 - 3m)$
m $2(3a + 2b)$	**n** $5(x + y)$	**o** $7(3x - 5y)$

2 Remove the grouping symbols.

a $x(x + 7)$	**b** $a(a - 1)$	**c** $m(m + 10)$	**d** $n(n - 3)$
e $a(2a + 1)$	**f** $b(3b - 5)$	**g** $x(5 + x)$	**h** $y(3 - y)$
i $2x(x + 3)$	**j** $3y(y - 1)$	**k** $5z(2 + z)$	**l** $10m(1 - m)$
m $3a(2a - 1)$	**n** $9x(2x + 7)$	**o** $8p(2 - 5p)$	**p** $7q(3 + 2q)$
q $x(a + b)$	**r** $y(2y + x)$	**s** $2m(m + n)$	**t** $5a(2a + 3b)$

3 Expand by removing the parentheses.

a $-2(x + 3)$	**b** $-3(a + 5)$	**c** $-2(y - 1)$	**d** $-5(p - 3)$
e $-7(3a + 2)$	**f** $-5(2x - 1)$	**g** $-7(3 + 2m)$	**h** $-4(7 - x)$
i $-(a + 1)$	**j** $-(3x + 7)$	**k** $-(8 - 2p)$	**l** $-(3a + 2b)$
m $-x(x + 10)$	**n** $-y(5 - y)$	**o** $-3x(2x + 7)$	**p** $-10n(8m - n)$

4 Expand and simplify:

a $4\left(\frac{x}{2} + 3\right)$	**b** $6\left(\frac{2x}{3} - 1\right)$	**c** $4\left(\frac{3x}{4} - \frac{x}{2}\right)$	**d** $10\left(\frac{2y}{5} - \frac{y}{2}\right)$

Investigation 4:05 | Checking algebraic simplification by substitution

investigation 4:05

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

If two algebraic expressions are equal they will always give the same value when any numbers are substituted for the pronumerals. This gives an easy way to check algebraic simplifications.

Examples

1 A student thinks that $2(x + 5)$ is the same as $2x + 5$. Substituting $x = 1$ gives $2(x + 5) = 12$ and $2x + 5 = 7$. As $12 \neq 7$ the expressions are not equal.
2 Use substitution to see if $(a + b)^2$ is the same as $a^2 + b^2$.
Substituting $a = 1$ and $b = 2$ gives $(a + b)^2 = 9$ while $a^2 + b^2 = 5$. So they are not the same.

Questions

Substitute a value for the pronumeral to show why the following pairs of expressions cannot be the same.

1 $2x + 3, \quad 5x$	**2** $7a - 7, \quad a$	**3** $6m \div 2m, \quad 3m$
4 $\frac{a}{3} + \frac{a}{2}, \quad \frac{2a}{5}$	**5** $a + a, \quad a^2$	**6** $3a + 4 - a + 2, \quad 2a + 2$
7 $\sqrt{a^2 + 1}, \quad a + 1$	**8** $(a + 2)^2, \quad a^2 + 4$	**9** $\frac{a + 3}{3}, \quad a + 1$

Assessment Grid for Investigation 4:05 | Checking algebraic simplification by substitution

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	A systematic approach has not been used and the results are confusing.	1	
			2	
	b	The student has attempted to use a systematic approach and to explain the results.	3	
			4	
	c	An organised approach has been used and conclusions regarding the patterns have been described.	5	
			6	
	d	All results have been correctly interpreted and an alternative strategy for simplification has been investigated.	7	
			8	
Criterion C Communication	a	Little or no working out is shown and the steps undertaken are difficult to follow.	1	
			2	
	b	Some working out is shown and the results have been interpreted using symbols and words.	3	
			4	
	c	Presentation is very good and the working out is easy to follow. The results have been interpreted fully using symbols and words.	5	
			6	
Criterion D Reflection in Mathematics	a	An attempt has been made to explain the steps undertaken and to check the results.	1	
			2	
	b	A justification for the method is given (even though one is provided) and results have been checked with some success.	3	
			4	
	c	A concise justification is given for the method and the reliability of findings; and the alternative strategy has been fully explained, with other possible applications or examples.	5	
			6	

4:06 | Simplifying Expressions with Grouping Symbols

prep quiz 4:06

Simplify:
1 $7x + 3x$
2 $4a^2 - a^2$
3 $4x + 3 + 2x + 5$
4 $2x + 7 - x - 5$
5 $3y^2 + 5y + 2y^2 - y$
6 $7 - 3a + 6 + 5a$

Expand:
7 $3(x - 7)$
8 $9(2 - 5y)$
9 $2a(a + 3)$
10 $-5(x + 7)$

worked examples

1 $5x + 3 + 2(x + 3) = 5x + 3 + 2x + 6$
$= 7x + 9$

2 $x(x - 1) - x^2 + 5 = x^2 - x - x^2 + 5$
$= -x + 5$

3 $2a(a + b) - a(3a - 4b) = 2a^2 + 2ab - 3a^2 + 4ab$
$= 6ab - a^2$

Exercise 4:06

Foundation Worksheet 4:06
Simplifications with expansions
1 Expand and simplify:
a $4(t + 3) + 2$
2 Expand and simplify:
a $4(t + 3) + 2 + 5t$
3 Expand and simplify:
a $4(t + 3) + 2(t + 7)$

1 Simplify the following expressions.

a $6(x + 2) + 3$
b $4(a + 6) + 7$
c $6 + 2(y + 1)$
d $8 + 3(p + 2)$
e $6a + 3(a + 4)$
f $4m + 3(2m + 3)$
g $2(a + 3) + 5a + 2$
h $2x + 7 + 6(2x + 3)$
i $3(2a + 5) + 6a + 1$
j $4 + 4(3m + 1) + 6$

2 Expand all grouping symbols and then simplify each expression.

a $3(x + 2) + 2(x + 1)$
b $5(y + 2) + 3(y + 4)$
c $3(m + 5) + 2(m + 4)$
d $x(x + 3) + 3(x + 1)$
e $a(a + 5) + 5(a + 5)$
f $n(2n + 1) + 3(2n + 1)$

3 Expand and simplify.

a $2(a - 4) + 6$
b $7(a - 3) + 25$
c $4(m + 6) - 10$
d $3(x + 3) - 11$
e $4(m - 3) - 3$
f $8 + 3(2p - 5)$
g $5(y - 2) + 3y - 7$
h $4(a - 1) + 6a - 5$
i $2x + 7 + 5(2x - 1)$

4 Expand all grouping symbols, then simplify each expression.

a $8 - 2(a + 4)$
b $20 - 3(6 + x)$
c $6a - 3(2a + 1)$
d $3m + 10 - 2(m + 1)$
e $10a - 4 - 5(a + 4)$
f $12y - 3(2y + 5) + 15$
g $6(m + 1) - 3(m + 2)$
h $5(x + 7) - 3(x + 4)$
i $2a(2a + 3b) - b(3a + 2b)$

5 Expand all grouping symbols, then simplify each expression.

a $9k - 2(3 - 4k)$
b $11m - (5 - 2m)$
c $8a + 7 - 2(3a - 4)$
d $6 - 2(y - 4) + 4y$
e $20 - 4(x - 2) - 2x$
f $8x - 3(1 - 2x) - 10$
g $9(a + 5) - 7(a - 3)$
h $t(t - 5) - 4(t - 5)$
i $4x(x - 2) - 3(x - 2)$
j $2m(m - 3) - 5(m - 3)$
k $4(y - 5) - (y - 3)$
l $2a(3a + 1) - 3(2a - 3)$
m $2a(a + b) - a(a + b)$
n $x(x - y) - 3(2x - y)$
o $2x(3x - y) - y(3x - 2)$

4:07 | Factorising Using Common Factors

prep quiz

4:07

Expand:

1 $2(x + 5)$ **2** $x(4x - 1)$

3 $3a(2a + 7)$ **4** $2m(m + 3n)$

Write down the 'factors' of:

5 12 **6** 20 **7** 30

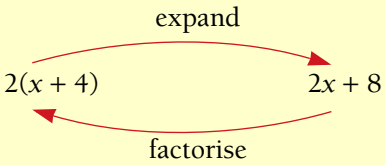
Write down the 'highest common factor' (HCF) of:

8 12 and 20 **9** 12 and 30 **10** 20 and 30

■ A factor of a given number is another number that will divide into the given number with no remainder.
eg {1, 2, 3, 6, 9, 18} is the set of factors of 18.

To factorise an algebraic expression we must determine the highest common factor (HCF) of the terms and insert grouping symbols, usually parentheses.

If we expand the expression $2(x + 4)$ we obtain $2x + 8$. To factorise $2x + 8$ we simply reverse this procedure. We notice that 2 is the highest common factor of $2x$ and 8, so 2 is written outside the parentheses and the remainder is written inside the parentheses,
ie $2x + 8 = 2(x + 4)$.



worked examples

1 $2x + 6 = 2 \times x + 2 \times 3$ (HCF is 2)
$= 2(x + 3)$

2 $10x + 15y = 5 \times 2x + 5 \times 3y$ (HCF is 5)
$= 5(2x + 3y)$

3 $2mn + m = m \times 2n + m \times 1$ (HCF is m)
$= m(2n + 1)$

4 $9ax + 6a = 3a \times 3x + 3a \times 2$ (HCF is $3a$)
$= 3a(3x + 2)$

5 $5x^2 - 10xy = 5x \times x - 5x \times 2y$ (HCF is $5x$)
$= 5x(x - 2y)$

6 $-4a - 8 = -4 \times a + -4 \times 2$ (HCF is -4)
$= -4(a + 2)$

7 $-a^2 + 3a = -a \times a - (-a) \times 3$ (HCF is $-a$)
$= -a(a - 3)$

8 $4xy + 6x^2 - 2xz$ (HCF is $2x$)
$= 2x \times 2y + 2x \times 3x - 2x \times z$
$= 2x(2y + 3x - z)$

$ab + ac = a(b + c)$ and $ab - ac = a(b - c)$

Exercise 4:07

1 Complete the following.

a $4x + 8 = 4(\quad)$ **b** $6a + 18 = 6(\quad)$ **c** $8a - 12 = 4(\quad)$

d $5x + 10y = 5(\quad)$ **e** $21x - 14y = 7(\quad)$ **f** $12pq - 15x = 3(\quad)$

g $mn - np = n(\quad)$ **h** $ab - ac = a(\quad)$ **i** $2ax + 4ay = 2a(\quad)$

j $y^2 - 5y = y(\quad)$ **k** $5at - 3a^2t = at(\quad)$ **l** $12m^2n - 6mn = 6mn(\quad)$

Foundation Worksheet 4:07

Factorising

1 What is the common factor of: 6 and 8?

2 Complete the following: $6a + 12 = 6(\ldots + 2)$

3 Factorise: $6a + 6$

2 Factorise the following by taking out the HCF.

a $2x + 10$	**b** $6a + 4$	**c** $7y + 21$	**d** $28 + 4x$
e $27 - 3y$	**f** $24x + 6$	**g** $9x - 45$	**h** $16 - 12a$
i $9x + 3y$	**j** $5a + 10b$	**k** $15m - 20n$	**l** $4b - 6a$
m $mp + mn$	**n** $ax + ay$	**o** $x^2 + xy$	**p** $p^2 - pq$
q $ap + 3a$	**r** $5x + ax$	**s** $4m - mn$	**t** $xt - t$

3 Factorise completely.

a $3ax + 6ay$	**b** $5mn - 10mp$	**c** $4ab - 6bc$	**d** $9pq - 6qr$
e $5x^2 - 10xy$	**f** $3ab + 6a^2$	**g** $10m^2 - 4mn$	**h** $12x^2 + 4xy$
i $abc + bcd$	**j** $apq - bpq$	**k** $xyz + xy$	**l** $mn - mnp$
m $x^2a - xay$	**n** $5ax - 10xy$	**o** $a^2p - 5ap$	**p** $xy^2 + xyz$
q $10ab - 15bc$	**r** $5x^2y - 3xy^2$	**s** $ap^2 - a^2p$	**t** $5ab - a^2b^2$

4 Factorise the following by taking out the negative common factor.

a $-2a - 6$	**b** $-5x - 15$	**c** $-8m - 12$	**d** $-10x - 5$
e $-8x + 4$	**f** $-3n + 9$	**g** $-7y + 35$	**h** $-6a + 4$
i $-x^2 - 3x$	**j** $-m^2 - m$	**k** $-3x^2 + 2x$	**l** $-5y^2 + 10y$
m $-4p - p^2$	**n** $-3x - 2x^2$	**o** $-m + 7m^2$	**p** $-4a + 18a^2$

5 Factorise each of the following.

a $ab + ac + ad$	**b** $3x + xy + xz$	**c** $m^2 - 3m + mn$	**d** $7a - ab + a^2$
e $p^2 + pq - 5p$	**f** $2x + 4y - 6z$	**g** $10a - 5b + 15c$	**h** $9x^2 + 6x - 12$
i $8 - 4x + 6x^2$	**j** $25 + 15y - 20y^2$	**k** $x^2y - 3xy + x$	**l** $2ab - 4ac + 10a$
m $3x^2 + 6xy - 9x$		**n** $x^2y + xy + xy^2$	
o $a^2b^2 + 3a^2b + 2ab^2$		**p** $mn + 4m^2n - 8mn^2$	

6 Examine this example.

If $3(x + 2) - a(x + 2)$ is to be factorised then $(x + 2)$ is common to both terms, so it can be taken out as a common factor:

ie $3(x + 2) - a(x + 2) = (x + 2)(3 - a)$

Similarly:

$$x(a + 1) + (a + 1) = x(a + 1) + 1(a + 1)$$
$$= (a + 1)(x + 1)$$

> **Note:**
> $(a + 1)(x + 1) = (x + 1)(a + 1)$

Now factorise these similar types.

a $a(a + 2) + 3(a + 2)$	**b** $m(m + 2) + 4(m + 2)$	**c** $x(x - 1) + 5(x - 1)$
d $b(b + 1) - 5(b + 1)$	**e** $7(y - 2) - y(y - 2)$	**f** $t(t - 7) - 9(t - 7)$
g $4(2m - 3) + 3m(2m - 3)$	**h** $2x(7x + 1) - 5(7x + 1)$	**i** $x(a + 3) - (a + 3)$
j $y(2y - 1) - (2y - 1)$	**k** $p(p - 3) - 3(p - 3)$	**l** $(5x + 3) - x(5x + 3)$

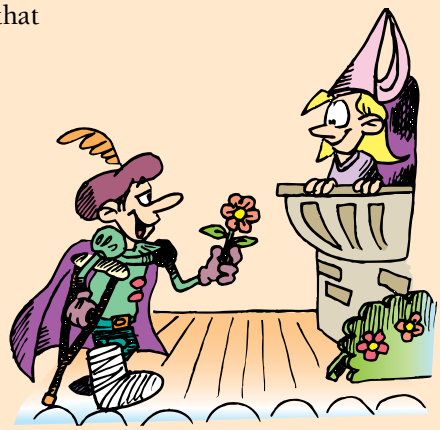
- If the width of one figure is x what is the width of:
  a 2 figures?
  b 4 figures?

fun spot

4:07

Fun Spot 4:07 | In the theatre, when do you tell someone to 'break a leg'?

Work out the answer to each part and put the letter for that part in the box that is above the correct answer.



Simplify:

A $5a + 8a$ **A** $13x - x$
A $x^2 + x^2$ **A** $4a \times 3$
A $16x \div 8$ **A** $2a \times 6a$
A $3(a + b) - 3b$
B $7x \times 2y$ **C** $ab + ba$
C $a^2 \div a$ **E** $2a \times 2b$
E $x + x + x$ **E** $10x \div 2x$
G $-8x - 2x$ **H** $-2x \times -5$

Expand and simplify where possible:

I $3(7x + 4)$ **I** $2(7 - 3x)$ **K** $-4(x - 3)$ **L** $-4(x + 3)$
N $x(x + 7)$ **N** $2a(3a - 1)$ **N** $-a(2a + 3)$ **O** $x(x + y)$
O $3(2x - 4) + 5(3x - 5)$ **R** $x(x + 9) + 4(x + 9)$
R $a(a + 7) - 2(a + 7)$ **S** $1(x - 3) - x(x - 3)$

Factorise:

S $3x + 3y$ **S** $3x + 9$ **T** $a + ax$ **T** $x^2 + x$
U $6a - 12$ **W** $-3x - 3$ **Y** $-3x + 3$ **Y** $4a^2 - 3a$

$a(4a - 3)$	$x^2 + xy$	$6(a - 2)$	$3(x + y)$	$13a$	$-3(x - 1)$	$14xy$	$a^2 + 5a - 14$	$4ab$	$12x$	$-4x + 12$	$2x^2$	$-4x - 12$	$3x$	$-10x$	$-3(x + 1)$	$10x$	5	$x^2 + 7x$

$12a$	$-2a^2 - 3a$	$2x$	a	$x(x + 1)$	$21x - 37$	$x^2 + 13x + 36$	$21x + 12$	$3(x + 3)$	$14 - 6x$	$6a^2 - 2a$	$3a$	$2ab$	$12a^2$	$-x^2 + 4x - 3$	$a(1 + x)$

- Algebra is important in the design and construction of buildings.

Mathematical terms 4

algebra
- A branch of mathematics where numbers are represented by symbols.

pronumeral
- A symbol used to represent a number.

like terms
- Are terms that have identical pronumeral parts.
  eg $6x - 15y + 3x$

collect like terms
- To simplify an algebraic expression containing many terms by addition and/or subtraction.
  eg $5x + 3 + 7x - 4$
  $= 12x - 1$

parentheses
- The name given to these grouping symbols: ( )

brackets
- The name given to these grouping symbols: [ ]

algebraic expression
- A group of terms and numbers that are joined by addition or subtraction signs.

denominator
- The bottom number of a fraction.

numerator
- The top number of a fraction.

cancel
- To simplify a fraction by dividing the numerator and denominator by any common factor.
  eg $\frac{21^{7}}{30_{10}} \begin{matrix}\div 3\\ \div 3\end{matrix}$ so $\frac{21}{30} = \frac{7}{10}$

factorise
- To write an expression as a product.
- The reverse of expanding.

expand
- To remove grouping symbols by multiplying each term inside the grouping symbols by the term outside.

Mathematical terms 4

1 Three darts are thrown and all land in the '20' sector. What are the possible total scores for the three darts if all darts can land on either the 20, double 20 or triple 20?
2 Three darts are thrown and all land in the 'x' sector. Write an algebraic expression for the possible total scores.
3 Three darts are thrown and all land in the same sector. The total score is 102. In what sector did the darts land?

diagnostic test

4

Diagnostic Test 4: | Algebraic Expressions

- Each part of this test has similar items that test a certain type of example.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
1 Simplify: **a** $5a + 2b - 3a + b$ **b** $5p^2 + 2p - 3p^2$ **c** $6ab - 4ba$ **d** $6a - 2x + 5 + x - 2a - 7$	4:01
2 Simplify: **a** $8 \times 7m$ **b** $5a \times 6b$ **c** $10y \times y$ **d** $-4n \times 2y$	4:02
3 Simplify: **a** $6a \div 2$ **b** $15xy \div 3x$ **c** $12ac \div 8ab$ **d** $-6x \div 18xy$	4:03
4 Simplify completely: **a** $\frac{3a}{5} + \frac{a}{5}$ **b** $\frac{5m}{7} - \frac{3m}{7}$ **c** $\frac{x}{6} + \frac{2x}{6}$ **d** $\frac{7p}{10} - \frac{3p}{10}$	4:04A
5 Simplify: **a** $\frac{a}{3} + \frac{a}{4}$ **b** $\frac{2m}{5} - \frac{m}{3}$ **c** $\frac{3x}{4} + \frac{x}{2}$ **d** $\frac{3n}{6} - \frac{n}{4}$	4:04A
6 Simplify completely: **a** $\frac{x}{3} \times \frac{y}{4}$ **b** $\frac{a}{5} \times \frac{a}{2}$ **c** $\frac{2a}{5} \times \frac{10}{a}$ **d** $\frac{ab}{2x} \times \frac{x}{b}$	4:04B
7 Simplify completely: **a** $\frac{m}{3} \div \frac{m}{4}$ **b** $\frac{a}{5} \div \frac{2}{a}$ **c** $\frac{3p}{4} \div \frac{p}{2}$ **d** $\frac{mn}{3} \div \frac{m}{n}$	4:04B
8 Expand: **a** $9(x + 7)$ **b** $6(5a - 2)$ **c** $p(p + 3)$ **d** $3a(5 - 2a)$	4:05
9 Expand: **a** $-3(x + 2)$ **b** $-2(m - 8)$ **c** $-5(3x + 4)$ **d** $-(7 - 2m)$	4:05
10 Simplify completely: **a** $2(a + 3) + 5$ **b** $x(2x - 7) + 3x^2$ **c** $9m - 2(1 - m)$ **d** $2(x + 5) + 4(x - 7)$	4:06
11 Factorise completely: **a** $5m + 10$ **b** $x^2 - 3x$ **c** $6ab + 15a$ **d** $-8y - 12$	4:07

Chapter 4 | Revision Assignment

assignment 4A

1 Simplify:

a $3x + 2x$ b $3x \times 2x$
c $7x - x$ d $x^2 + x^2$
e $12x \div 6$ f $12x \div 6x$
g $2x + 3y$ h $2x \times 3y$

2 Simplify:

a $3ab \times 2b$ b $12a^2b \div 6a$
c $3ab + 2ab$ d $3a^2 - a$
e $4x + 3y + 2x$ f $12 + 6x + 3$
g $9a - 3 - 2a + 4$ h $x^2 - 3x + 2x + 3x^2$

3 Expand:

a $3(x + 5)$ b $2(x - 7)$
c $-3(x + 2)$ d $-4(1 - 3x)$
e $a(a + 3)$ f $x(y + z)$
g $3p(p - q)$ h $5x(2x - 7)$

4 Factorise fully:

a $3a + 15$ b $6m + 9$
c $15 - 5y$ d $ax - 3x$
e $2x + 6xy$ f $4x^2 - 2x$
g $9ab - 6bc$ h $6x^2 - 9x + 3xy$

5 Expand and simplify:

a $3(x + 3) + x + 4$
b $2(2y + 5) + 3(2y + 5)$
c $a - 4 + 2(a + 5)$
d $4(a - 3) + 2(1 - a)$
e $3(2m - 6) - (m - 4)$
f $3x(x - 2) - 2(x - 2)$

6 Simplify:

a $\frac{a}{3} + \frac{a}{3}$ b $\frac{a}{5} + \frac{a}{4}$
c $\frac{7m}{10} + \frac{m}{10}$ d $\frac{3y}{4} + \frac{2y}{3}$
e $\frac{5d}{4} - \frac{3d}{4}$ f $\frac{x}{2} - \frac{x}{5}$
g $\frac{3a}{4} - \frac{a}{2}$ h $\frac{3m}{5} - \frac{3m}{20}$

7 Simplify:

a $\frac{a}{4} \times \frac{a}{3}$ b $\frac{3a}{4} \times \frac{2b}{5}$
c $\frac{2mn}{3} \times \frac{9}{2n}$ d $\frac{ab}{4} \times \frac{a}{3b}$

8 Simplify:

a $\frac{m}{2} \div \frac{m}{6}$ b $\frac{A}{5} \div \frac{2A}{3}$
c $\frac{2a}{3} \div \frac{a}{6}$ d $\frac{m}{3} \div \frac{m}{12n}$

9 Determine whether each of the following simplifications is correct by substituting numbers for the pronumerals. (Finding just one number for which the statement is not true means the simplification is incorrect.)

For example: $3(x + 2) = 3x + 2$
For $x = 1$, $3(1 + 2) = 3 \times 1 + 2$
$9 = 5$
Hence simplification is incorrect.

a $5(a + 4) = 5a + 4$
b $-3(a - 2) = -3a - 6$
c $4(p - 3) + 2(p + 4) = 6p - 4$
d $5(a + 1) - 3(a - 2) = 2a - 1$

10 Find the error in each of the following expansions and rewrite each correctly.

a $4(3a - 4) = 12a - 4$
b $-3(x - 2) = -3x - 6$
c $2(a + 3) - 3(a + 4)$
$= 2a + 6 - 3a - 12$
$= a - 6$
d $4(3a - 2) - 3(2a + 1)$
$= 12a - 2 - 6a - 3$
$= 6a - 5$

Technology Applications Drag and Drops

1 Addition and subtraction in algebra
2 Multiplying and dividing in algebra
3 Addition and subtraction of algebraic fractions
4 Multiplication and division of algebraic fractions
5 Grouping symbols
6 Simplifying expressions with grouping symbols
7 Factorising using common factors

Chapter Review Questions

assignment 4B

Chapter 4 | Working Mathematically

1 Use ID Card 4 on page xvi to give the mathematical term for:

a 1 b 2 c 3 d 4 e 5
f 6 g 7 h 8 i 9 j 11

2 a What geometric shape has inspired the design of this coffee cup?

b What would you estimate the capacity of the cup to be?

3 Diane and Garry married and had three children. Each child married and had three children. Assuming that no one has died, how many people are now in this extended family altogether?

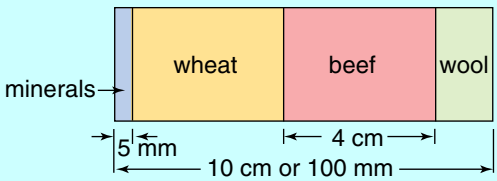
4 The numerals 1 to 10 are written on ten separate cards, one on each card.

a How many pairs of cards have a sum of 10?

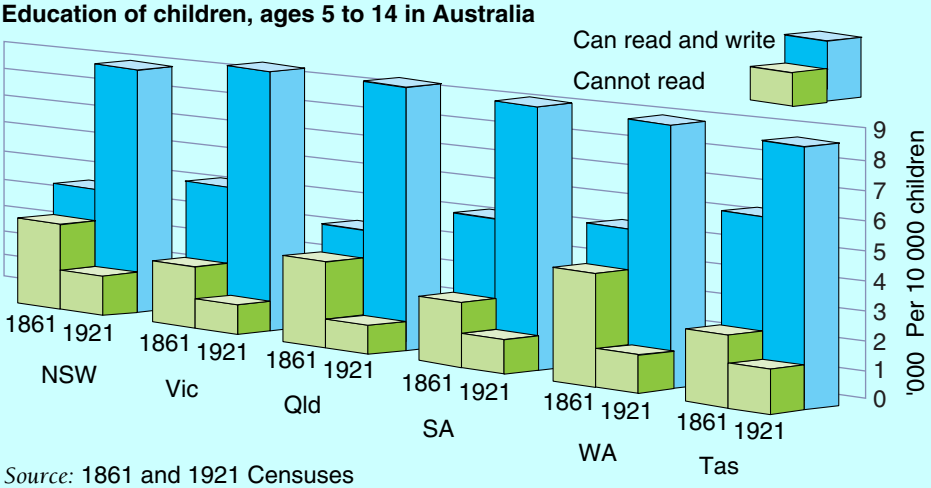
b How many groups of three cards are there that have a sum of 18?

5 A particular country's exports are shown in the bar graph below (reduced in size). Find what percentage of the country's exports are taken up by:

a beef b minerals.



6 **Education of children, ages 5 to 14 in Australia**



Source: 1861 and 1921 Censuses

a In 1861, which state had the greatest number per 10 000 children that could read and write? What percentage was this?

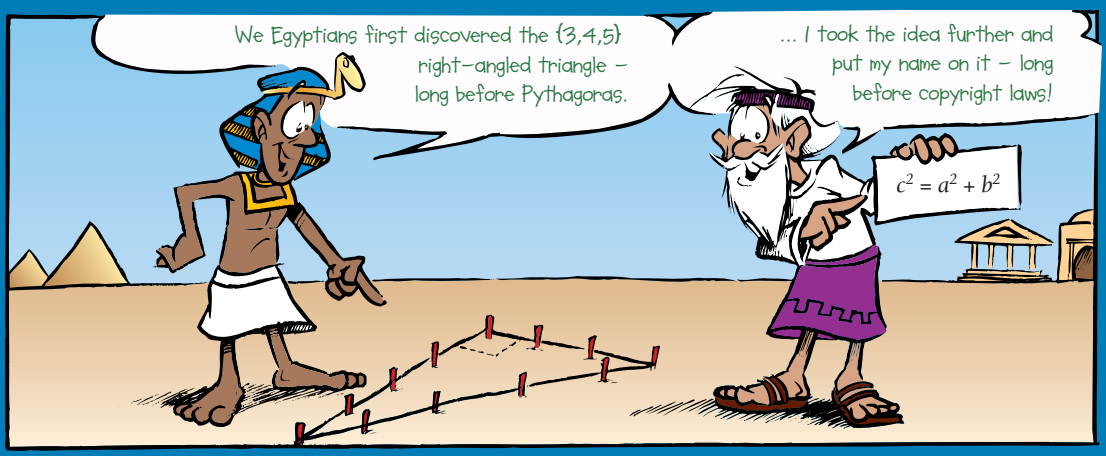
b In 1921, which state had the greatest percentage of children that could read and write? What percentage was this?

c Which state had 4000 per 10 000 children that could read and write in 1861? About how many in that state could not read in 1861?

d Consider Western Australia in 1861. Approximately what percentage could read and write? Approximately what percentage could not read? (To determine this, measure the height of this column and measure this height on the scale.)

5

Pythagoras' Theorem



Chapter Contents

Learning Outcomes

Students will be able to:

- Use Pythagoras' Theorem to calculate the third side of a right-angled triangle given the other two sides.
- Use Pythagoras' Theorem to solve problems using triangles.

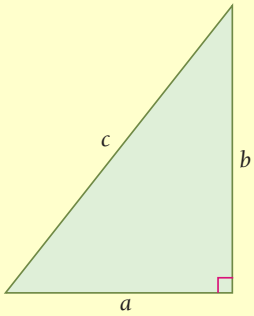
Areas of Interaction

Environment, Homo Faber, Approaches to Learning

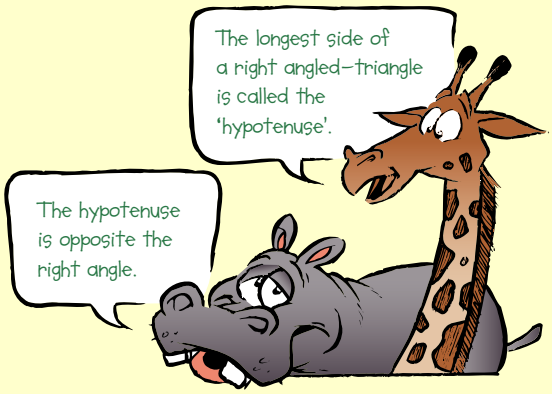
The Egyptians made use of the fact that a triangle that had sides 3 units, 4 units and 5 units in length was always a right-angled triangle. However, it was a religious group led by the Greek philosopher Pythagoras, around 550 BC, that discovered the relationship which exists between the sides of all right-angled triangles.

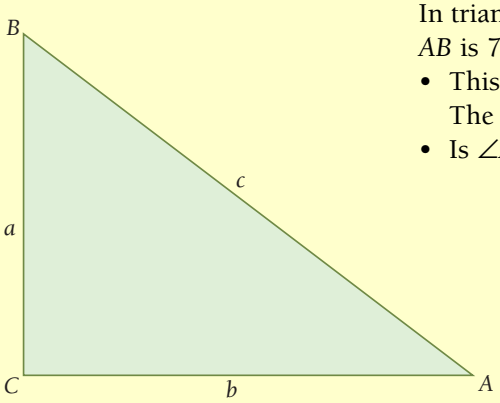
5:01 | Investigating Right-angled Triangles

As we investigate right-angled triangles we will call the side lengths a, b and c, where c will always stand for the length of the longest side. We call the longest side of a right-angled triangle the **hypotenuse**.



a	b	c
3 cm	4 cm	5 cm





In triangle ABC (also called ΔABC), $BC = 4{\cdot}5$ cm, $CA = 6$ cm and AB is $7{\cdot}5$ cm.

- This is also a {3, 4, 5} triangle if we let each unit be $1{\cdot}5$ cm. The side lengths are: $3 \times 1{\cdot}5$ cm, $4 \times 1{\cdot}5$ cm and $5 \times 1{\cdot}5$ cm.
- Is $\angle ACB$ a right angle? Which side is the hypotenuse?

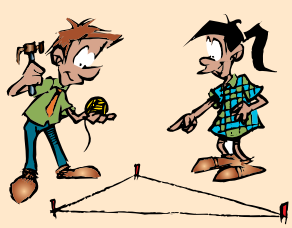
a	b	c
$4{\cdot}5$ cm	6 cm	$7{\cdot}5$ cm
$3 \times 1{\cdot}5$ cm	$4 \times 1{\cdot}5$ cm	$5 \times 1{\cdot}5$ cm

investigation
5:01A

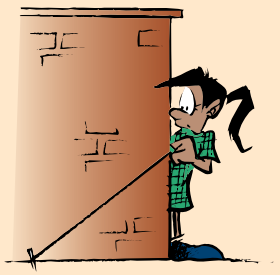
Investigation 5:01A | Right-angled triangles

Please use the Assessment Grid on page 94 to help you understand what is required for this Investigation.

1 Take three lengths of rope or string 3 m, 4 m and 5 m in length. Use them to form a triangle and measure the largest angle. Is it a right angle?



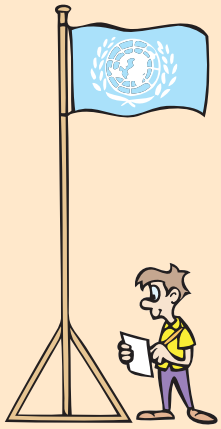
2 Measure 1·5 m up and 2 m along the bottom of a wall that stands on flat ground. How long is the distance between these two points? Use string to measure this distance. Is the triangle a {3, 4, 5} triangle?



3 Draw or find some right-angled triangles in the school grounds. Make the sides any length you wish. For each triangle, measure the side lengths a, b and c (where c is the longest side) and enter these in a table like the one below. Make sure you include the three already in the table.

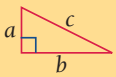
4 Can you recognise a pattern in the table? If so, describe it in a sentence.

a	3	6	5			
b	4	8	12			
c						



Exercise 5:01

Foundation Worksheet 5:01
Pythagoras' Theorem A

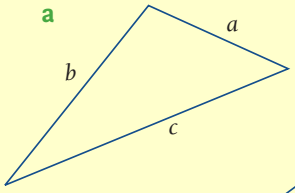


Measure each side of this triangle. Work out a^2, b^2 and c^2. Does $c^2 = a^2 + b^2$?

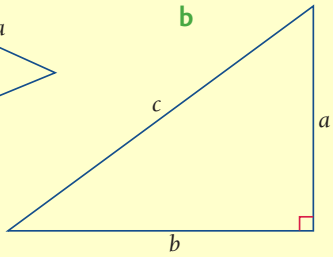
1 For each of the triangles drawn below, measure the sides to the nearest millimetre and complete the table.

a	b	c	a^2	b^2	c^2	$a^2 + b^2$

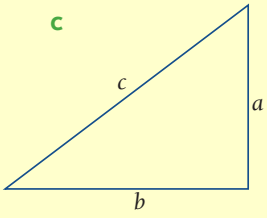
a



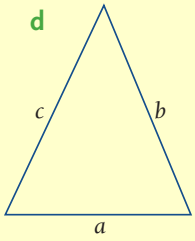
b



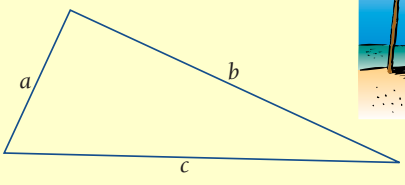
c

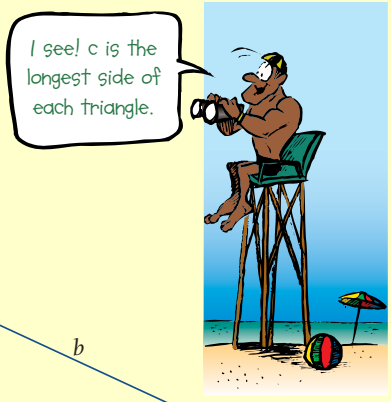


d



e





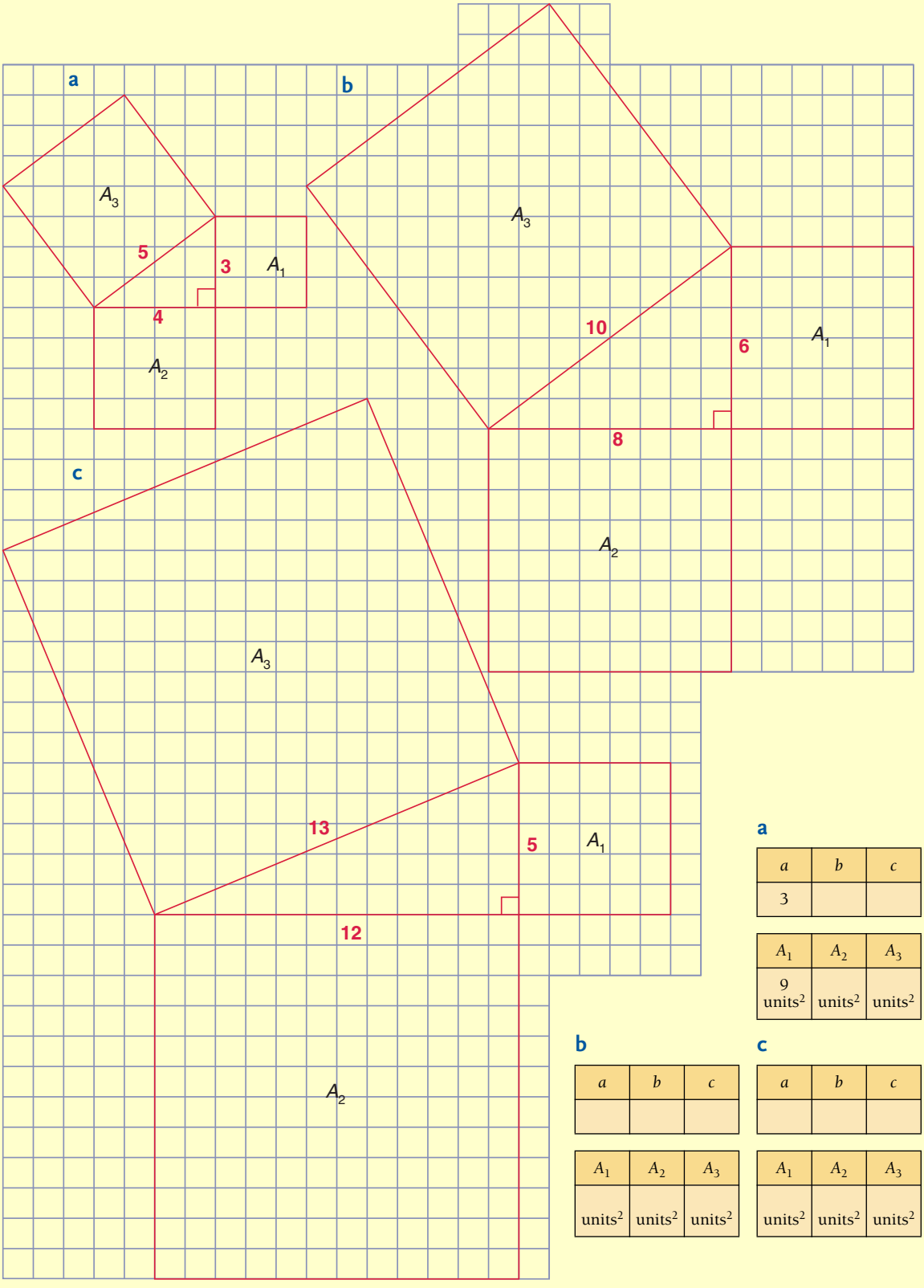
2 In which of the triangles in Question 1 does $c^2 = a^2 + b^2$? What type of triangles are they?

Assessment Grid for Investigation 5:01A | Right-angled triangles

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	Following instructions proved difficult as did applying skills to routine situations.	1	
			2	
	b	An organised approach has been used and the student has made a contribution to the group.	3	
			4	
	c	An organised approach has been used and the table has been extended to help describe the patterns found.	5	
			6	
	d	The correct pattern has been recognised and justified clearly using words and/or symbols.	7	
			8	
Criterion C Communication	a	No working out is shown and no contribution has been made to the group.	1	
			2	
	b	Working out is shown and demonstrates a clear progression through the investigation.	3	
			4	
	c	Presentation is good and the final question is answered clearly using words and/or symbols.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt has been made to explain the steps undertaken in the investigations.	1	
			2	
	b	An explanation of the steps undertaken is given and results have been checked for reasonableness.	3	
			4	
	c	A detailed explanation has been given for questions 3 and 4, the accuracy of the results has been justified and extended to make further predictions.	5	
			6	

3 A square has been drawn on each of the sides of these right-angled triangles. Complete the tables for each triangle. Can you find a rule that links a, b and c?



a

a	b	c
3		

A_1	A_2	A_3
9 units^2	units^2	units^2

b

a	b	c

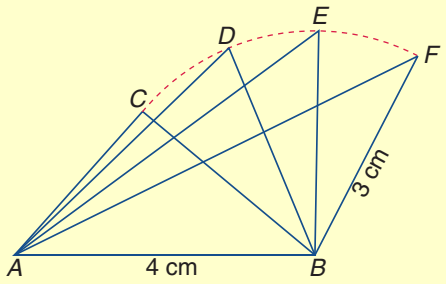
A_1	A_2	A_3
units^2	units^2	units^2

c

a	b	c

A_1	A_2	A_3
units^2	units^2	units^2

4



A sequence of triangles has been formed with a common base AB by gradually increasing the size of $\angle B$ while keeping the side BC constant in length. The vertex C moves along a circle to the positions D, E and F.

a Complete the table for the triangles named.

Triangle	AB^2	*(right side)*^{2}	*(left side)*^{2}
ΔABC			
ΔABD			
ΔABE			
ΔABF			

b For what triangle does AB^2 + (right side)^2 = (left side)^2?

c What is the size of $\angle B$ for the triangle in **b**?

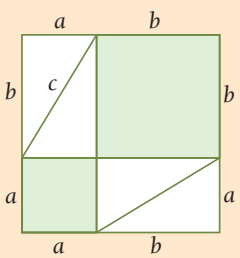
While answering the questions above you should have discovered, like the Pythagoreans, the relationship between the sides of a right-angled triangle.

investigation

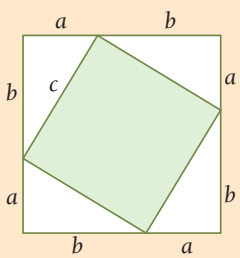
5:01B

Investigation 5:01B | Pythagoras' Theorem

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.



I



II

Construct two identical squares as shown in the diagram. Use any value you like for a and b (eg $a = 6$ cm, $b = 8$ cm). The coloured squares are built on the three sides of a right-angled triangle.

a Is the area of square I equal to the area of square II?

b Do the triangles in square I take up the same areas as the triangles in square II?

c If the triangles are removed from squares I and II, are the remaining parts equal in area?

d If the triangles are removed from square I, what is the area of the two remaining squares?

e If the triangles are removed from square II, what is the area of the remaining square?

f Does $c^2 = a^2 + b^2$?

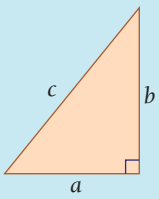
g If $a = 30$ and $b = 40$, what is the value of c?

Complete: In any right-angled triangle the square on the . . . . . side is equal to the . . . . . of the squares on the other two . . . . . .

Assessment Grid for Investigation 5:01B | Pythagoras' Theorem

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	The instructions have not been followed and the student demonstates difficulty applying skills.	1	
			2	
	b	The instructions have generally been followed correctly and an attempt has been made to explain the results.	3	
			4	
	c	The instructions have been followed correctly and the theorem has been correctly applied with some conclusions.	5	
			6	
	d	The theorem has been explicitly recognised and clearly justified, with further applications shown through the final question.	7	
			8	
Criterion C Communication	a	No working out is shown. Presentation is poor and there is little or no use of symbols and/or diagrams.	1	
			2	
	b	Working out is shown. Presentation is good with some structure to the work.	3	
			4	
	c	Results are well communicated using correct symbols and/or diagrams. Work is well structured.	5	
			6	
Criterion D Reflection in Mathematics	a	An attempt has been made to explain the results at each step.	1	
			2	
	b	The results at each step are explained and have been checked for reasonableness.	3	
			4	
	c	A detailed understanding of the purpose of the investigation has been shown and the significance of the results has been explained and extended in alternate contexts.	5	
			6	



Pythagoras' Theorem

- **In any right-angled triangle the square of the longest side is equal to the sum of the squares of the two shorter sides.**
  For the triangle shown, the theorem can be written as:

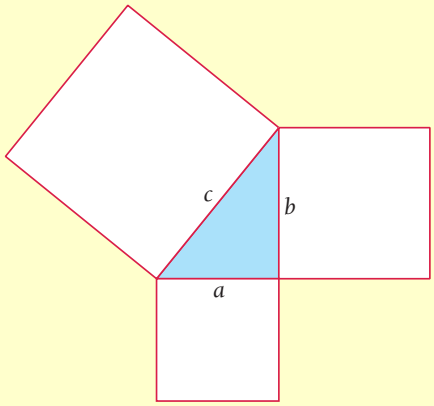
$$c^2 = a^2 + b^2$$

Converse

- **If a triangle obeys the equation above, it must be right-angled.**

Class activity

The Pythagorean Theorem can also be illustrated by the following geometric method.
Construct a right-angled triangle and construct squares on each side of the triangle as shown in the diagram.



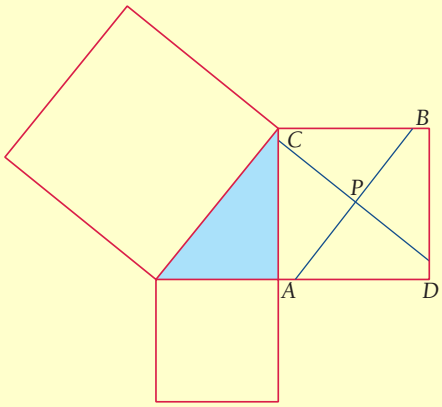
We have seen that for all right-angled triangles

$$c^2 = a^2 + b^2.$$

Since the areas of the squares constructed on the sides are a^2, b^2 and c^2 respectively, the theorem can be restated as:

The square on the hypotenuse is equal to the sum of the squares on the two smaller sides.

The larger square can be made from the two smaller squares by dissecting the middle square and rearranging these pieces with the small square. The dissection is shown in the following diagram. Copy the dissection and find how the pieces can be arranged to form the large square.



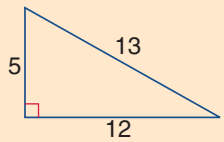
- P is the centre of the square.
- AB is drawn through P, parallel to the hypotenuse.
- CD is drawn through P, perpendicular to the hypotenuse.

5:02 | Using Pythagoras' Theorem: Calculating the Hypotenuse

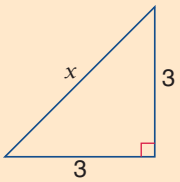
prep quiz 5:02

For the triangles given in **1** to **3**, select the correct statements from **A**, **B** or **C**.

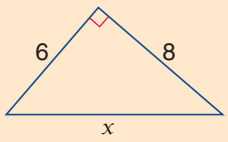
1 **A** $5^2 + 12^2 = 13^2$
B $(5 + 12)^2 = 13^2$
C $13^2 + 5^2 = 12^2$



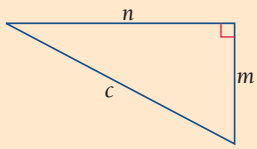
2 **A** $3 + 3 = x$
B $3^2 + 3^2 = x^2$
C $(3 + 3)^2 = x^2$



3 **A** $8^2 = 6^2 + x^2$
B $x^2 = 6^2 + 8^2$
C $x = 6 + 8$



4 Write Pythagoras' Theorem for the triangle given.



5 If $4^2 = 16$, then $\sqrt{16} = \ldots$.?

6 If $25^2 = 625$, then $\sqrt{625} = \ldots$.?

7 If $\sqrt{25} = 5$, then $5^2 = \ldots$.?

8 $3^2 + 4^2 = \ldots$.?

9 $\sqrt{36} = \ldots$.?

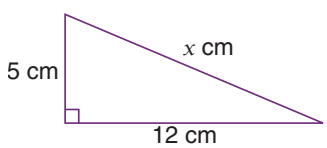
10 Can 2, 3 and 4 be the side lengths of a right-angled triangle?

- Pythagoras' Theorem is used to calculate the lengths of the sides of right-angled triangles. To calculate one side, the other two sides must be known.

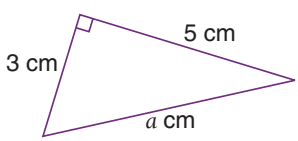
worked examples

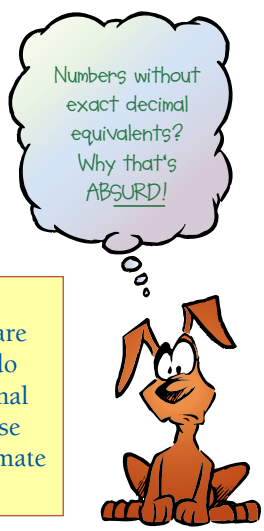
Calculate the length of the hypotenuse in each of the following triangles.

1



2





Solutions

1 $x^2 = 5^2 + 12^2$
$= 25 + 144$
$= 169$
$x = \sqrt{169}$
$= 13$
Hypotenuse is 13 cm.

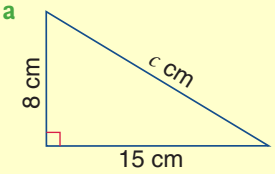
2 $a^2 = 3^2 + 5^2$
$= 9 + 25$
$= 34$
$a = \sqrt{34}$
$a \doteq 5{\cdot}8$ (to 1 dec. pl.)
Hypotenuse is 5·8 cm.

Important Notice
Numbers like $\sqrt{34}$ are called surds. Surds do not have exact decimal equivalents, so we use decimals to approximate their value.

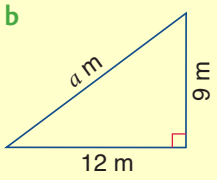
Exercise 5:02A (No calculator is needed for this exercise)

1 Use Pythagoras' Theorem to write an equation that shows how the sides of each triangle are related to each other.

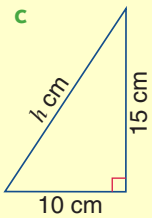
a



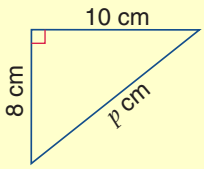
b



c



d



2 Complete the following table.

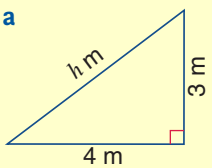
Number	3	4	5	6	7	8	9	10	12	13	15	16	17	18	20	24	25	26	30
Square	9	16	25																

3 Use the table in Question **2** to find:

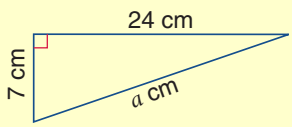
a $\sqrt{25}$ **b** $\sqrt{100}$ **c** $\sqrt{169}$ **d** $\sqrt{225}$ **e** $\sqrt{900}$ **f** $\sqrt{625}$ **g** $\sqrt{289}$

4 Find the length of the hypotenuse in each of the following. (Use the table in Question **2**.)

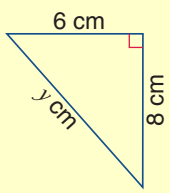
a



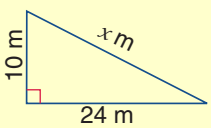
b



c



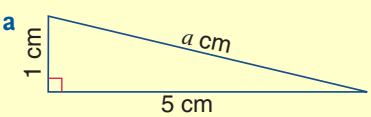
d



5 Find the length of the hypotenuse in each of the following. (Use the table below to evaluate the surds correct to 1 decimal place.)

Square root	$\sqrt{5}$	$\sqrt{8}$	$\sqrt{10}$	$\sqrt{13}$	$\sqrt{17}$	$\sqrt{18}$	$\sqrt{20}$	$\sqrt{26}$	$\sqrt{29}$	$\sqrt{32}$	$\sqrt{34}$
Approximate value	2·2	2·8	3·2	3·6	4·1	4·2	4·5	5·1	5·4	5·7	5·8

a



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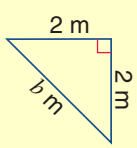

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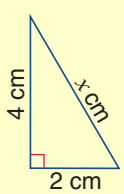

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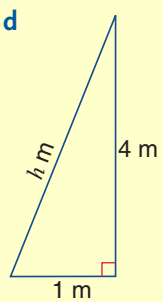

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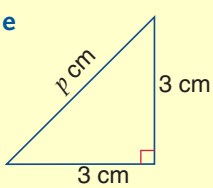

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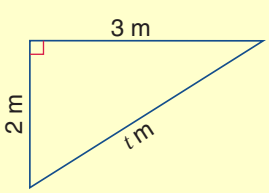

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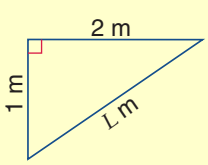
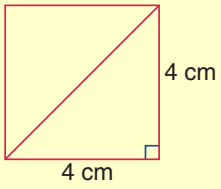


6 Use Pythagoras' Theorem and the tables in Questions **2** and **5** to find the length of the diagonal and the perimeter of each square or rectangle.

a



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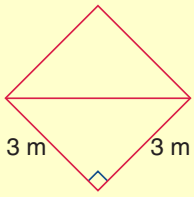

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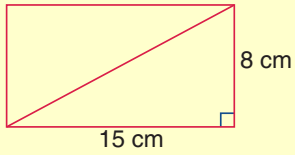
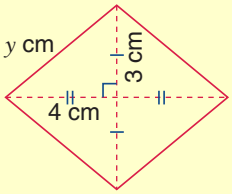


7 Use Pythagoras' Theorem to find the value of pronumerals in each of the following. (You will need to use the tables in Questions **2** and **5**.)

a



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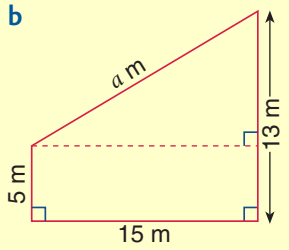

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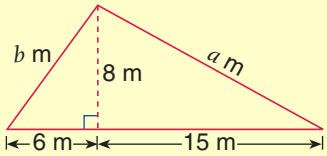
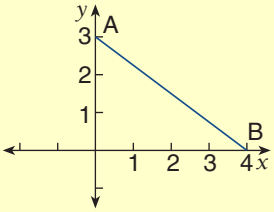
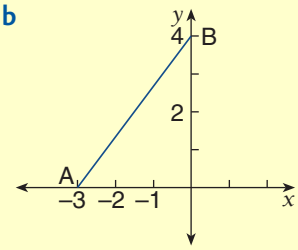


8 Find the length of the interval AB in each of the following.

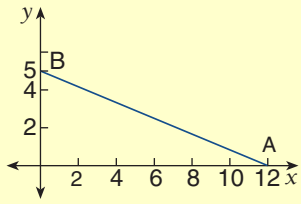
a



b



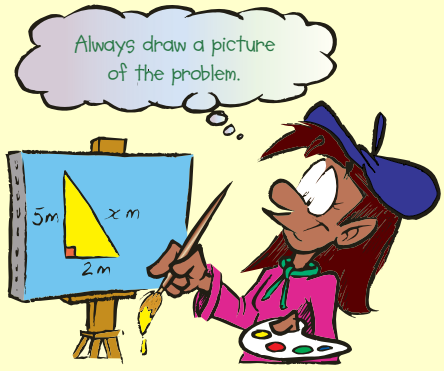
c



9 **a** The foot of a ladder is 1 m from a vertical wall. If the ladder reaches 5 m up the wall, find the length of the ladder, to the nearest centimetre.

b Two roads meet at right angles. One person walks 6 km along one road while another person walks 8 km along the other road. How far apart are they?

c A radio mast is supported by three wires. Each wire is attached to the mast 5 m from the bottom. The wires meet the ground at a point 2 m from the bottom of the mast. Find the length of each wire to the nearest tenth of a metre.



Exercise 5:02B (Calculators are needed)

Foundation Worksheet 5:02

Pythagoras' Theorem

1 

Here $c^2 = 2^2 + 3^2$
$= 13$
$\therefore c = \sqrt{13}$
$= \ldots\ldots$

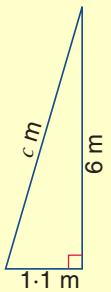
1 Use the calculator to simplify:

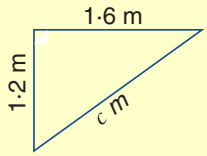
a 16^2 **b** $2{\cdot}5^2$ **c** $6{\cdot}8^2$

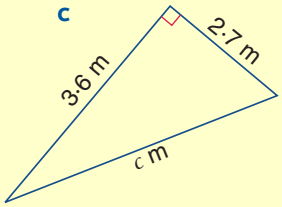
d $\sqrt{289}$ **e** $\sqrt{1681}$ **f** $\sqrt{33{\cdot}64}$

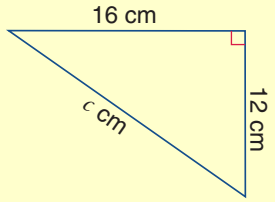
g $\sqrt{96}$ (to 1 dec. pl.) **h** $\sqrt{148}$ (to 2 dec. pl.)

2 Use Pythagoras' Theorem to calculate the length of the hypotenuse in each of the following.

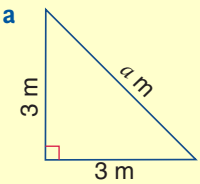
a 

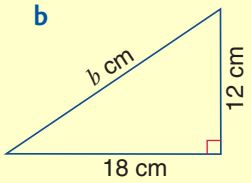
b 

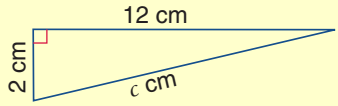
c 

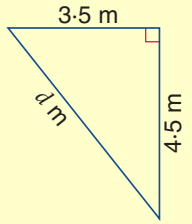
d 

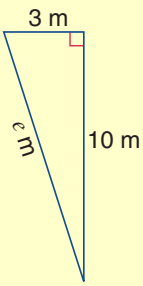
3 Find the value of the pronumeral in each of the following, correct to 1 decimal place.

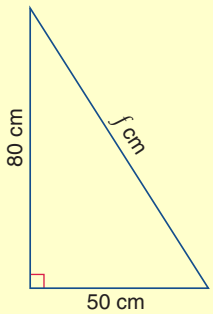
a 

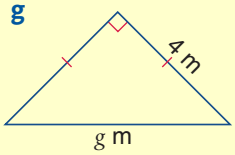
b 

c 

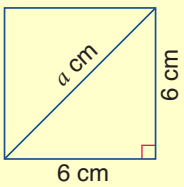
d 

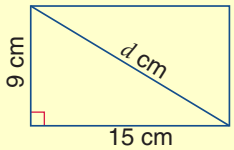
e 

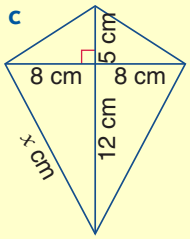
f 

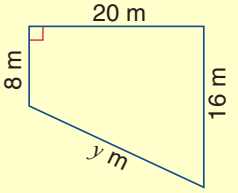
g 

4 Find the value of the pronumeral in each of the following. Give answers correct to one decimal place where necessary.

a 

b 

c 

d 

e Find the perimeter of the kite in **c**.

Investigation 5:02 | Pythagorean triads

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

A **Pythagorean triad** is a set of three numbers that obey Pythagoras' Theorem.

If $5^2 + 12^2 = 13^2$, then {5, 12, 13} is a Pythagorean triad.

1 Use algebra to show that $3x$, $4x$, $5x$ is a Pythagorean triad.

Use this result to complete the table below.

a	b	c
3	4	5
6	8	. . .
9	12	. . .
. . .	16	20
. . .	36	45
3·6	. . .	6

2 This table shows a special set of Pythagorean triads.

a Can you complete the next three lines of the table?

b The numbers in the table can be found using the formula $2n + 1$, $2n^2 + 2n$, $2n^2 + 2n + 1$.

Use this formula to find the triads starting with:

i 27 **ii** 51 **iii** 101

a	b	c
3	4	5
5	12	13
7	24	25
9	40	41
11	. . .	. . .
13	. . .	. . .
15	. . .	. . .

There are many Pythagorean triads but often they are hard to figure out. Of course, you can make more triads by simply multiplying an existing one by a constant, such as:

$$\{3, 4, 5\} \times 2 \Rightarrow \{6, 8, 10\}$$

The formula below, however, turns out triads by substituting odd numbers, starting at 3, for n.

$\frac{n^2 \pm 1}{2}$
'±' means 'plus or minus'

Formula: n, $n^2 - 1$, $n^2 + 1$.

If we put in 3 we get $\frac{3^2 \pm 1}{2}$ → $\frac{3^2 + 1}{2} = 5$ and $\frac{3^2 - 1}{2} = 4$

∴ the triad is {3, 4, 5}.

If we put in 11 we get $\frac{11^2 \pm 1}{2}$ → $\frac{11^2 + 1}{2} = 61$ and $\frac{11^2 - 1}{2} = 60$

∴ the triad is {11, 60, 61}.

Check!
$11^2 + 60^2 = 121 + 3600$
$= 3721$
Now $61^2 = 3721$
So $11^2 + 60^2 = 61^2$

3 Substitute further odd numbers for n and turn out some more triads.
Check that they actually work by testing them with Pythagoras' Theorem.

4 Make a list of Pythagorean triads that are made up of three numbers which are all less than 100.

Assessment Grid for Investigation 5:02 | Pythagorean triads

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	An organised approach has not been used and the student had a problem applying skills to routine tasks.	1	
			2	
	b	An organised approach has been used and an attempt has been made to follow patterns and complete the tables.	3	
			4	
	c	An organised approach has been used and the table used to help describe the patterns found.	5	
			6	
	d	The correct pattern has been recognised and justified clearly using completed tables and correct responses to questions 3 and 4.	7	
			8	
Criterion C Communication	a	No working out is shown and the presentation is poor with little or no use of symbols and/or diagrams.	1	
			2	
	b	Working out is shown and the student has attempted to interpret the results.	3	
			4	
	c	Working out and presentation are good and results are well interpreted. Clear statements have been made using words and symbols.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt has been made to explain the processes involved and to check the results.	1	
			2	
	b	The processes used to complete the tables are explained and results have been checked for reasonableness.	3	
			4	
	c	A detailed explanation has been given for questions 3 and 4, demonstrating a sound ability to generate further triads.	5	
			6	

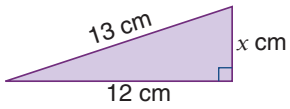
5:03 | Using Pythagoras' Theorem: Calculating One of the Short Sides

Pythagoras' Theorem can also be used to calculate the length of either short side.
The examples below show how this is done.

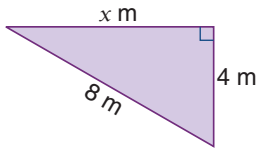
worked examples

Find the value of x in each of the following.

1



2



Solutions

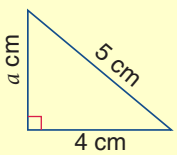
1 $13^2 = 12^2 + x^2$
$169 = 144 + x^2$
$25 = x^2$
$\therefore\ x = \sqrt{25}$
$= 5$

2 $8^2 = x^2 + 4^2$
$64 = x^2 + 16$
$48 = x^2$
$\sqrt{48} = x$
$x = \sqrt{48}$
$\doteqdot 6{\cdot}93$ m
(correct to 2 dec. pl.)

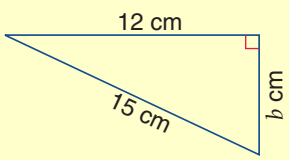
Exercise 5:03A (No calculator is needed for this exercise)

1 Use Pythagoras' Theorem to write an equation that shows how the sides of each triangle are related to each other.

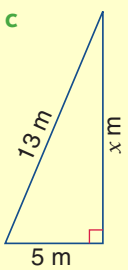
a



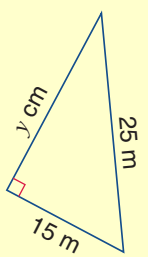
b



c



d



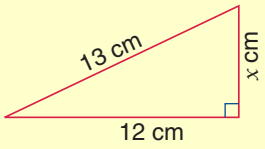
2 Use the following table to answer the questions below.

Number	3	4	5	6	7	8	9	10	12	13	15	16	17	18	20	24	25	26	30
Square	9	16	25	36	49	64	81	100	144	169	225	256	289	324	400	576	625	676	900

a $\sqrt{9}$ **b** $\sqrt{144}$ **c** $\sqrt{400}$ **d** $\sqrt{324}$ **e** $\sqrt{64}$ **f** $\sqrt{81}$

3 Complete the working for each of the following. Use the table from Question **2** in all exercises.

a



$13^2 = 12^2 + x^2$

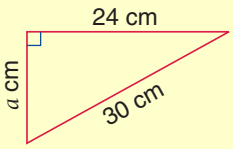
$169 = 144 + x^2$

$x^2 = \ldots\ldots\ldots$

$x = \sqrt{\ldots\ldots\ldots}$

$x = \ldots\ldots\ldots$

b



$30^2 = 24^2 + a^2$

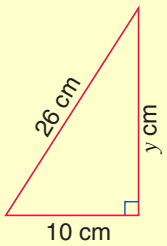
$900 = 576 + a^2$

$a^2 = \ldots\ldots\ldots$

$a = \sqrt{\ldots\ldots\ldots}$

$a = \ldots\ldots\ldots$

c



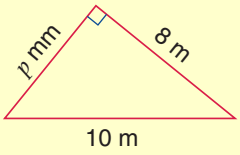
$26^2 = y^2 + 10^2$

$\ldots\ldots = y^2 + \ldots\ldots$

$y^2 = \ldots\ldots$

$y = \ldots\ldots$

d



$10^2 = p^2 + 8^2$

$\ldots\ldots = p^2 + \ldots\ldots$

$p^2 = \ldots\ldots$

$p = \ldots\ldots$

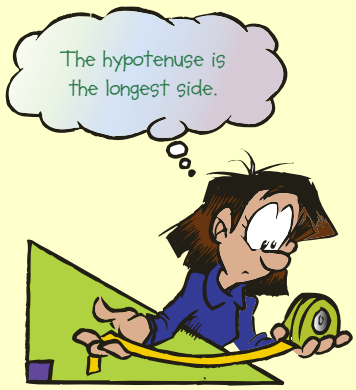
4 Using the setting out shown in Question **3**, find the values of the pronumerals in Question **1**.

5 **a** The hypotenuse of a right-angled triangle is 20 cm long. If one of the short sides is 12 cm long, what is the length of the other side?

b The longest side of a right-angled triangle is 17 cm. If one of the other sides is 8 cm in length, find the length of the third side.

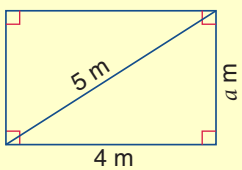
c The two longer sides of a right-angled triangle are 26 cm and 24 cm in length. How long is the shortest side?

d In a right-angled triangle the longest side is 15 cm long. The shortest side is 9 cm long. How long is the third side?

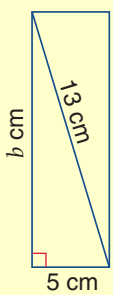


6 Find the value of the pronumeral in each of the following.

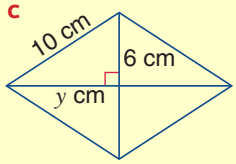
a



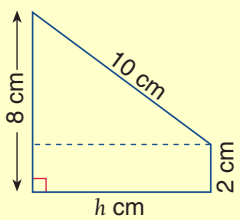
b



c



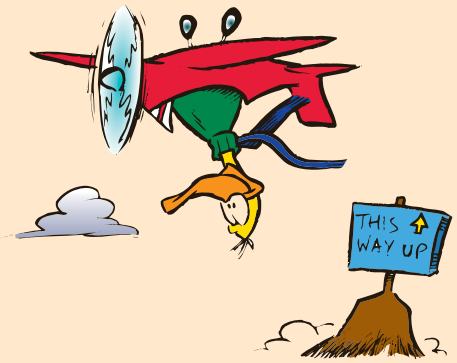
d



e Find the perimeter of the rectangle in **b**.

fun spot
5:03

Fun Spot 5:03 | What happens when ducks fly upside down?



Answer each question and put the letter for that question in the box above the correct answer.

The Pythagorean triad {3, 4, 5} has been multiplied by a constant to produce these triads. Find the missing number in each triad.

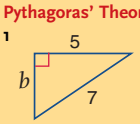
Q 6, 8, ...	**A** . . . ., 12, 15	**T** 12, ..., 20
C 2·1, 2·8, ...	**P** 1·8, ..., 3	**E** . . . ., 2, 2·5
U 33, ..., 55	**Y** 42, 56, ...	**U** 27, ..., 45
H . . . ., 8·8, 11	**K** 3·75, ..., 6·25	

16	6·6	1·5	70	10	36	9	3·5	5	44	2·4

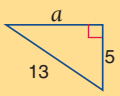
Exercise 5:03B (Calculators are needed)

Foundation Worksheet 5:03

Pythagoras' Theorem

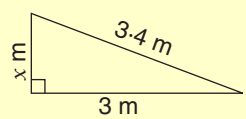
1 

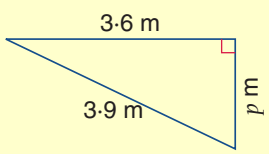
$b^2 + 5^2 = 7^2$
$b^2 + 25 = 49$
$\therefore b^2 = \ldots.$
$\therefore b = \sqrt{\ldots.}$
$= \ldots.$

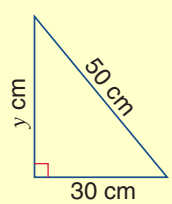
2 

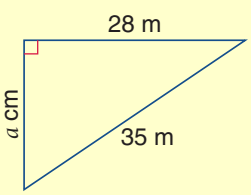
$a^2 + 5^2 = 13^2$
$a^2 + \ldots. = \ldots.$
$\therefore a^2 = \ldots.$

1 Find the value of the pronumeral in each of the following.

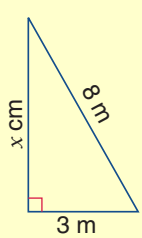
a 

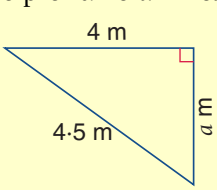
b 

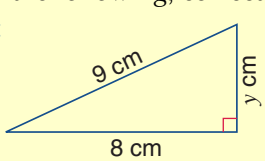
c 

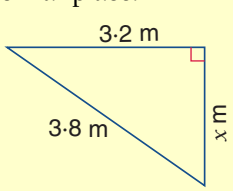
d 

2 Find the value of the pronumeral in each of the following, correct to 1 decimal place.

a 

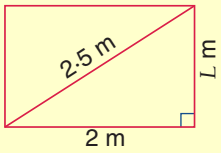
b 

c 

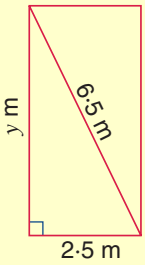
d 

3 Find the value of the pronumeral in each of the following. Give answers correct to 1 decimal place where necessary.

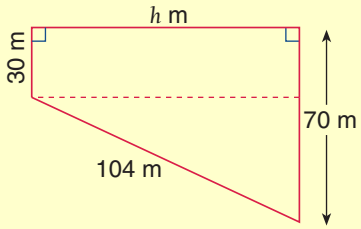
a



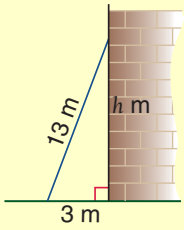
b



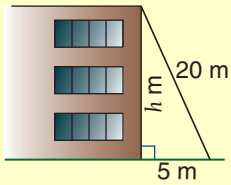
c



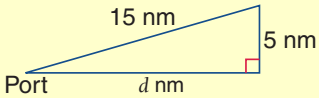
4 **a** A ladder that is 13 m long is placed so that its foot is 3 m from the base of a vertical wall. How far does the ladder reach up the wall?



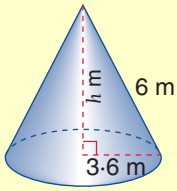
b A piece of wire 20 m long has one end attached to the top of a vertical building. The other end is fixed to the ground at a distance of 5 m from the building. Find the height of the building, correct to 2 decimal places.



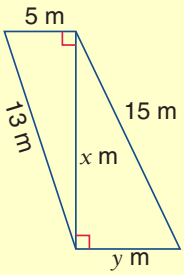
c A ship is 15 nautical miles (nm) from port. After sailing 5 nm due south, it is directly east of the port. How far is the ship then from the port?



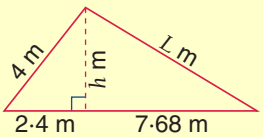
5 **a** The diagram shows a cone with a radius of 3·6 m and a slant height of 6 m. Calculate the height, h, of the cone.



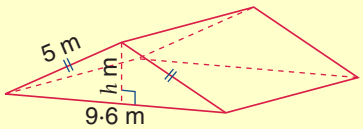
b Calculate x and y.



c Find the values of h and L.



d Find the value of h.



6 **a** The perimeter of a square is 16 cm. Find the length of its diagonal, correct to 1 decimal place.

b A rectangle's length is twice its breadth. Find the length of its diagonal (to 1 dec. pl.) if its perimeter is 90 cm.

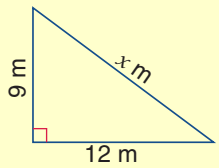
5:04 | Miscellaneous Exercises Involving Pythagoras' Theorem

The following exercise consists of a mixture of questions in which Pythagoras' Theorem will need to be used. In some questions you will be asked to calculate the length of the hypotenuse, while in others you will need to calculate the length of the shorter side.

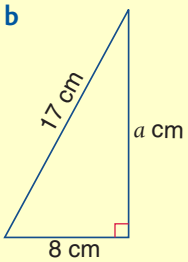
Exercise 5:04 (Calculators are needed)

1 Find the value of the pronumeral in each of the following.

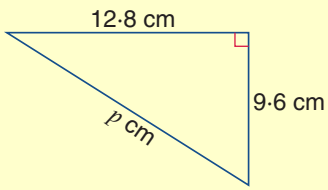
a



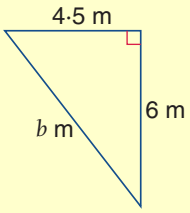
b



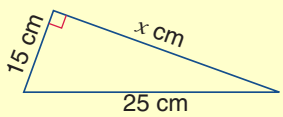
c



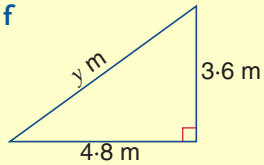
d



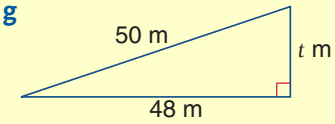
e



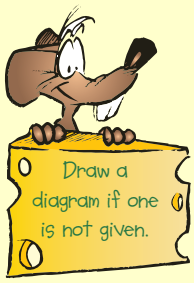
f



g

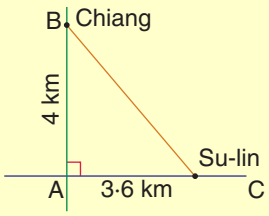


2 **a** Fiona measured the two shorter sides of a right-angled triangle and found them to be 16 cm and 30 cm. She then measured the longest side. What should its length have been?



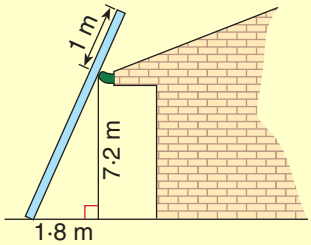
b Gino had an extension ladder that was 12 m long. If he placed the ladder against a wall so that the foot was 2 m from the building, how far up the wall would it reach, to the nearest centimetre?

c



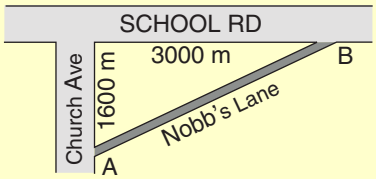
Chiang and Su-lin start at A. Chiang walks towards B while Su-lin walks towards C. How far apart are they when Chiang has walked 4 km and Su-lin 3·6 km? Answer correct to 1 decimal place.

d Larry Bagnell needs to repair the guttering on his house. From the diagram calculate the length of the smallest ladder he needs to hire, if the ladder has to extend at least 1 m past the gutter.



e Angelika is building a barbecue on a rectangular concrete slab. The slab is to be 2·4 m long and 1·8 m wide. What should the length of the diagonal be?

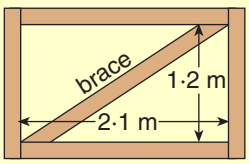
f



School Rd and Church Ave meet at right angles as shown in the diagram. Peter Perfect wants to go from A to B. What distance does he save by taking the shortcut down Nobb's Lane?

g Is a triangle with sides 3·6 m, 10·5 m and 11·1 m in length right-angled?

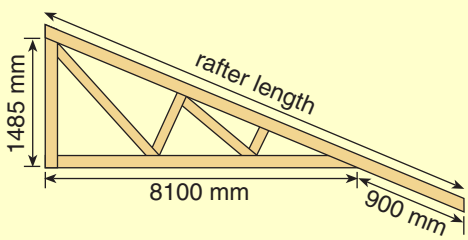
h



The diagram shows a gate. Calculate the length of timber that would need to be bought to make the brace.

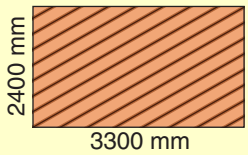
i Two roads intersect at right angles. Two drivers, Bill and Bev, leave the intersection at the same time. Bill drives down one road at an average speed of 60 km/h while Bev drives down the other road at an average speed of 80 km/h. Find how far each person has driven in half an hour and how far apart they are at that time.

j

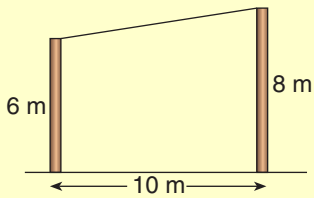


Part of a roof truss is shown in the diagram. Calculate the length of the rafter to the nearest ten millimetres.

k Jill and John decide to cover their bathroom wall with cedar boards as shown in the diagram. What is the longest length board that needs to be bought so that there would be no joins?

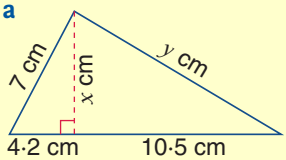


l Two posts are 6 m tall and 8 m tall. If these posts are 10 m apart on level ground and a string is stretched from the top of one post to the top of the other, find the length of the string, to the nearest centimetre.

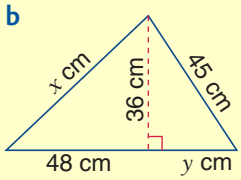


3 Use Pythagoras' Theorem to find the values of the pronumerals in each of the following.

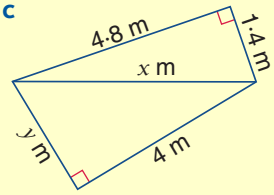
a



b



c



Start with a triangle in which two sides are known.

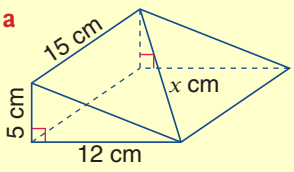
4 **a** Find the perpendicular distance from the corner of the triangle to the opposite side if the side is 25 cm long.

b Find the length of the diagonal if the length of the window is 48 cm and its breadth is 35 cm.

c The diameter of the cone in this hat is 18 cm. Its slant height, s cm, is 21 cm. What is the height of the hat?

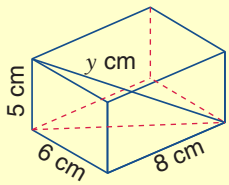
d The legs of this ladder are of equal length and are 1 metre apart. The slant height of the ladder is 2·4 metres. How high above the ground does the ladder reach?

5 **a**



The figure is a triangular prism, Find the value of x, correct to 2 decimal places.

b

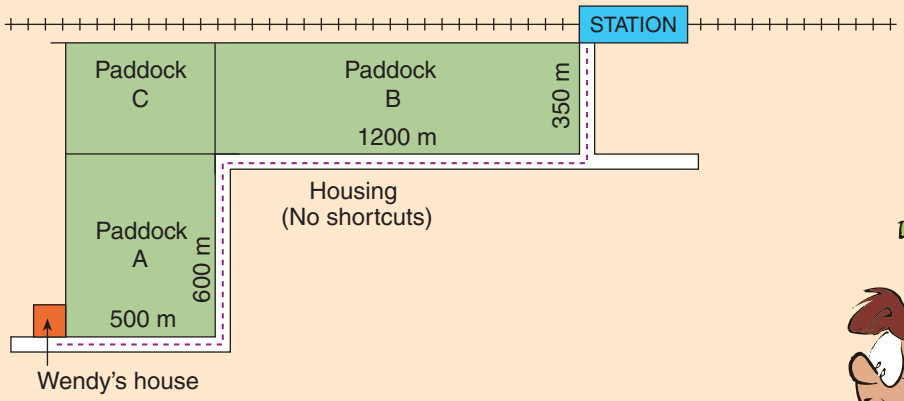


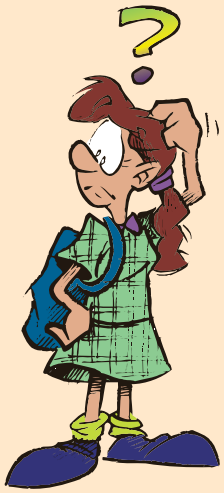
The figure is a rectangular prism. Find the value of y, correct to 2 decimal places.

Investigation 5:04 | Pythagoras and speed

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Wendy usually walks to the railway station along roads shown in the diagram below.





Recently, after sleeping in, she needed to get to the station in a hurry and decided to take short cuts through the paddocks. Wendy can run at 10 km/h and walk at 5 km/h. If she has only 18 minutes before the train leaves, should she:

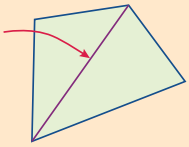
i walk (arriving on time and unflustered)?
ii run (because she will be late otherwise)?
iii walk part of the way and run the rest?
iv change her plans (as she just can't make it)?

Explain your answer in sentences.

Mathematical terms 5

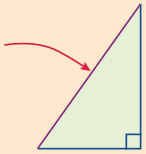
diagonal

- A straight line that crosses a polygon to join two of its vertices.



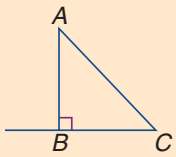
hypotenuse

- The longest side of a right-angled triangle.



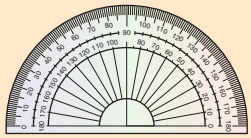
perpendicular

- Meeting at right angles. eg AB is perpendicular to BC, ie $AB \perp BC$



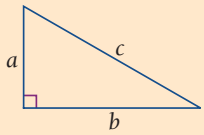
protractor

- A geometrical instrument used to measure angles.



Pythagoras' theorem

- In any right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.



$c^2 = a^2 + b^2$

Assessment Grid for Investigation 5:04 | Pythagoras and speed

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	The distance Wendy walks along the road has been found.	1	
			2	
	b	The short cuts have been found, with some mistakes.	3	
			4	
	c	Correct short cuts have been found and each of the distances Wendy could travel has been examined, with some consideration of time and speed factors.	5	
			6	
	d	The optimal solutions for distance and speed have been found and justified clearly with full working.	7	
			8	
Criterion C Communication	a	Working out is hard to follow and no explanations are given.	1	
			2	
	b	Working is well set out, some explanations are given and diagrams have been used.	3	
			4	
	c	What is done is communicated well and working out is well structured.	5	
			6	
Criterion D Reflection in Mathematics	a	Some explanations are given.	1	
			2	
	b	Methods are justified and results have been checked to ensure that they are reasonable.	3	
			4	
	c	The methods and results are fully explained and an alternative approach has been explored and compared.	5	
			6	

diagnostic test

5

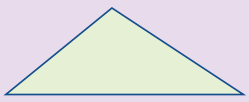
Diagnostic Test 5: | Investigating Pythagoras' Theorem

- Each section of the test has similar items that test a certain type of example.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

Section

1 State whether the given triangle is right-angled. — 5:01

a

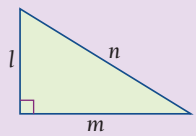


b

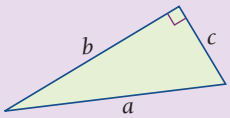
c

2 In each of the following, list the sides in order of length from shortest to longest. — 5:01

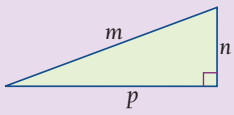
a



b

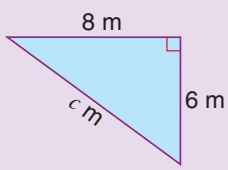


c

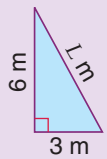


3 Use Pythagoras' Theorem to write an equation that shows how the sides of each triangle are related to each other. — 5:02

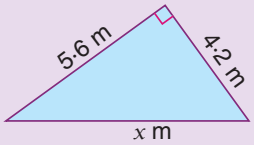
a



b



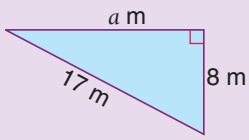
c



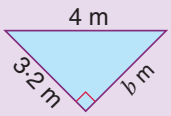
4 Find the length of the unknown side in each of the triangles in Question **3**. — 5:02

5 Use Pythagoras' Theorem to write an equation that shows how the sides of each triangle are related to each other. — 5:03

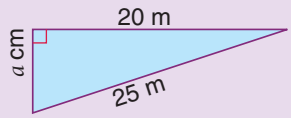
a



b



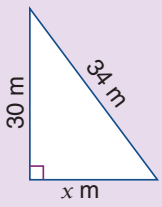
c



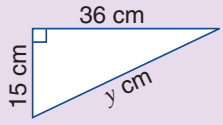
6 Find the length of the unknown side in each of the triangles in Question **5**. — 5:03

7 Find the length of the unknown side in each of the triangles below. — 5:04

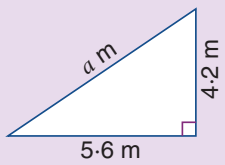
a



b



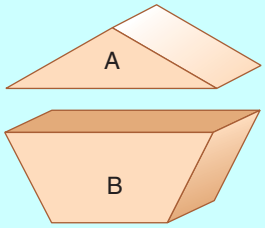
c



Chapter 5 | Revision Assignment

assignment 5A

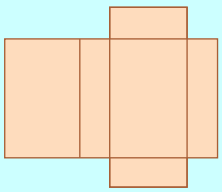
1



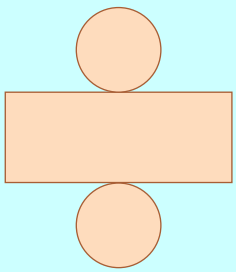
- **a** Name solid A.
- **b** Name solid B.
- **c** Calculate the number of faces, edges and vertices in solid B.
- **d** Name the cross-sectional shape of the solid formed by placing solid A on top of solid B.

2 The nets of three solids are given below. Name the solid formed from each net.

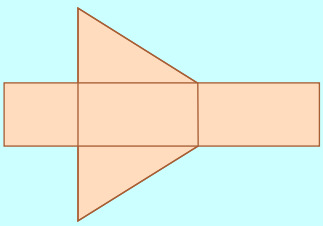
a



b



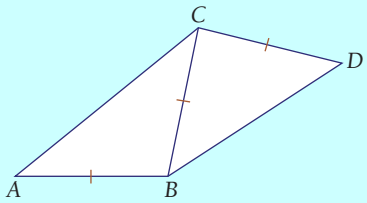
c



3 Classify the following angles as acute, obtuse or reflex.

- **a** 78°
- **b** 155°
- **c** 98°
- **d** 185°
- **e** 278°

4



- **a** What type of angle is $\angle ABC$?
- **b** What type of angle is $\angle BAC$?
- **c** By estimating, choose the angle that is closest in size to a right angle.
- **d** Name an obtuse-angled isosceles triangle.

5 Change these Roman numerals into our own numerals.

- **a** XLVIII
- **b** CIX
- **c** MDLXV
- **d** MCMXCIII

6 Complete the following conversions.

- **a** 5 m = ☐ mm
- **b** 2·5 m = ☐ cm
- **c** 5·2 g = ☐ mg
- **d** 560 kg = ☐ t
- **e** 3300 mL = ☐ L

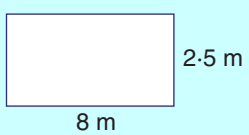
7 A property is rectangular in shape. It is 110 m long and 22 m wide. Calculate the cost of fencing the block if fencing is $8.40 a metre.

8 Write each time below as a digital time.

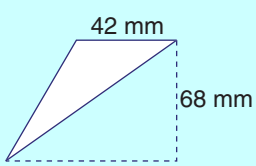
- **a** 20 past 11 in the morning
- **b** a quarter to 8 in the evening

9 Calculate the areas of the figures shown.

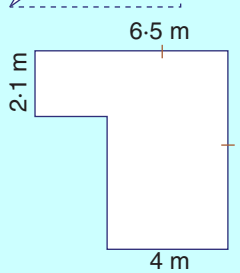
a



b

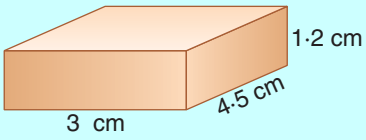


c

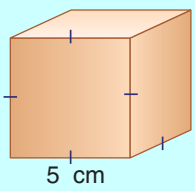


10 Calculate the volume of the figures shown.

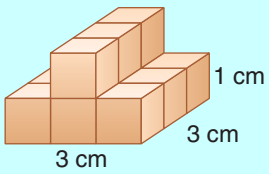
a



b



c



assignment 5B

Chapter 5 | Working Mathematically

1 Eight dots are drawn on a page. If no three dots are in the same straight line, how many intervals can be drawn that join any two dots?

2 Fred Shipshape built a sailing ship to mark Australia's 200th anniversary of European settlement. He wanted to use two words in naming it, but both words could not be the same. How many names were possible if the first name had to be either Endeavour, Australia, National, Young or Shining, and the second name had to be either Endeavour, Australia, Pride, Supreme, Courage or Star?

3 Five times Kate's age plus twice Sarah's age is equal to 25. List the possible ages for these two girls.

4 On a digital clock, how many times between 12 noon and 12 midnight will the readout be a palindromic number? (See Appendix A:02D, for a definition of palindromic numbers.)

5 How many integers are less than 5 units from the number two? (An integer is any whole number, positive, negative or zero.)

6 Drinks cost $1.20 each and ice-creams 80c each. The drinks and ice-creams for our group cost $12.40. If 12 items were purchased, how many drinks did we buy?

Technology Applications Drag and Drops
1 Number properties
2 Language and symbols
3 Special sets of numbers

Technology Applications Activities
5:04 Pythagorean triads

Technology Applications Animations
Pythagoras' Theorem

Chapter Review Questions

6

Indices

Chapter Contents

Learning Outcomes

Students will be able to:

- Apply index laws to simplify and evaluate arithmetic expressions.
- Apply the index laws to simplify algebraic expressions.

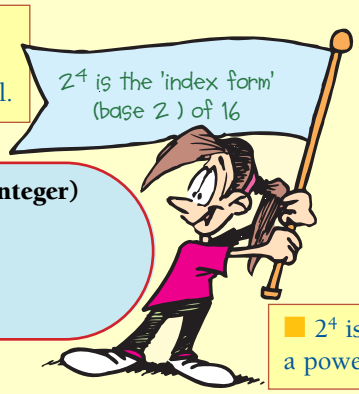
Areas of Interaction

Approaches to Learning, Homo Faber

6:01 | Review of Indices

■ $2 \times 2 \times 2 \times 2 = 16$
$2^4 = 16$

- 2 is called the base.
- 4 is called the index.
- 16 is called the basic numeral.



■ 2^4 is called a power of 2.

$x^n = \underbrace{x \times x \times x \times \ldots \times x \times x}_{n \text{ factors}}$ (where n is a positive integer)

For: x^n x is the base, n is the index

worked examples

1 Express the following in index form.
a $3 \times 3 \times 3 \times 3$ **b** 6×6
c $m \times m \times m$

2 Rewrite in expanded form.
a 2^5 **b** 7^3
c a^5

3 Find the basic numeral for:
a 4^3 **b** 13^5
c $(-4)^2$

4 Express as a product of its prime factors:
a 72 **b** 3500

- What is the largest power of 2 that can be displayed on a 10-digit screen?

Solutions

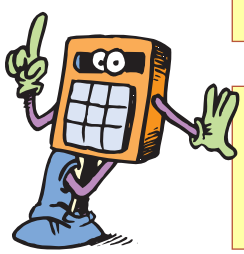
1 **a** $3 \times 3 \times 3 \times 3 = 3^4$ **b** $6 \times 6 = 6^2$ **c** $m \times m \times m = m^3$

2 **a** $2^5 = 2 \times 2 \times 2 \times 2 \times 2$ **b** $7^3 = 7 \times 7 \times 7$ **c** $a^5 = a \times a \times a \times a \times a$

3 **a** $4^3 = 4 \times 4 \times 4$
$= 64$

b $13^5 = 13 \times 13 \times 13 \times 13 \times 13$
Using the calculator to evaluate 13^5
PRESS 13 x^y 5 =
$13^5 = 371\,293$

c $(-4)^2 = -4 \times -4$
$= 16$



■ Another name for an INDEX is an EXPONENT.

■ Remember the x^y button. Enter the base x first, press x^y then enter the index y.

4 To find the prime factors of a number, we continue to break down each set of factors into further factors until each one is a prime number. Examine these solutions carefully.

a $72 = 9 \times 8$

$= 3 \times 3 \times 4 \times 2$

$= 3 \times 3 \times 2 \times 2 \times 2$

This product of prime numbers can be written as $3^2 \times 2^3$.

$72 = 3^2 \times 2^3$

b $3500 = 35 \times 100$

$= 7 \times 5 \times 10 \times 10$

$= 7 \times 5 \times 5 \times 2 \times 5 \times 2$

$3500 = 7 \times 5^3 \times 2^2$

Exercise 6:01

1 Write each expression in index form.

a $2 \times 2 \times 2 \times 2$ **b** 3×3

c $5 \times 5 \times 5$ **d** $7 \times 7 \times 7 \times 7 \times 7$

e $10 \times 10 \times 10$ **f** $9 \times 9 \times 9 \times 9$

g $x \times x$ **h** $a \times a \times a \times a$

i $n \times n \times n$ **j** $m \times m \times m \times m \times m$

k $p \times p \times p \times p \times p \times p$ **l** $y \times y$

m $4 \times 4 \times 4$ **n** $t \times t \times t \times t$

o $x \times x \times x \times x \times x$

2 Rewrite in expanded form.

a 2^3 **b** 4^2 **c** 6^5

d 10^4 **e** 7^3 **f** 3^1

g a^3 **h** x^4 **i** y^2

j m^5 **k** n^7 **l** p^3

3 Determine the basic numeral for:

a 2^5 **b** 3^4 **c** 7^2

d 11^2 **e** 10^4 **f** 6^3

g 2^8 **h** 5^7 **i** 8^5

j 9^5 **k** 4^{12} **l** 3^{15}

m $2^6 \times 5^3$ **n** $3^4 \times 7^3$ **o** $9^3 \times 5^4$ **p** $12^3 \times 7^3$

4 Write as powers of 10:

a 1000 **b** 100 000 **c** one million **d** one billion

5 Write as powers of 2:

a 64 **b** 256 **c** 16 384 **d** 1 048 576

6 Write as powers of 3:

a 27 **b** 243 **c** 59 049 **d** 4 782 969

7 Express the following integers as products of their prime factors in index form.

a 36 **b** 200 **c** 196 **d** 80

8 Find the value of x in the following. These are called exponential equations.

a $8^x = 64$ **b** $3^x = 27$ **c** $10^x = 100\,000$ **d** $2^x = 16$

9 As a reward for service, a man asked his king for the amount of rice that would be needed to place one grain of rice on the first square of a chessboard, 2 on the second, 4 on the third, 8 on the fourth, doubling each time, until the 64th square is considered. How many grains of rice would be needed for the 32nd square?

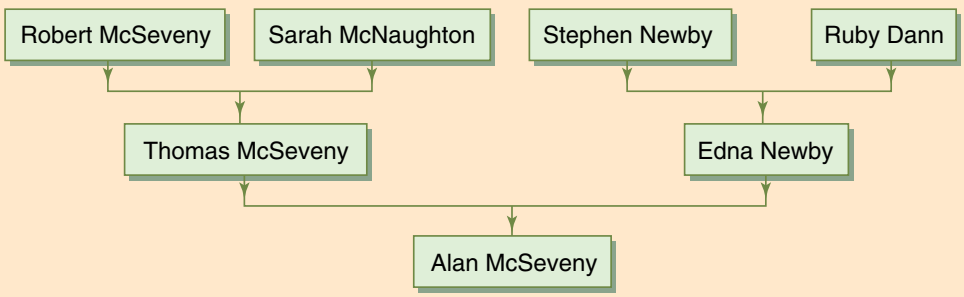
10 For question **9**, would the sum of the grains of rice on the first 31 squares be more or less than the number of grains of rice on the 32nd square? (*Hint:* Consider the rectangle below to represent the rice on the 32nd square. Colour the fraction of the rectangle equal to the amount of rice on the 31st square, then the 30th square, etc.)

challenge

6:01

Challenge 6:01 | Family trees

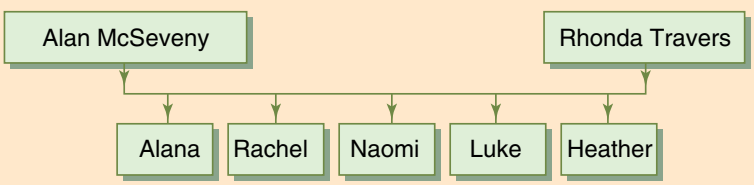
A part of Alan's family tree is drawn below.



1 How many great-grandparents would Alan have had?

2 Estimate the number of generations you would need to go back to before over one million boxes would be required to show that generation. Would this mean that in that generation there would be over 1 000 000 different ancestors?

3 Estimate Alan's total number of ancestors in the previous 20 generations.



It is much harder to estimate the number of descendants Alan will have in any one generation as a lot of assumptions will need to be made. Will all of his five children marry? Will there be wars, diseases or population control in the future?

4 Estimate how many generations would be needed before Alan has a total number of descendants in excess of 1 000 000.

6:02 | The Index Laws

prep quiz 6:02

Write in index form:	**1** $5 \times 5 \times 5 \times 5$	**2** $a \times a \times a$	
Rewrite as a product:	**3** 2^4	**4** m^5	
Evaluate the following:	**5** 2^2	**6** 2^3	**7** $2^2 \times 2^3$
	8 3^3	**9** $3^3 \div 3^2$	**10** $3^3 \div 3^3$

Multiplication using indices

Note the following:

$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3)$
$= 3^6 \ [= 3^{4+2}]$

$x^5 \times x^3 = (x \times x \times x \times x \times x) \times (x \times x \times x)$
$= x^8 \ [= x^{5+3}]$

You should note the law given below.

Law 1 **When multiplying terms, ADD the indices** $x^m \times x^n = x^{m+n}$

Division using indices

$3^5 \div 3^2 = \dfrac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}}$
$= 3^3 \ [= 3^{5-2}]$

$x^4 \div x^3 = \dfrac{x \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}}$
$= x^1 \ [= x^{4-3}]$

You should note the law given below.

Law 2 **When dividing terms, SUBTRACT the indices** $x^m \div x^n = x^{m-n}$

Powers of indices

$(3^3)^2 = 3^3 \times 3^3$
$= 3^{3+3}$ [Using Law 1]
$= 3^6 \ [= 3^{3 \times 2}]$

$(x^5)^4 = x^5 \times x^5 \times x^5 \times x^5$
$= x^{5+5+5+5}$ [Using Law 1]
$= x^{20} \ [= x^{5 \times 4}]$

You should note the law given below.

Law 3 **For powers of a power, MULTIPLY the indices** $(x^m)^n = x^{mn}$

If we simplify the division $x^n \div x^n$, using the second law above:

$x^n \div x^n = x^{n-n}$
$= x^0$

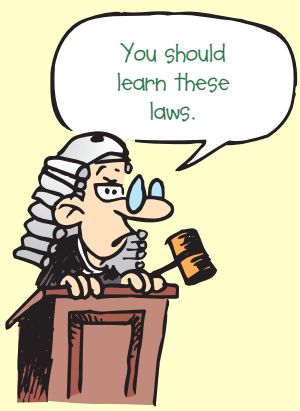
But any expression divided by itself must equal 1.

$x^n \div x^n = 1$

Therefore x^0 must be equal to 1.

$x^0 = 1$

Law 4 $x^0 = 1$



worked examples

1 Simplify:

a $3^2 \times 3^5$ **b** $x^3 \times x^2$ **c** $6m^2n \times mn^4$

2 Simplify:

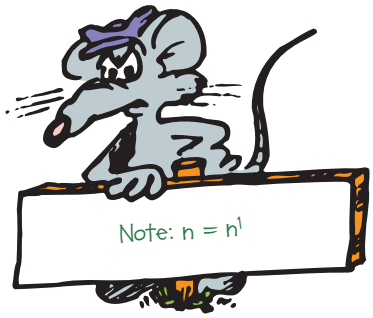
a $x^7 \div x^2$ **b** $15a^5 \div 3a^2$ **c** $20a^3b^2 \div 10ab$

3 Simplify:

a $(a^4)^2$ **b** $(2a^4)^3$ **c** $(p^4)^3 \div (p^2)^4$

4 Simplify 7^0

Solutions



1 Using index law 1:

a $3^2 \times 3^5 = 3^{2+5}$
$= 3^7$

b $x^3 \times x^2 = x^{3+2}$
$= x^5$

c $6m^2n \times mn^4 = 6 \times m^{2+1} \times n^{1+4}$
$= 6m^3n^5$

2 Using index law 2:

a $x^7 \div x^2 = x^{7-2}$
$= x^5$

b $15a^5 \div 3a^2 = \dfrac{15a^5}{3a^2}$
$= \dfrac{15}{3} \times \dfrac{a^5}{a^2}$
$= 5 \times a^{5-2}$
$= 5a^3$

c $20a^3b^2 \div 10ab$
$= \dfrac{20a^3b^2}{10ab}$
$= \dfrac{20}{10} \times \dfrac{a^3}{a} \times \dfrac{b^2}{b}$
$= 2a^2b$

3 Using index law 3:

a $(a^4)^2 = a^{4\times 2}$
$= a^8$

b $(2a^4)^3 = 2^3 \times (a^4)^3$
$= 8 \times a^{4\times 3}$
$= 8a^{12}$

c $(p^4)^3 \div (p^2)^4 = p^{12} \div p^8$
$= p^4$

4 Using index law 4:

$7^0 = 1$

With practice many of the steps in the above solutions can be left out.

Exercise 6:02

Foundation Worksheet 6:02

The index laws

1 Evaluate:

a 2^3 **b** 3^4 **c** 5^3

2 Use your calculator to evaluate:

a 6^3 **b** 4^5 **c** 8^4

1 Use a calculator to complete the tables below.

Power	11^6	6^8	8^{10}	5^6
Answer				
	8^7	7^8	5^4	3^5

Expression	$7^3 \times 7^5$	$5^8 \div 5^2$	$(11^3)^2$	56^0	$3^{10} \div 3^5$	$8^2 \times 8^5$	$(13)^0$	$(6^2)^4$
Answer								

2 Use the tables in question 1 to help you write *true* or *false* for:

a $7^3 \times 7^5 = 7^8$ **b** $5^8 \div 5^2 = 5^4$ **c** $5^8 \div 5^2 = 5^6$

d $(11^3)^2 = 11^6$ **e** $56^0 = 1$ **f** $8^2 \times 8^5 = 8^7$

g $3^{10} \div 3^5 = 3^5$ **h** $3^{10} \div 3^5 = 3^2$ **i** $(6^2)^4 = 6^8$

3 Simplify these products, writing answers in index form.

a $10^2 \times 10^3$ **b** 10×10^2 **c** $10^3 \times 10^3$

d $5^2 \times 5^4$ **e** $2^3 \times 2^2$ **f** $7^5 \times 7$

g $3^2 \times 3^3 \times 3^4$ **h** $2 \times 2^2 \times 2^5$ **i** $10^7 \times 10 \times 10^2$

4 Simplify these quotients, writing answers in index form.

a $10^2 \div 10$ **b** $10^6 \div 10^3$ **c** $10^5 \div 10^4$

d $5^8 \div 5^4$ **e** $7^5 \div 7^2$ **f** $3^{10} \div 3$

g $2^3 \div 2^2$ **h** $5^3 \div 5^3$ **i** $2^8 \div 2^2$

5 Simplify these powers, writing answers in index form.

a $(10^2)^3$ **b** $(10^3)^3$ **c** $(10^6)^2$

d $(2^3)^4$ **e** $(2^2)^2$ **f** $(2^7)^5$

g $(3^4)^2$ **h** $(5^3)^5$ **i** $(7^2)^4$

j $(2^3 \times 3^2)^2$ **k** $(7^3 \times 11^4)^2$ **l** $(3 \times 2^2)^4$

6 Simplify:

a $x^3 \times x^2$ **b** $y^4 \times y^2$ **c** $m^3 \times m^3$

d $m \times m^4$ **e** $p^5 \times p^5$ **f** $a \times a$

g $y^3 \times y^4$ **h** $x^2 \times x$ **i** $m^2 \times m^5$

j $3y^4 \times y$ **k** $m^6 \times 3$ **l** $5x^3 \times 3$

7 Simplify:

a $x^6 \div x^3$ **b** $x^6 \div x^2$ **c** $x^6 \div x$

d $m^3 \div m$ **e** $y^5 \div y^2$ **f** $m^4 \div m^2$

g $y^6 \div y^6$ **h** $x^5 \div x^5$ **i** $y^8 \div y^2$

j $6m^6 \div 3$ **k** $4y^8 \div 2$ **l** $20x^5 \div 5$

8 Simplify:

a $(x^2)^3$	**b** $(y^4)^2$	**c** $(a^3)^5$
d $(m^2)^0$	**e** $(x^0)^3$	**f** $(a^7)^0$
g $(y^3)^2$	**h** $(a^6)^3$	**i** $(x^3)^3$
j $(2x)^3$	**k** $(3x^2)^2$	**l** $(5m^2)^4$

6:02

Investigation 6:02 | Exploring index notation

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Searching for patterns is part of mathematics and being able to explain concepts is important if our ideas are to be shared.

- Find pairs of terms that can be multiplied to give 2^7. Explain the relationship between the members of any pair.
- Find pairs of terms that can be divided to give $3x^2$. Explain the difference between the members of any pair.
- Explain the difference between $7x^0$ and $(7x)^0$.
- Explain why $3^2 \times 3^4 \neq 9^6$.
- List all the pairs of expressions that could be multiplied together to give $10xy^2$. (Use only whole numbers.)
- Fold a sheet of A4 paper in half as many times as you can. How does the number of regions increase with each new fold. Write a formula for the number of regions (R) for n folds.

Assessment Grid for Investigation 6:02 | Exploring index notation

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	An organised approach has not been used to find patterns. The student has had difficulty following instructions.	1	
			2	
	b	An organised approach has been used and an attempt has been made to describe the patterns discovered in words or symbols.	3	
			4	
	c	An organised approach has been used and conclusions regarding the patterns have been described in words or symbols.	5	
			6	
	d	The correct formula has been found and explained fully using points 3 and 4, and justification is given for the patterns found.	7	
			8	
Criterion C Communication	a	No working out is shown. Presentation is poor with no structure to the work.	1	
			2	
	b	Working out is shown with some interpretation of the results. There is some structure to the work.	3	
			4	
	c	Results are interpreted with the correct use of symbols. Work is well structured and shows clear progression.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt has been made to describe the method used and to check the given terms for correctness.	1	
			2	
	b	The given terms have been checked with some success and a justified explanation is given for points 1 to 4.	3	
			4	
	c	Detailed, accurate justifications have been given, including an explanation for the differences in points 3 and 4 with applications of the patterns found to further examples.	5	
			6	

6:03 | Negative Indices

All the indices seen so far have been positive integers or zero.
If we had $2^3 \div 2^5$, the answer, according to the second index law should be 2^{3-5},
ie $2^3 \div 2^5 = 2^{-2}$.

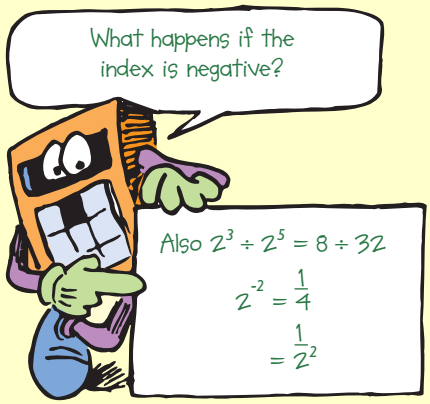
But this could also be written in this way:

$$\frac{2^3}{2^5} = \frac{\cancel{2^1} \times \cancel{2^1} \times \cancel{2^1}}{\cancel{2_1} \times \cancel{2_1} \times \cancel{2_1} \times 2 \times 2}$$

$$= \frac{1}{2 \times 2}$$

$$= \frac{1}{2^2}$$

So: $2^{-2} = \frac{1}{2^2}$



In general, the meaning of a negative index can be summarised by the rules: $x^{-m} = \frac{1}{x^m}$, $(x \neq 0)$

x^{-m} is the reciprocal of x^m, since $x^m \times x^{-m} = 1$

Examples

$x^{-3} = \frac{1}{x^3}$

$x^3 \times x^{-3} = x^0$

$= 1$

worked examples

1 Simplify the following:
a 3^{-2} **b** 5^{-1} **c** $(\frac{1}{4})^{-2}$ **d** $(\frac{2}{3})^{-3}$

2 Evaluate, using the calculator:
a 2^{-3} **b** $(\frac{1}{3})^{-2}$

Solutions

1 **a** $3^{-2} = \frac{1}{3^2}$

$= \frac{1}{9}$

b $5^{-1} = \frac{1}{5^1}$

$= \frac{1}{5}$

c $(\frac{1}{4})^{-2} = \frac{1}{\left(\frac{1}{4}\right)^2}$

$= \frac{1}{\left(\frac{1}{16}\right)}$

$= 16$

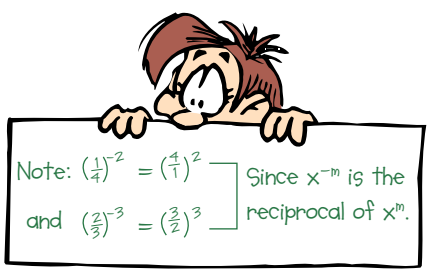
d $(\frac{2}{3})^{-3} = \frac{1}{\left(\frac{2}{3}\right)^3}$

$= \frac{1}{\left(\frac{8}{27}\right)}$

$= \frac{27}{8}$

$= 3\frac{3}{8}$

$\frac{1}{\left(\frac{1}{16}\right)} = 1 \div \frac{1}{16} = 1 \times \frac{16}{1} = 16$



2 The x^y key can also be used for negative indices by entering y as a negative number. Examine the following:

a Press: 2 x^y 3 $^+/_-$ = answer: 0·125, ie $\frac{1}{8}$ or 2^{-3}

b Press: 1 ÷ 3 = x^y 2 $^+/_-$ = answer: 9, ie $(\frac{1}{3})^{-2}$

Exercise 6:03

Foundation Worksheet 6:03

Negative indices

1 Write down the value of:
a 4^{-1} **b** 2^{-3} **c** 3^{-2}

2 Write with a negative index.
a $\frac{1}{4}$ **b** $\frac{1}{2^3}$ **c** $\frac{1}{3^2}$

1 Write down the value of each of the following.

a 3^{-1} **b** 5^{-1} **c** 2^{-1}
d 6^{-2} **e** 4^{-2} **f** 10^{-3}
g 2^{-4} **h** 10^{-4} **i** 5^{-2}

2 Write each with a negative index.

a $\frac{1}{11}$ **b** $\frac{1}{3}$ **c** $\frac{1}{5}$ **d** $\frac{1}{7}$
e $\frac{1}{3^3}$ **f** $\frac{1}{5^4}$ **g** $\frac{1}{2^8}$ **h** $\frac{1}{7^2}$
i $\frac{1}{10^2}$ **j** $\frac{1}{10^3}$ **k** $\frac{1}{10^6}$ **l** $\frac{1}{10^5}$

3 Write *true* or *false* for:

a $1024 = 2^{10}$ **b** $8 = 2^4$ **c** $3^{-2} = \frac{1}{9}$ **d** $2(3)^2 = 36$
e $2(3)^{-1} = \frac{1}{6}$ **f** $4^{-1} = \frac{1}{2}$ **g** $2^{-1} < 1$ **h** $-2^8 = (-2)^8$

4 Simplify, leaving answers as powers of ten.

a $10^2 \div 10^5$ **b** $10^{-2} \times 10^3$ **c** $10 \div 10^6$ **d** $10^{-1} \times 10^{-2}$
e $\frac{10^2 \times 10^3}{10^6}$ **f** $\frac{(10^{-2})^2}{10^2}$ **g** $\frac{10}{10^2 \div 10^7}$ **h** $\frac{10^4 \times 10^{-2}}{(10^3)^2}$

5 Write each without a negative index.

a a^{-1} **b** x^{-1} **c** m^{-1} **d** y^{-1}
e x^{-3} **f** y^{-2} **g** x^{-4} **h** m^{-6}
i $2x^{-1}$ **j** $5a^{-3}$ **k** $10y^{-2}$ **l** $36q^{-4}$

6 Rewrite each using a negative index to avoid having a fraction.

a $\frac{1}{x}$ **b** $\frac{1}{x^2}$ **c** $\frac{1}{x^3}$ **d** $\frac{1}{x^4}$
e $\frac{5}{y^2}$ **f** $\frac{3}{a}$ **g** $\frac{10}{m^4}$ **h** $\frac{75}{x^3}$
i $\frac{x}{y^2}$ **j** $\frac{m}{a^3}$ **k** $\frac{3a}{b^2}$ **l** $\frac{4x}{y}$

7 Write each as an integer, fraction or mixed number.

a $\left(\frac{1}{2}\right)^{-1}$ **b** $\left(\frac{1}{3}\right)^{-1}$ **c** $\left(\frac{2}{3}\right)^{-1}$ **d** $\left(\frac{1}{10}\right)^{-1}$

e $\left(\frac{1}{2}\right)^{-2}$ **f** $\left(\frac{1}{3}\right)^{-2}$ **g** $\left(\frac{2}{3}\right)^{-2}$ **h** $\left(\frac{3}{10}\right)^{-2}$

8 Rewrite each expression with a positive index.

a x^{-2} **b** $a^{-}5$ **c** $3x^{-1}$ **d** $5m^{-2}$

e $(x+1)^{-2}$ **f** $(3+a)^{-1}$ **g** $(6x)^{-2}$ **h** $4(x+2)^{-1}$

9 Evaluate the following, using your calculator. Leave your answers in decimal form.

a 2^{-3} **b** 4^{-2} **c** 5^{-2} **d** 8^{-2}

e 2^{-6} **f** 4^{-3} **g** $(0{\cdot}5)^{-3}$ **h** $(0{\cdot}2)^{-2}$

i $(0{\cdot}05)^{-2}$ **j** $(2{\cdot}5)^{-3}$ **k** $(0{\cdot}1)^{-5}$ **l** $(0{\cdot}625)^{-2}$

- The formula for the volume of a sphere is:
  $V = \frac{4}{3}\pi r^3$
  where $\pi \doteq 3{\cdot}142$ and r is the radius of the sphere.

Investigation 6:03 | Zero and negative indices

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

1 What does it mean to have a zero or negative index?
Complete these tables writing answers less than 1 as fractions.

Divide each answer by 10 to reduce the power.

Power of 10	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Answer	10 000	1000							

÷10

- As the power of 10 decreases, does the answer decrease?

$10^4 \div 10 = 10^3$
$10^3 \div 10 = 10^2$
$10^2 \div 10 = 10^1$
$10^1 \div 10 = 10^0$
$10^0 \div 10 = 10^{-1}$
. . . . . = . . .

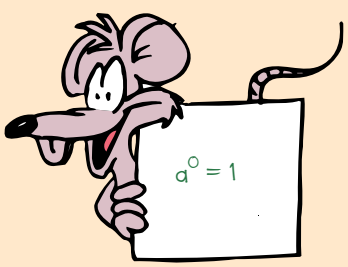
Divide each answer by 5 to reduce the power.

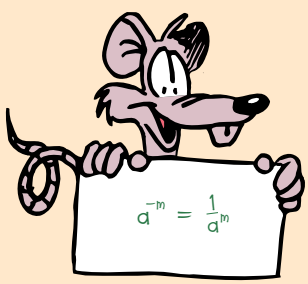
Power of 5	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}	5^{-4}
Answer	625								

÷5

- As the power of 5 decreases, does the answer decrease?

$5^4 \div 5 = 5^3$
$5^3 \div 5 = 5^2$
$5^2 \div 5 = 5^1$
$5^1 \div 5 = 5^0$
$5^0 \div 5 = 5^{-1}$
. . . . . = . . .





2 Use the tables above to write *true* or *false* for:

a $10^{-1} = \frac{1}{10}$ **b** $5^0 = 1$ **c** $5^{-1} = \frac{1}{5}$

d $10^{-2} = \frac{1}{10^2}$ **e** $5^{-3} = \frac{1}{5^3}$ **f** $10^0 = 1$

g $10^{-4} = \frac{1}{10^4}$ **h** $5^{-2} = \frac{1}{5^2}$ **i** $10^{-3} = \frac{1}{10^3}$

Assessment Grid for Investigation 6:03 | Zero and negative indices

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	An organised approach has not been used to find patterns.	1	
			2	
	b	An organised approach has been used and some patterns have been found.	3	
			4	
	c	An organised approach has been used and conclusions regarding the patterns found have been described in words or symbols.	5	
			6	
	d	The correct patterns have been justified and an ability to use them meaningfully to predict other results has been demonstrated.	7	
			8	
Criterion C Communication	a	No working out is shown and only oral explanations can be provided.	1	
			2	
	b	Clear explanations are given. These are supported by working out that is well presented.	3	
			4	
	c	The work is well structured and presented and demonstrates a clear understanding of the task.	5	
			6	
Criterion D Reflection in Mathematics	a	An attempt has been made to describe how results were obtained. Results have been checked with limited success.	1	
			2	
	b	There is an explanation of how results were obtained, and the results have been checked with some success.	3	
			4	
	c	Detailed, accurate justifications of the methods and results have been given, particularly with respect to the power of zero, with applications of the index operations to further examples.	5	
			6	

Mathematical terms 6

base

- The term which is operated on by the index.
  eg for x^n, x is the base
  for 5^3, 5 is the base.

exponent

- Another term for a power or index.
- Equations which involve a power are called *exponential* equations.
  eg $3^x = 27$

index

- A number indicating how many of a base term need to be multiplied together.
  eg for x^n, n is the index
  $x^n = \underbrace{x\ x\ x\ x\ x\ x\ .\ .\ .\ .\ .\ x\ x}_{n \text{ factors}}$
- The plural of index is *indices*.

negative indices

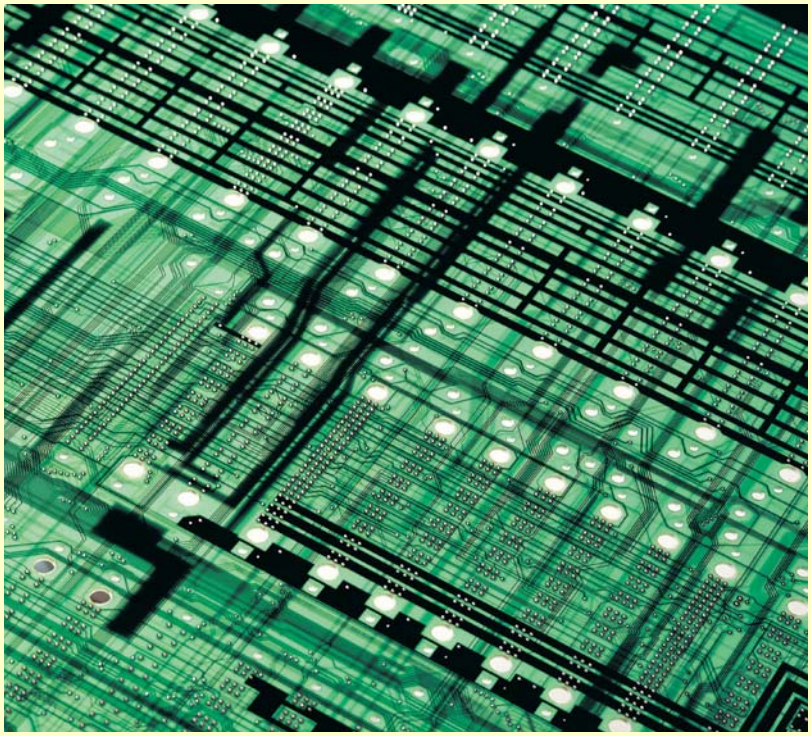
- Indicate the reciprocal of a term.
  eg $x^{-1} = \frac{1}{x}$, $x^{-n} = \frac{1}{x^n}$
  ie $5^{-1} = \frac{1}{5}$, $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

power

- Another term for an index or exponent.

zero index

- A term or number with a zero index is equal to 1.
  eg $x^0 = 1$, $4^0 = 1$



Mathematical terms 6

Technology Applications Drag and Drops

- A computer's memory is measured in gigabytes. In computing, a gigabyte is 2^{30} bytes. Write this number in scientific notation correct to 1 significant figure.

diagnostic test

6

Diagnostic Test 6: | Indices

- These questions reflect the important skills introduced in this chapter.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

				Section
1	Express in index form:			6:01
	a $3 \times 3 \times 3 \times 3$	**b** 5×5	**c** $m \times m \times m$	
2	Evaluate:			6:01
	a 3^2	**b** 2^4	**c** 10^3	
3	Simplify:			6:02
	a $3^2 \times 3^5$	**b** $x^3 \times x^2$	**c** $m^2n \times mn^4$	
4	Simplify:			6:02
	a $x^7 \div x^2$	**b** $a^5 \div a^2$	**c** $a^3b^2 \div ab$	
5	Simplify:			6:02
	a $(a^4)^2$	**b** $(x^3)^4$	**c** $(2a^4)^3$	
6	Simplify:			6:02
	a 7^0	**b** $5p^0$	**c** $x^3 \div x^3$	
7	Simplify:			6:03
	a 3^{-2}	**b** 5^{-1}	**c** $(\frac{2}{3})^{-3}$	
8	Simplify, writing answers without negative indices:			6:03
	a $x^7 \times x^{-3}$	**b** $x^2 \div x^4$	**c** $(3x^{-1})^2$	

Chapter 6 | Revision Assignment

assignment 6A

1 Simplify, writing the answers in index form:

a $a^2 \times a^3$ **b** $a^2b \times ab$

c $3a^2b \times 4ab^2$ **d** $3^2 \times 3^3$

e $a^6 \div a^3$ **f** $4^7 \div 4$

g $(3^2)^4$ **h** $(x^2)^3$

i $(a^3)^2 \times a^5$ **j** $m^7 - (m^2)^3$

2 Express in simplest form:

a $(2x^2)^0$ **b** $6x^0$

c $(5x^3)^3$ **d** $(10a^2)^3$

3 Use your calculator to evaluate:

a 2^{10} **b** 3^{12}

c $5^5 \times 6^6$ **d** $7^3 \times 4^5$

4 Find the value of n, if:

a $2^n = 128$ **b** $3^n = 243$

c $10^n = 100\,000\,000$

5 Simplify and evaluate:

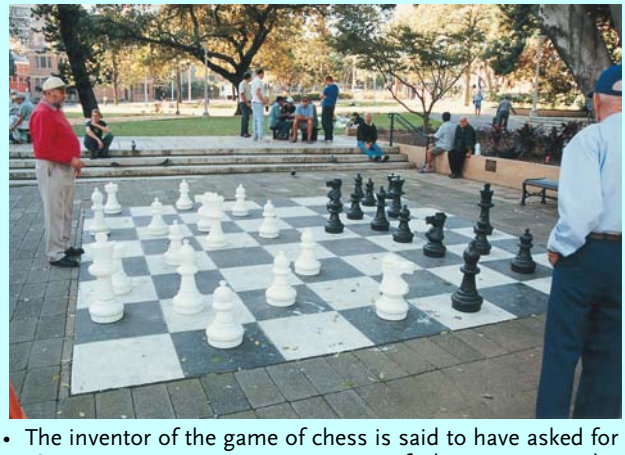
a $3^2 \times 3^5$ **b** $10^7 \div 10^4$

c $(2^4)^2$

6 Evaluate (writing answers in fraction form):

a 5^{-1} **b** 2^{-4}

c 3^{-2} **d** 10^{-3}



- The inventor of the game of chess is said to have asked for 18 400 000 000 000 000 000 grains of wheat as a reward. What is this number in scientific notation?

1 Index laws
2 Negative indices

Technology Applications Drag and Drops

Chapter Review Questions

assignment 6B

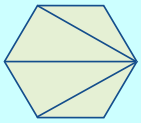
Chapter 6 | Working Mathematically

1 Use ID Card 7 on page xix to identify:

a 5 b 8 c 17 d 18 e 19
f 20 g 21 h 22 i 23 j 24

2 Use ID Card 6 on page xviii to identify numbers 1 to 12.

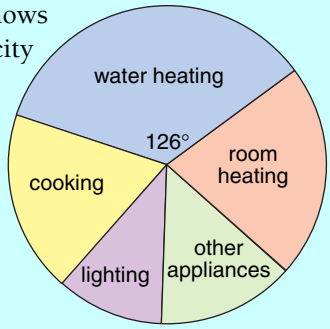
3 a How many diagonals can be drawn from one vertex of a regular hexagon? How many vertices has a hexagon?



b Each diagonal joins two vertices and a diagonal cannot be drawn from a vertex to the two adjacent vertices or to itself. The number of diagonals of a hexagon is $\frac{6(6-3)}{2}$. How many diagonals has:

i a regular octagon?
ii a regular decagon?
iii a regular polygon that has 30 sides?

4 a The sector graph shows the usage of electricity in the average household. What percentage of electricity is used in heating water?



b Use a protractor to measure each angle on the graph.

c Construct a column graph to represent the information shown in this graph.

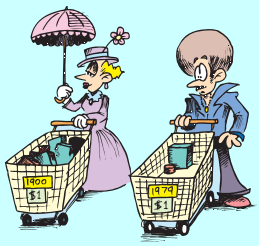
5 Tom was given a cheque for an amount between $31 and $32. The bank teller made a mistake and exchanged dollars and cents on the cheque. Tom took the money without examining it and gave 5 cents to his son. He now found that he had twice the value of the original cheque. If he had no money before entering the bank, what was the amount of the cheque?

6 a Look at the graph below. In which year did the dollar have its greatest purchasing power? What was the dollar called at that time?

b How many times more purchasing power did the dollar have in 1900 than in 1978/79?

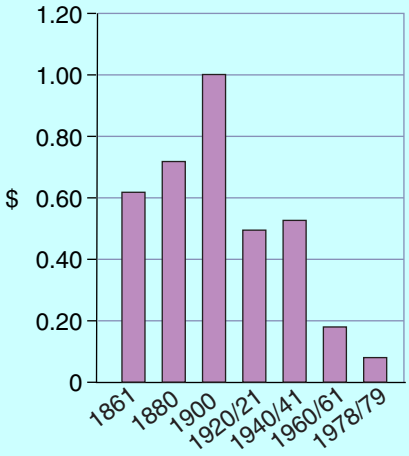
c Why doesn't $1 buy as much now as in the past?

d Give some reasons why the purchasing power of the dollar may have been less before 1900 than in 1900.



e Compared to 1900, what was the purchasing power of the dollar in:

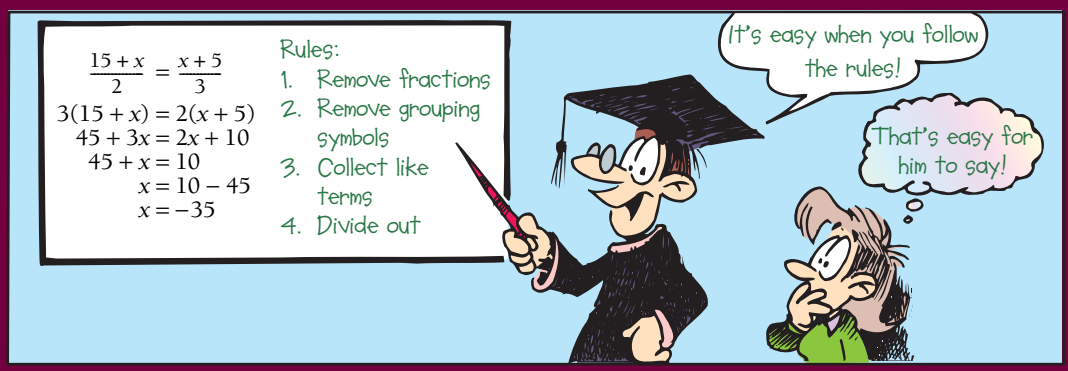
i 1940/41? ii 1960/61? iii 1920/21?



THE DOLLAR IN YOUR POCKET
(Base: 1900 = $1.00)
The dollar, valued in terms of the basket of goods it could buy in 1900, was only worth 6c in 1978/79, so the goods that could be purchased for $1.00 in 1900 would cost about $15.70 in 1978/79.

7

Equations and Inequations



Chapter Contents

Learning Outcomes

Students will be able to:

- Use algebraic techniques to solve linear equations and simple inequalities.
- Solve linear simultaneous equations using graphical and analytical methods.

Areas of Interaction

Approaches to Learning, Homo Faber

- Equations are number sentences where one or more of the numbers is missing or unknown. Because it is unknown, the number is represented by a pronumeral.
- When we solve an equation, we are trying to find the numerical value of the pronumeral that makes the sentence true. With some equations it is easy to find this value or solution. With harder equations more work has to be done before the solution is found.
- A solution is correct if it gives a true number sentence when it replaces the pronumeral in the equation. We say that the solution *satisfies* the equation.

An equation is a number sentence where one or more of the numbers have been replaced by a pronumeral.

7:01 | Solving Equations by Inspection

Often we can just 'see' the answer to an equation. When we write down the answer we are solving the equation by inspection.

Make the number sentences true by finding the missing number.

1 $\square + 6 = 8$ **2** $5 - \triangle = 2$ **3** $3 \times \oplus = 36$ **4** $10 \div \star = 2$

Find another way of writing:

5 $x \div 6$ **6** $5 \times y$ **7** $\frac{1}{2}y$ **8** $\frac{p}{4}$

Which of the following values makes the sentence true?

9 $\square + 3 = 15$ $\{11, 12, 13\}$ **10** $9 - \triangle = 5$ $\{4, 5, 2\}$

worked examples

1 Find the value of the pronumeral in each of the following:

a $x + 6 = 8$ **b** $6a = 18$ **c** $\frac{p}{4} = 3$ **d** $y - 3 = 7$

Solution 1

a

x	+	6	=	8
2	+	6	=	8

$\therefore x = 2$

b

6	×	a	=	18
6	×	3	=	18

$\therefore a = 3$

c

p	÷	4	=	3
12	÷	4	=	3

$\therefore p = 12$

d

y	−	3	=	7
10	−	3	=	7

$\therefore y = 10$

- Notice how we compare the equation with an equivalent number sentence.
- The parts in the boxes are equal.
- Often these can be solved mentally. There is no need to write the number sentence underneath the equation.
- Always *check* your solution by *substituting* back into the equation.
- If in trouble, see if you can express the equation in words,
  eg $\frac{p}{4} = 3$ means 'what number does 4 go into 3 times?'

2 An equation and a solution are given. Check to see whether each solution is correct or incorrect by substituting the value into the equation.

a $x + 9 = 11$, $x = 3$ **b** $5x = 35$, $x = 7$ **c** $y - 3 = 11$, $y = 8$ **d** $\frac{a}{6} = 9$, $a = 54$

Solution 2

2 a $x + 9 = 11$
If $x = 3$ then
$x + 9 = 3 + 9$
$= 12$
But $12 \neq 11$
$\therefore x = 3$ is incorrect

b $5x = 35$
If $x = 7$ then
$5x = 5 \times 7$
$= 35$
Now $35 = 35$
$\therefore x = 7$ is correct

c $y - 3 = 11$
If $y = 8$ then
$y - 3 = 8 - 3$
$= 5$
But $5 \neq 11$
$\therefore y = 8$ is incorrect

d $\frac{a}{6} = 9$
If $a = 54$
then $\frac{a}{6} = \frac{54}{6}$
$= 9$
Now $9 = 9$
$\therefore a = 54$ is correct

Exercise 7:01

Foundation Worksheet 7:01
Solving equations by inspection
1 Solve:
a $y + 5 = 8$ **b** $m - 7 = 10$ **c** $x - 1 = -5$
2 Solve:
a $5m = 30$ **b** $6y = 42$ **c** $4x = 3$

1 Solve the following equations.

a $x + 5 = 10$ **b** $y + 7 = 20$
c $12 + y = 16$ **d** $9 = 3 + x$
e $x - 1 = 5$ **f** $7 - y = 1$
g $6 = 8 - p$ **h** $x - 10 = 20$
i $5 \times y = 30$ **j** $10p = 100$ **k** $7x = 28$ **l** $120 = 6m$
m $x \div 6 = 3$ **n** $\frac{y}{4} = 5$ **o** $8 \div p = 4$ **p** $\frac{6}{p} = 2$

2 Find the value of the pronumeral in each of the following equations.

a $6x = 66$ **b** $\frac{m}{3} = 9$ **c** $y - 7 = 7$ **d** $6 + p = 13$
e $3 \div b = 3$ **f** $a + 3 = 3$ **g** $\frac{1}{3}x = 5$ **h** $2y = 3$
i $14 - p = 6$ **j** $\frac{12}{x} = 1$ **k** $20 = 16 + m$ **l** $12 = 20 - t$
m $\frac{x}{3} = 8$ **n** $\frac{1}{2}p = 7$ **o** $30 - x = 15$ **p** $5 + a = 12$

3 Each question contains an equation and a solution. Check to see if the solution is correct or incorrect by substituting the solution into the equation. (Remember if it's correct you will obtain a true number sentence.)

a $x + 9 = 14$
$x = 5$

b $y + 7 = 11$
$y = 5$

c $12 = a + 7$
$a = 6$

d $16 + b = 20$
$b = 4$

e $p - 3 = 6$
$p = 9$

f $3 - x = 4$
$x = 2$

g $10 = m - 7$
$m = 3$

h $a - 4 = 4$
$a = 0$

i $3x = 18$
$x = 6$

j $5y = 45$
$y = 8$

k $60 = 5p$
$p = 10$

l $4a = 1$
$a = \frac{1}{4}$

m $\frac{1}{2}m = 7$
$m = 14$

n $\frac{y}{3} = 9$
$y = 3$

o $\frac{b}{6} = 2$
$b = \frac{1}{3}$

p $4a = 2$
$a = 2$

4 Under each equation is a set of solutions. Only one of these is correct. Find the correct solution by substitution.

a $x + 27 = 53$
{25, 26, 27}

b $6 + y = 4$
{2, 0, −2}

c $12 = p - 4$
{8, 16, 10}

d $3b = 69$
{24, 23, 30}

e $\frac{x}{4} = 8$
{2, 32, 16}

f $\frac{6}{x} = 3$
{18, $\frac{1}{2}$, 2}

g $138 = m + 69$
{70, 45, 69}

h $\frac{1}{2}a = 7$
{$3\frac{1}{2}$, 14, 7}

5 The left-hand column gives a set of equations, **a** to **h**. The right-hand column gives the solutions to these equations, A to H. Match each of the equations **a** to **h** with its solution.

a $x + 7 = 10$

b $7y = 28$

c $\frac{p}{6} = 10$

d $12 \div x = 12$

e $30 = 32 - p$

f $16 - m = 5$

g $7 + p = 12$

h $\frac{1}{2}p = 20$

A 3
B 40
C 1
D 60
E 2
F 4
G 11
H 5

- Computers are used to solve many equations.

7:02 | Inverse Operations

prep quiz 7:02

Write an algebraic expression for the following:

1 add 7 to x
2 multiply c by 4
3 divide y by 3
4 take 9 away from a
5 multiply m by 4 and add 8
6 subtract 5 from x and multiply the result by 4

What is the opposite of:

7 adding 6?
8 subtracting 11?
9 multiplying by 3?
10 dividing by 7?

Arrow diagrams can be used to show how the expression has been made. The operation which has caused the change is written above the arrow.

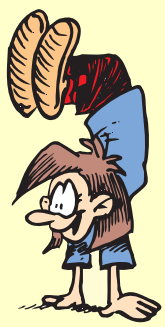
When solving equations we need to reverse the steps to get back to the pronumeral. We need to do the 'inverse' operations. For instance, if:

$x \xrightarrow{+3} x+3$ then $x+3 \xrightarrow{-3} x$

$x \xrightarrow{-3} x-3$ then $x-3 \xrightarrow{+3} x$

$x \xrightarrow{\times 3} 3x$ then $3x \xrightarrow{\div 3} x$

$x \xrightarrow{\div 3} \frac{x}{3}$ then $\frac{x}{3} \xrightarrow{\times 3} x$



■ 'Inverse' means opposite.

$2 \xrightarrow{+3} 5$

$5 \xrightarrow{-3} 2$

Adding 3 and subtracting 3 are inverse operations.

worked examples

1 Note how each expression has been built from the pronumeral.

a $y \xrightarrow{-7} y-7$

b $a \xrightarrow{\times 3} 3a \xrightarrow{+2} 3a+2$

c $q \xrightarrow{+5} q+5 \xrightarrow{\times 3} 3(q+5)$

d $m \xrightarrow{\div 2} \frac{m}{2} \xrightarrow{-1} \frac{m}{2}-1$

e $n \xrightarrow{-5} n-5 \xrightarrow{\div 3} \frac{n-5}{3}$

2 Note the inverse operations used to get back to the pronumeral.

a $7m \xrightarrow{\div 7} m$

b $5x+2 \xrightarrow{-2} 5x \xrightarrow{\div 5} x$

c $2(y+3) \xrightarrow{\div 2} y+3 \xrightarrow{-3} y$

d $\frac{a-2}{4} \xrightarrow{\times 4} a-2 \xrightarrow{+2} a$

Exercise 7:02

1 Copy and complete the following arrow diagrams by writing the correct operation above the arrow.

a $x \xrightarrow{\quad} 5x$ **b** $a \xrightarrow{\quad} a+7$ **c** $q \xrightarrow{\quad} q-4$ **d** $m \xrightarrow{\quad} \frac{m}{3}$

e $a \xrightarrow{\quad} 4a \xrightarrow{\quad} 4a+1$ **f** $x \xrightarrow{\quad} x-8 \xrightarrow{\quad} 3(x-8)$

g $y \xrightarrow{\quad} -7y \xrightarrow{\quad} -7y+2$ **h** $m \xrightarrow{\quad} m+3 \xrightarrow{\quad} \frac{m+3}{10}$

i $a \xrightarrow{\quad} \frac{a}{7} \xrightarrow{\quad} \frac{a}{7}+6$ **j** $x \xrightarrow{\quad} x+9 \xrightarrow{\quad} -6(x+9)$

2 Complete these arrow diagrams, showing how to get back to the pronumeral.

a $6n \xrightarrow{\quad} n$ **b** $\frac{a}{2} \xrightarrow{\quad} a$ **c** $x+9 \xrightarrow{\quad} x$ **d** $p-4 \xrightarrow{\quad} p$

e $8n-7 \xrightarrow{\quad} 8n \xrightarrow{\quad} n$ **f** $2+9y \xrightarrow{\quad} 9y \xrightarrow{\quad} y$

g $4-5x \xrightarrow{\quad} -5x \xrightarrow{\quad} x$ **h** $\frac{m}{4}-1 \xrightarrow{\quad} \frac{m}{4} \xrightarrow{\quad} m$

i $\frac{x+2}{4} \xrightarrow{\quad} x+2 \xrightarrow{\quad} x$ **j** $5+\frac{a}{4} \xrightarrow{\quad} \frac{a}{4} \xrightarrow{\quad} a$

3 Use arrow diagrams to show how the following were made from the pronumeral.

a $7m$ **b** $3y-5$ **c** $8a+1$ **d** $-3a+2$

e $5(p+4)$ **f** $9+4m$ **g** $10(x-7)$ **h** $7-5c$

i $\frac{x}{4}+1$ **j** $\frac{m+6}{2}$ **k** $\frac{a-8}{7}$ **l** $6+\frac{c}{3}$

4 Complete the following arrow diagrams.

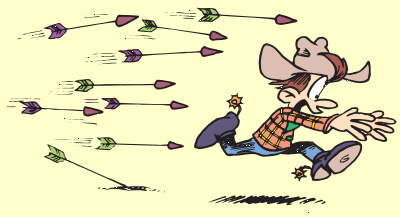
a $x \xrightarrow{+4} \square$ **b** $m \xrightarrow{\times 5} \square$ **c** $t \xrightarrow{\div 2} \square$

d $y \xrightarrow{-4} \square$ **e** $a \xrightarrow{\times 2} \square \xrightarrow{+1} \square$ **f** $x \xrightarrow{+2} \square \xrightarrow{\times 9} \square$

g $n \xrightarrow{\times(-6)} \square \xrightarrow{-7} \square$ **h** $y+6 \xrightarrow{-6} \square$ **i** $a-3 \xrightarrow{+3} \square$

j $9n \xrightarrow{\div 9} \square$ **k** $\frac{x}{4} \xrightarrow{\times 4} \square$ **l** $3+7x \xrightarrow{-3} \square \xrightarrow{\div 7} \square$

m $\frac{a+3}{5} \xrightarrow{\times 5} \square \xrightarrow{-3} \square$ **n** $5-3n \xrightarrow{-5} \square \xrightarrow{\div(-3)} \square$



5 What operations must be performed on x to get the following expressions?

a $2x+7$ **b** $-6x-2$ **c** $9x-1$

d $4(x-6)$ **e** $7(x+5)$ **f** $3(4+x)$

6 What must be done to these to get a?

a $3a + 10$ **b** $15 - 11a$ **c** $20a - 200$

d $\frac{a}{3}$ **e** $6 + \frac{a}{5}$ **f** $\frac{a+3}{4}$

7 What order of inverse operations must be performed to get back to the pronumeral?

a $\frac{6a-1}{4}$ **b** $10(7x + 2)$ **c** $\frac{7-2a}{3}$ **d** $8(4 - 5x)$

e $\frac{5b}{3} - 4$ **f** $6 + \frac{10m}{11}$ **g** $\frac{7(4-3a)}{10}$ **h** $\frac{6(2a-3)}{5}$

7:03 | Solving Simple Equations

prep quiz 7:03

Simplify the following:

1 $9x \div 9$ **2** $x + 2 - 2$ **3** $a - 5 + 5$ **4** $\frac{m}{3} \times 3$

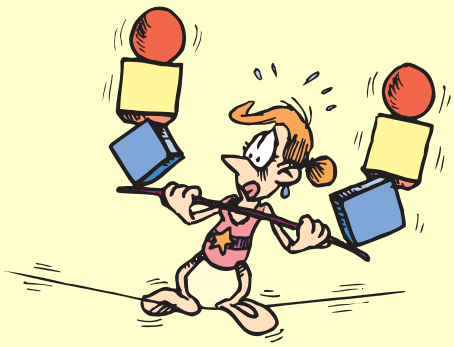
What is the 'inverse' of:

5 multiplying by 7? **6** adding 8? **7** subtracting 1? **8** dividing by 4?

Complete these arrow diagrams.

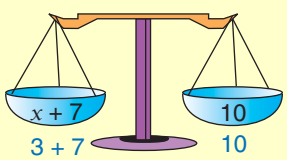
9 $7m + 10 \rightarrow 7m \rightarrow m$ **10** $3a - 2 \rightarrow 3a \rightarrow a$

If one equation can be changed into another by performing the same operation on both sides, then the equations are said to be equivalent.



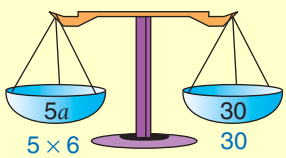
Solving equations is like balancing scales.
With equations, we know that one side is equal to the other. The solution of the equation is the value of the pronumeral that 'balances' the equation.

$x + 7 = 10$



$x = 3$ balances the scale
so $x = 3$ is the solution.

$5a = 30$

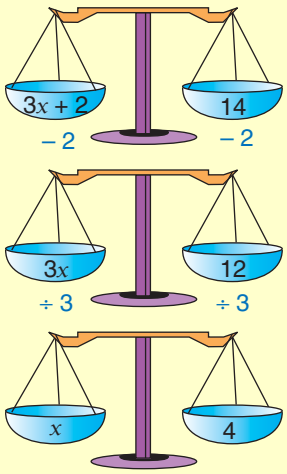


$a = 6$ balances the scale
so $a = 6$ is the solution.

Often, solving an equation requires us to change the equation into a simpler one. We can do this by adding (+), subtracting (–), multiplying (×) or dividing (÷) both sides of the equation by the same number.

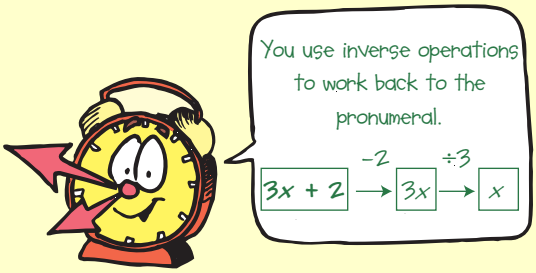
Study the solution to the equations on the next page. Note that the sides remain balanced because we perform the same operation on both sides.

$3x + 2 = 14$

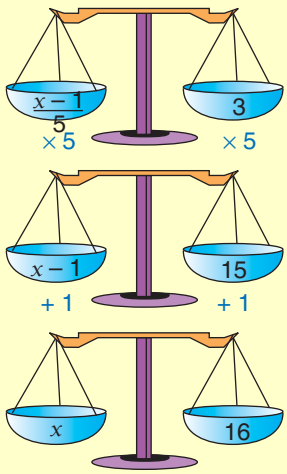


$x = 4$ is the solution.

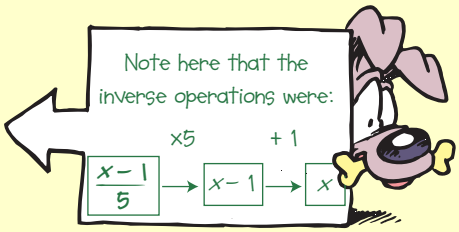
■ We need to perform operations that will leave only the pronumeral on one side of the equation.

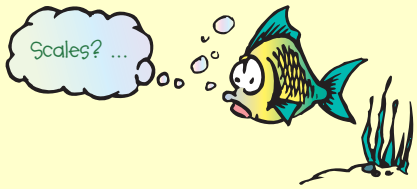


$\frac{x-1}{5} = 3$



$x = 16$ is the solution.





In the examples below inverse operations have been used to solve the equations.

worked examples

1 These solutions involve only one step.

a $m + 17 = 28$

$-17 \quad -17$

$m = 28 - 17$

$\therefore m = 11$

b $a - 13 = 31$

$+13 \quad +13$

$a = 31 + 13$

$\therefore a = 44$

c $3p = 5$

$\div 3 \quad \div 3$

$p = \frac{5}{3}$

$\therefore p = 1\frac{2}{3}$

d $\frac{x}{4} = 8$

$\times 4 \quad \times 4$

$x = 8 \times 4$

$\therefore x = 32$

2 These solutions involve two steps.

a $5m - 4 = 16$
$+4 \quad +4$
$5m = 20$
$\div 5 \quad \div 5$
$\frac{5m}{5} = \frac{20}{5}$
$\therefore\ m = 4$

b $8a + 6 = 15$
$-6 \quad -6$
$8a = 9$
$\div 8 \quad \div 8$
$\frac{8a}{8} = \frac{9}{8}$
$\therefore\ a = 1\frac{1}{8}$

c $1 - 3b = 7$
$-1 \quad -1$
$-3b = 6$
$\div -3 \quad \div -3$
$\frac{-3b}{-3} = \frac{6}{-3}$
$\therefore\ b = -2$

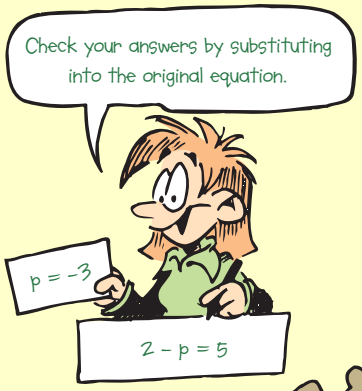
d $12 = 5x + 4$
$-4 \quad -4$
$8 = 5x$
$\div 5 \quad \div 5$
$\frac{8}{5} = \frac{5x}{5}$
$\therefore\ 1\frac{3}{5} = x$

Exercise 7:03

Foundation Worksheet 7:03
Solving simple equations
1 Solve:
a $m + 15 = 32$ **b** $n - 12 = 27$ **c** $3m = 72$
2 Solve:
a $2a + 3 = 9$ **b** $4x - 1 = 3$ **c** $7n - 6 = 8$

1 Solve the following equations.

a $x + 72 = 138$ **b** $y + 37 = 68$
c $72 + m = 145$ **d** $725 = a + 473$
e $p - 64 = 237$ **f** $55 = x - 96$
g $x - 125 = 716$ **h** $a + 412 = 917$
i $\frac{x}{6} = 32$ **j** $\frac{p}{14} = 20$
k $\frac{a}{7} = 92$ **l** $\frac{m}{50} = 13$
m $10m = 96$ **n** $7m = 58$
o $36y = 728$



2 The answer to an equation can be checked by substituting it into the equation. Check to see if the answer to each question below is correct.

a $x + 96 = 123$
$x = 27$

b $18 + y = 21$
$y = 5$

c $m - 13 = 26$
$m = 13$

d $2 - q = 0$
$q = 2$

e $100 - y = 76$
$y = 24$

f $14m = 20$
$m = \frac{10}{7}$

g $-4y = 12$
$y = -3$

h $\frac{x}{2} = 9$
$x = 18$

i $\frac{m}{2} = 1$
$m = 2$

Example
$136 - x = 72$
$x = 64$
LHS $= 136 - 64$
$= 72$
$=$ RHS

3 Solving these equations involves two steps. Clearly show each step in your working. (All the answers are integers.)

a $4x + 1 = 21$	**b** $3a + 2 = 32$	**c** $6m + 7 = 31$
d $3n - 8 = 19$	**e** $5k - 1 = 44$	**f** $2t - 4 = 196$
g $11 + 5a = 26$	**h** $10 + 3w = 25$	**i** $12 + 4q = 16$
j $10 - 3x = 10$	**k** $15 - 2m = 15$	**l** $20 - 5q = 0$
m $10 = 2x - 6$	**n** $7 = 5y - 28$	**o** $-6 = 2 - 4a$
p $6 - x = -7$	**q** $-3 - x = -2$	**r** $3 = 4 - x$

4 The solutions to these equations involve fractions.

a $4x + 1 = 4$	**b** $8a + 5 = 10$
c $2m + 4 = 5$	**d** $7n + 2 = 8$
e $3p - 3 = 2$	**f** $2q - 1 = 2$
g $5n - 5 = 4$	**h** $6y - 3 = 1$
i $5 + 3k = 12$	**j** $1 + 3x = 9$
k $9 + 3a = 10$	**l** $4 + 2a = 4$
m $4m + 6 = 3$	**n** $5p + 7 = 4$
o $1 - 2a = 6$	**p** $8 - 3a = 1$

■ $3a + 2 = 6$
$-2 \quad -2$
$3a = 4$
$\div 3 \quad \div 3$
$a = \frac{4}{3}$ or $1\frac{1}{3}$

5 Now try this set of equations which are either one-step or two-step types.

a $7x = 35$	**b** $m + 9 = 24$	**c** $11x + 1 = 89$	**d** $12 - n = 0$
e $3a + 4 = 40$	**f** $a - 6 = -7$	**g** $y + 3 = 1$	**h** $3m - 1 = 5$
i $-8x = 16$	**j** $5 + 3n = 10$	**k** $3x = 2$	**l** $7 - q = 10$
m $6 - 2a = 12$	**n** $3p + 5 = -4$	**o** $420 - 2x = 20$	**p** $10 = 7 - 2p$

- You must perform the same operation on both sides of an equation.

Investigation 7:03 | Solving equations using a spreadsheet

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

A spreadsheet such as Excel can be used to solve equations using the 'formula bar'. To solve an equation such as $2x + 5 = 11$ we need to enter the numbers into the appropriate 'cells' and tell the spreadsheet how to arrive at the answer for x.

- The equation $2x + 5 = 11$ is of the form $ax + b = c$.
- Enter the letters a, x, b and c in the first row as shown. These will act as headings.
- The numbers 2, 5 and 11 are then placed in cells A2, C2 and D2.
- To solve $2x + 5 = 11$ we would need to complete these steps:

$$2x + 5 = 11$$
$$2x = 11 - 5$$
$$x = \frac{11 - 5}{2}$$

- Matching the 'cells' with these numbers we would have:

$$x = \frac{D2 - C2}{A2}$$

- In cell B2 we type '=(D2 – C2)/A2'. This also shows in the formula bar.
- When we press ENTER the answer 3 appears in cell B2.

B2 | fx | =(D2–C2)/A2

	A	B	C	D	F
1	a	x	b	c	
2	2	3	5	11	
3					
4					

	A	B	C	D
1	a	x	b	c
2	2	3	5	11
3				
4				

We can now change any of the values for a, b or c and the value for x, ie the answer will automatically change.

Try entering other values including negative numbers and decimals.

You can also try this with other simple equations of your own.

Assessment Grid for Investigation 7:03 | Solving equations using a spreadsheet

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (C, D) for this investigation				Achieved ✓
Criterion C Communication	a	No working out is shown and presentation is poor with little or no use of diagrams or symbols.	1	
			2	
	b	Working out of the solution steps are shown with some explanation. Presentation is good with some structure.	3	
			4	
	c	Working out of the solution steps is shown and these steps are well structured and well communicated, showing clear progression.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt has been made to explain how the solution steps were found and to check the results.	1	
			2	
	b	Justification of how the solution steps were arrived at is given and the results have been checked for reasonableness with some success.	3	
			4	
	c	A precise and reasoned justification is given for the solution steps, and several alternate equation types have been efficiently applied in a variety of contexts to demonstrate understanding and extend the task.	5	
			6	

7:04 | Equations with Pronumerals on Both Sides

prep quiz 7:04

Simplify the following:

1 $7x - 6x$ **2** $9a + a$ **3** $5y - 5y$ **4** $-p + p$

What operation should be written above each arrow?

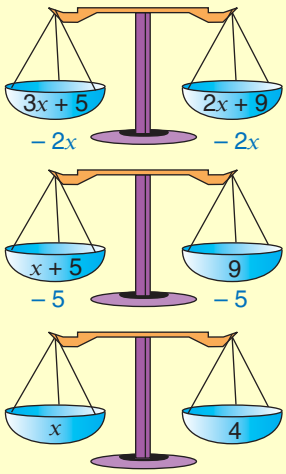
5 $6x + 9 \rightarrow 6x$ **6** $5a - 2 \rightarrow 5a$ **7** $\frac{x + 11}{10} \rightarrow x + 11$

Complete these arrow diagrams using the operation above each arrow.

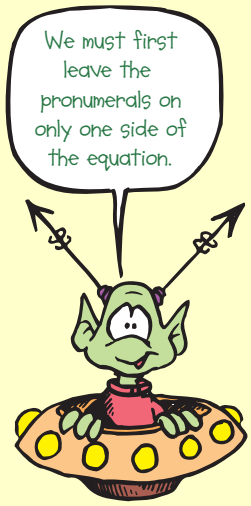
8 $14 + 2x \xrightarrow{-2x} \square$ **9** $3x + 4 \xrightarrow{-3x} \square$ **10** $7 - 2x \xrightarrow{+2x} \square$

It is possible to have pronumerals on both sides of an equation. As well as adding or subtracting numerals, we may also need to add or subtract pronumerals.

$3x + 5 = 2x + 9$

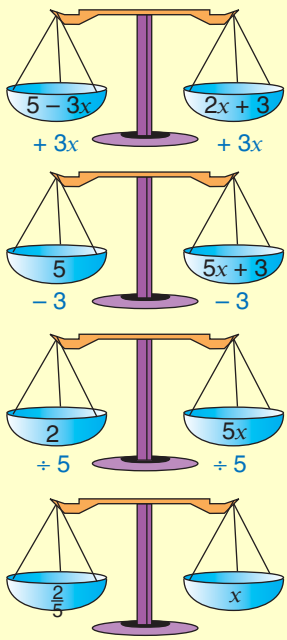


So $x = 4$ is the solution.



Hint: Leave the pronumerals on the side where the term will be positive.

$5 - 3x = 2x + 3$



So $x = \frac{2}{5}$ is the solution.

worked examples

1 $3a + 5 = 2a + 7$
$-2a \quad -2a$
$a + 5 = 7$
$-5 \quad -5$
$\therefore a = 2$

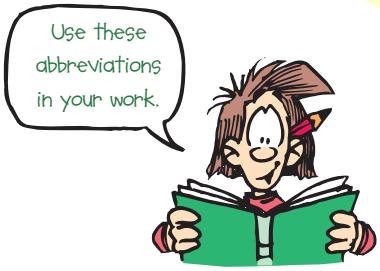
2 $5x - 3 = 2x + 9$
$-2x \quad -2x$
$3x - 3 = 9$
$+3 \quad +3$
$3x = 12$
$\div 3 \quad \div 3$
$\therefore x = 4$

3 $2 - 2x = 8 + x$
$+2x \quad +2x$
$2 = 8 + 3x$
$-8 \quad -8$
$-6 = 3x$
$\div 3 \quad \div 3$
$-2 = x$
$\therefore x = -2$

Therefore! $\therefore$

continued →→→

4 By substituting the given solution, see if it is correct or incorrect.
$5a - 9 = 3 - a$
$a = 5$



Solution

LHS means left-hand side.

$\text{LHS} = 5a - 9$
$= 5 \times 5 - 9$
$= 16$
$\therefore$ LHS $\neq$ RHS
$\therefore$ $a = 5$ is incorrect.

$\text{RHS} = 3 - a$
$= 3 - 5$
$= -2$

RHS means right-hand side.

Exercise 7:04

Foundation Worksheet 7:04
Equations and pronumerals on both sides
1 Solve:
a $2x + 3 = x + 5$ **b** $5n - 2 = 4n + 3$
2 Solve:
a $6a + 3 = 3a + 9$ **b** $7m - 2 = 3m + 6$

1 Solve the following equations. (The answers are all integers.)

a $3a + 2 = 2a + 9$
b $3x + 3 = 2x + 10$
c $5m + 7 = 4m + 8$
d $4p - 8 = 3p + 2$
e $6q - 3 = 5q + 5$
f $9n - 5 = 8n + 1$
g $8n = 6n + 12$
h $9x = 6x + 3$
i $6x = x + 10$
j $7m + 8 = 8m - 4$
k $a + 11 = 2a + 5$
l $9q + 4 = 8q$
m $8x + 3 = x + 31$
n $6m + 1 = 3m + 16$
o $10x - 5 = x + 4$
p $8y - 5 = y + 9$
q $3x + 9 = 7x + 1$
r $6p + 1 = 9q - 5$
s $x + 8 = 3x - 10$
t $2m + 11 = 6m + 3$
u $x + 5 = x - 1$
v $3x + 4 = 9 - 2x$
w $15 - 2m = 6m - 1$
x $9 - 5m = m - 21$

2 Substitute the given solution to see if it is correct or incorrect.

a $9m - 5 = 4m + 10$, $m = 3$
b $2p + 6 = 9p + 10$, $p = 5$
c $3q + 3 = 2q + 1$, $q = -2$
d $11a + 8 = 5a + 20$, $a = 2$
e $8q - 3 = 5q + 9$, $q = 4$
f $x + 7 = 5x - 4$, $x = 2$
g $2n + 1 = n - 2$, $n = -3$
h $2m + 5 = 8m - 7$, $m = 2$
i $8a + 2 = 3a - 3$, $a = -1$
j $5x = 12x - 14$, $x = 2$
k $11a + 8 = 13a$, $a = 4$
l $27x = 50 + 2x$, $x = 2$

3 Solve these equations.

a $5x + 10 = x + 2$
b $m + 8 = 3m + 5$
c $4p + 7 = p + 11$
d $6m = 2m + 1$
e $4a = a - 1$
f $8n + 6 = n + 6$
g $9a - 2 = 3a + 5$
h $7n - 9 = 9n + 1$
i $7q - 5 = 5q + 4$
j $8n + 7 = 4n + 17$
k $11a + 1 = 5a + 6$
l $4k - 7 = k - 10$
m $2a + 7 = 4a + 15$
n $x + 16 = 7x + 3$
o $3q + 7 = 6q - 1$
p $3m = 7m - 3$
q $4a = 11 - 3a$
r $x - 4 = 7x + 4$
s $2y = 5y + 60$
t $5 - 2x = 3 + x$
u $11 - m = 24 - 4m$

7:05 | Equations with Grouping Symbols

prep quiz 7:05

Rewrite these expressions without grouping symbols.

1 $7(x+4)$ **2** $2(a-3)$ **3** $5(4a+9)$ **4** $6(2p-7)$ **5** $-3(x-4)$

Solve these one-step equations.

6 $x+9=4$ **7** $x-8=-2$ **8** $3p=-27$ **9** $10p=5$ **10** $6+x=-1$

If you remember how to 'expand' grouping symbols, these equations are no harder than the ones you have already seen. Look at these worked examples.

worked examples

1 Expand the grouping symbols and then solve the equation.

a $2(x+3)=8$

$2x+6=8$

$-6 \quad -6$

$2x=2$

$\div 2 \quad \div 2$

$\therefore x=1$

b $5(a-3)=3$

$5a-15=3$

$+15 \quad +15$

$5a=18$

$\div 5 \quad \div 5$

$\therefore a=\frac{18}{5}$ or $3\frac{3}{5}$

c $3(2m-4)=4m-6$

$6m-12=4m-6$

$-4m \quad -4m$

$2m-12=-6$

$+12 \quad +12$

$2m=6$

$\div 2 \quad \div 2$

$\therefore m=3$

2 Expand each set of grouping symbols and then solve the equations.

a $3(a+7)=4(a-2)$

$3a+21=4a-8$

$-3a \quad -3a$

$21=a-8$

$+8 \quad +8$

$29=a$

$\therefore a=29$

b $3(x+4)+2(x+5)=4$

$3x+12+2x+10=4$

Collect like terms.

$5x+22=4$

$-22 \quad -22$

$5x=-18$

$\div 5 \quad \div 5$

$\therefore x=-3\frac{3}{5}$

Exercise 7:05

Foundation Worksheet 7:05

Equations with grouping symbols

1 Expand these grouping symbols.

a $5(x+3)$ **b** $7(a-4)$ **c** $9(2y+3)$

2 Solve these equations.

a $2(a+3)=8$ **b** $5(m-1)=10$ **c** $7(2n+1)=21$

1 Expand the grouping symbols and then solve each equation. (Answers are all integers.)

a $3(a+2)=18$ **b** $2(x+4)=10$

c $5(6+m)=45$ **d** $6(x-4)=6$ **e** $4(x-1)=20$ **f** $7(x-3)=70$

g $5(2y+1)=25$ **h** $3(2p-7)=9$ **i** $10(6+5x)=10$ **j** $8(1+x)=9x+4$

k $2(5a+3)=8a+10$ **l** $7(2a+3)=10a+33$ **m** $9(2x-1)=x+42$

n $6(m-4)=2m+8$ **o** $2(7y-5)=15y-16$ **p** $2a+5=3(a-1)$

q $12x+6=6(x+2)$ **r** $8-2x=3(x-4)$

2 Solve each equation.

a $5(a+1)=8$ **b** $2(m-3)=-3$ **c** $4(x-7)=3$
d $3(3-x)=12$ **e** $5(7-n)=15$ **f** $3(7-y)=9$
g $5(2a+3)=20$ **h** $2(5x-1)=15$ **i** $3(4x-5)=1$
j $4(x-2)=x-2$ **k** $5(n+3)=3n+11$ **l** $3(x-10)=x+20$
m $2-x=3(2x-1)$ **n** $8+7x=3(3x-1)$ **o** $8n-16=4(3n-7)$
p $7a=6(2a-1)$ **q** $2n=3(4n-10)$ **r** $5x=4(2x+9)$

3 Find the solution to each equation by expanding all grouping symbols first.

a $3(a+2)=2(a+1)$ **b** $5(x-1)=4(x+2)$
c $5(p-2)=4(p+2)$ **d** $3(q+2)=2(q+5)$
e $3(m+1)=5(m-1)$ **f** $6(x+2)=4(x+6)$
g $2(a-7)=5(a-4)$ **h** $7(t+2)=4(t+5)$
i $3(2a+1)=5(a+2)$ **j** $4(3p-1)=5(2p+1)$
k $6(t+7)=4(t+10)+8$ **l** $5(2a-1)=3(a+6)-7$
m $3(2+m)=5(2-m)+6m$ **n** $6(p+3)=5(2-p)+7p-12$

4 Solve each equation. Use Worked Example 2(b) as a guide.

a $3(a+2)+a+5=15$ **b** $5(m-1)+2m=2$
c $2(m+3)+5(m+2)=23$ **d** $3(x+2)+2(x-3)=10$
e $5(p+1)+2(p+4)=20$ **f** $4(t-2)+2(t+5)=14$
g $4(2a+3)+2(a-5)=22$ **h** $2(2m+3)+3(m-5)=5$
i $5(a-3)+3(2+3a)=19$ **j** $7(a+5)+2(6-3a)=1$

5 Try solving these equations, but first read the warning sign!

a $3(a+2)-2(a+1)=6$
b $5(m+3)-4(m+2)=10$
c $5(n+4)-3(n-2)=30$
d $6(a+2)-4(a-1)=20$
e $4(a+3)-(a+2)=13$
f $4(p+5)-(p+3)=23$
g $5(2a+1)-2(a-4)=2$
h $6(2x+5)-5(3x+2)=10$

Warning!
Remember how to expand with a negative term:
$-2(x+4)=-2x-8$
or
$-3(a-1)=-3a+3$

fun spot
7:05

Fun Spot 7:05 | If I have 7 apples in one hand and 4 in the other, what have I got?

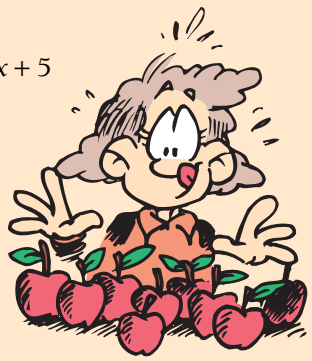
Solve each equation and put the letter for that part in the box above the correct answer.

I $x+12=7$ **A** $5x=2$ **H** $x-7=-2$

D $\frac{x}{10}=5$ **N** $5x+30=15$ **G** $15-3x=2x+5$

B $\frac{5x+21}{8}=3$ **S** $5(3x+8)+6(10-2x)=109$

$x=\frac{3}{5}$	$x=-5$	$x=2$	$x=5$	$x=\frac{2}{5}$	$x=-3$	$x=50$	$x=3$



7:06 | Equations with Fractions (1)

Simplify:

1 $-4+4$ 2 -3×-1 3 $-7+4$ 4 $\frac{15}{4}$ 5 $16\div -5$

6 $6\times\frac{p}{6}$ 7 $\frac{12p}{p}$ 8 $4\times\frac{(x+7)}{4}$ 9 $4\times\frac{5m}{4}$ 10 $4\left(3-\frac{p}{4}\right)$

prep quiz 7:06

Remove the fractions by multiplying by the denominator of the fraction.

worked examples

Find the value of the pronumeral in each of the following equations.

1 $\frac{y}{6}-1=3$ 2 $\frac{x+7}{4}=8$ 3 $\frac{3x-1}{5}=7$

Solutions

1 $\frac{y}{6}-1=3$

$\times 6 \quad \times 6$

$6\left(\frac{y}{6}-1\right)=6\times 3$

$\frac{6y}{6}-6=18$

$y-6=18$

$\therefore y=24$

2 $\frac{x+7}{4}=8$

$\times 4 \quad \times 4$

$\frac{4(x+7)}{4}=8\times 4$

$x+7=32$

$\therefore x=25$

3 $\frac{3x-1}{5}=7$

$\times 5 \quad \times 5$

$\frac{5(3x-1)}{5}=7\times 5$

$3x-1=35$

$3x=36$

$\therefore x=12$

1 **Make sure you multiply both sides of the equation by the same number.**
2 **Make sure you remove the parentheses correctly.**
$4(3+2)=4\times 3+4\times 2$
so
$4(x+2)=4\times x+4\times 2$
$=4x+8$

Exercise 7:06

1 Solve the following equations.

a $\frac{y}{4}+3=9$ **b** $\frac{y}{5}+3=7$ **c** $4+\frac{p}{6}=10$

d $7=\frac{m}{5}+4$ **e** $\frac{a}{4}-1=2$ **f** $13-\frac{m}{4}=6$

Foundation Worksheet 7:06

Equations with fractions

1 Solve:

a $\frac{x}{3}=2$ **b** $\frac{a}{5}=-2$ **c** $\frac{2m}{3}=6$

2 Simplify:

a $5\times\frac{(a+2)}{5}$ **b** $6\left(\frac{x}{6}-3\right)$

g $16 = \frac{m}{2} - 9$ **h** $5 = 8 - \frac{p}{6}$ **i** $\frac{5p}{2} = 4$

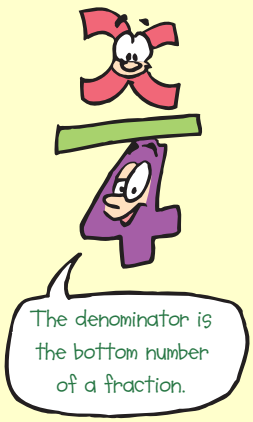
j $\frac{3m}{4} = 6$ **k** $15 = \frac{5p}{3}$ **l** $\frac{3}{5}m = 7$

m $\frac{p+3}{4} = 8$ **n** $\frac{5m+6}{3} = 12$ **o** $\frac{7+x}{4} = 2$

p $\frac{15-x}{4} = 3$ **q** $\frac{3m+4}{2} = 5$ **r** $\frac{6+5p}{4} = 9$

s $\frac{12a-3}{5} = 6$ **t** $\frac{7-3x}{3} = 1$ **u** $\frac{5a-2}{4} = -3$

v $\frac{-x-2}{4} = 5$ **w** $\frac{10+3p}{7} = 9$



2 Check to see if the given solution is correct or incorrect.

a $\frac{x}{4} + 7 = 8$, $x = 4$ **b** $\frac{y}{3} - 4 = 2$, $y = 10$ **c** $6 + \frac{m}{4} = 9$, $m = 12$ **d** $4 = \frac{m}{6} - 1$, $m = 30$

e $\frac{5m}{4} = 3$, $m = 3$ **f** $\frac{m+4}{2} = 5$, $m = 3$ **g** $\frac{p-5}{4} = 3$, $p = 17$ **h** $\frac{15-p}{4} = -1$, $p = 19$

i $\frac{3m+2}{4} = 1$, $m = \frac{2}{3}$ **j** $\frac{4m-6}{3} = 8$, $m = 6\frac{1}{2}$ **k** $\frac{1+3p}{2} = -1$, $p = -1$ **l** $\frac{9-5p}{5} = 8$, $p = -8$

3 Solve these equations which have pronumerals on both sides.

a $\frac{2m}{3} - 4 = m$ **b** $\frac{x}{5} + 3 = 2x$ **c** $p - 5 = \frac{p}{2}$

d $3x - 6 = \frac{3x}{4}$ **e** $\frac{x}{4} = 3 - x$ **f** $\frac{3n}{2} = 2n - 5$

g $\frac{a+3}{2} = a$ **h** $\frac{2x-1}{3} = 3x$ **i** $\frac{2y+3}{3} = 4y$

j $\frac{1-3p}{6} = p$ **k** $\frac{4-x}{5} = x + 2$ **l** $\frac{3b-1}{4} = 2 - b$

- Solve equations one step at a time.

Fun Spot 7:06 | Who holds up submarines?

Solve each equation and put the letter for that part in the box above the correct answer.

I $\frac{m}{4} = 8$

I $\frac{x}{5} = 4$

U $\frac{2n}{3} = 5$

E $\frac{5y}{3} = 2$

Y $\frac{x}{4} + 1 = 5$

S $\frac{a}{6} + 7 = 8$

L $\frac{x}{2} - 8 = 5$

H $\frac{m}{6} - 5 = 3$

L $\frac{a+2}{3} = 4$

D $\frac{n-3}{5} = 2$

T $\frac{3y+1}{5} = 2$

Q $\frac{7m-4}{2} = 5$

B $\frac{12+3x}{4} = 6$

4	20	10	26	16	3	48	1·2	6	2	7·5	32	13

7:07 | Graphing Inequations

prep quiz 7:07

1 3×-2 **2** -4×-3 **3** $-12 \div 6$

4 $-6 \div -3$ **5** $-6 > 2$, *true* or *false*? **6** $-3 < -4$, *true* or *false*?

7 What is the meaning of '>'? **8** Graph the set $\{1, 3, 5\}$ on a number line.

Insert either < or > to make each of the following sentences true.

9 $-2 \square -4$ **10** $-4 \square 6$

An inequation is a number sentence where the 'equals' sign has been replaced by an inequality sign. The most common inequality signs are:

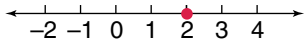
$>$	$<$	$\geqslant$	$\leqslant$
'is greater than'	'is less than'	'is greater than or equal to'	'is less than or equal to'

Inequations, unlike equations, usually have more than one solution. For instance:

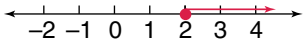
- the equation $x + 6 = 10$ has one solution, namely $x = 4$.
- the inequation $x + 6 > 10$ has an infinite number of solutions. The numbers $4\frac{1}{2}$, 8, 9·5, 30 are some solutions. The full set of solutions is written as $x > 4$.

The solutions of inequations are often graphed on a number line.

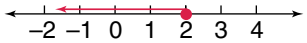
worked examples

1 

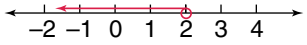
This shows the solution $x = 2$.

2 

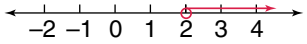
This shows the solution $x \geqslant 2$.

3 

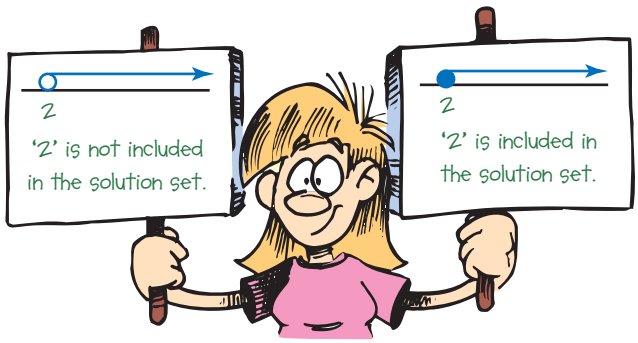
This shows the solution $x \leqslant 2$.

4 

This shows the solution $x < 2$.

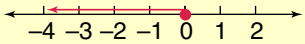
5 

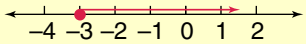
This shows the solution $x > 2$.

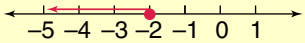


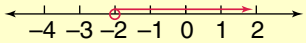
Exercise 7:07

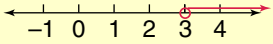
1 State the set of values of x which have been graphed below.

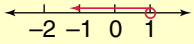
a 

b 

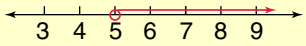
c 

d 

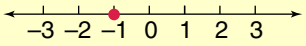
e 

f 

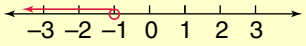
g



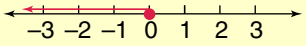
h



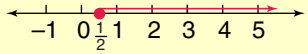
i



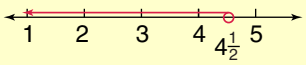
j



k



l



2 Graph each of the following solutions on a separate number line.

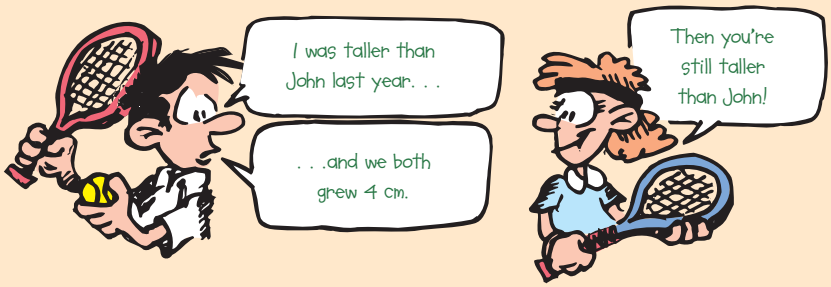
a $x \leqslant 2$ **b** $x > 1$ **c** $x \geqslant -2$ **d** $x < 0$

e $x > -4$ **f** $x > 3$ **g** $x \leqslant -1$ **h** $a < 1$

i $m \geqslant -5$ **j** $p = 3$ **k** $a \leqslant 3\frac{1}{2}$ **l** $b > -1\frac{1}{2}$

7:08 | Solving Inequations

Practical Activity 7:08 | Operating on inequations



- When we solved equations we saw that performing the same operation on both sides gave an equivalent equation.
- Use the examples below to investigate what happens with inequations.

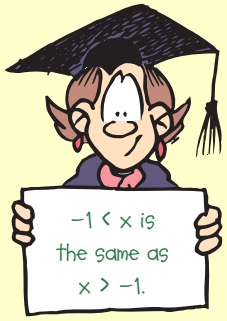
Each of the following inequations is true. Perform the operation indicated on both sides of each inequation and state if the inequation remains true.

1	**a**	$6 > 2$ Add 4	**b**	$-6 < -4$ Add 4	**c**	$6 > -4$ Add 4	**d**	$-2 < 4$ Add 4
2	**a**	$6 > 2$ Subtract 4	**b**	$-6 < -4$ Subtract 4	**c**	$6 > -4$ Subtract 4	**d**	$-2 < 4$ Subtract 4
3	**a**	$6 > 2$ Multiply by 2	**b**	$-6 < -4$ Multiply by 2	**c**	$6 > -4$ Multiply by 2	**d**	$-2 < 4$ Multiply by 2
4	**a**	$6 > 2$ Divide by 2	**b**	$-6 < -4$ Divide by 2	**c**	$6 > -4$ Divide by 2	**d**	$-2 < 4$ Divide by 2
5	**a**	$6 > 2$ Multiply by -3	**b**	$-6 < -4$ Multiply by -3	**c**	$6 > -4$ Multiply by -3	**d**	$-2 < 4$ Multiply by -3
6	**a**	$6 > 2$ Divide by -2	**b**	$-6 < -4$ Divide by -2	**c**	$6 > -4$ Divide by -2	**d**	$-2 < 4$ Divide by -2

- Write down the results of your investigation. Compare these with the results obtained by others in your class.

From the practical activity, you should have found that multiplying or dividing a true inequation by a negative numeral did not produce a true inequation. This is the very important difference between equations and inequations.

To allow for this, the inequality sign must be reversed when an inequation is multiplied or divided by a negative numeral.



When multiplying or dividing an inequation by a negative numeral, the inequality sign must be reversed to obtain an equivalent inequality.

worked examples

Solve the following inequations.

1 $2x + 3 < 6$

2 $\frac{x}{2} - 3 \leqslant 7$

3 $5 - 3x > 6$

4 $-\frac{1}{3}x < 5$

5 $2(1 - 2x) \leqslant 6$

Solutions

1
$$\begin{aligned} 2x + 3 &< 6 \\ -3 \quad &-3 \\ \therefore 2x &< 3 \\ \div 2 \quad &\div 2 \\ \therefore x &< \tfrac{3}{2} \\ \therefore x &< 1\tfrac{1}{2} \end{aligned}$$

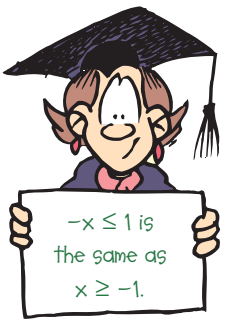
2
$$\begin{aligned} \frac{x}{2} - 3 &\leqslant 7 \\ +3 \quad &+3 \\ \therefore \frac{x}{2} &\leqslant 10 \\ \times 2 \quad &\times 2 \\ \therefore 2 \times \frac{x}{2} &\leqslant 10 \times 2 \\ \therefore x &\leqslant 20 \end{aligned}$$

3
$$\begin{aligned} 5 - 3x &> 6 \\ -5 \quad &-5 \\ \therefore -3x &> 1 \\ \div(-3) \quad &\div(-3) \\ &\text{(reverse sign)} \\ \therefore x &< -\tfrac{1}{3} \end{aligned}$$

4
$$\begin{aligned} -\tfrac{1}{3}x &< 5 \\ \times(-3) \quad &\times(-3) \\ &\text{(reverse sign)} \\ \therefore -\tfrac{1}{3}x \times (-3) &> 5 \times (-3) \\ \therefore x &> -15 \end{aligned}$$

5
$$\begin{aligned} 2(1 - 2x) &\leqslant 6 \\ \therefore 2 - 4x &\leqslant 6 \\ -2 \quad &-2 \\ -4x &\leqslant 4 \\ \div -4 \quad &\div -4 \\ \therefore x &\geqslant -1 \end{aligned}$$

OR
$$\begin{aligned} 2(1 - 2x) &\leqslant 6 \\ \therefore 2 - 4x &\leqslant 6 \\ +4x \quad &+4x \\ \therefore 2 &\leqslant 6 + 4x \\ -6 \quad &-6 \\ \therefore -4 &\leqslant 4x \\ \div 4 \quad &\div 4 \\ \therefore -1 &\leqslant x \\ \therefore x &\geqslant -1 \end{aligned}$$



Exercise 7:08

Foundation Worksheet 7:08

Solving inequations

1 Show on a number line:

a $x > 3$ **b** $x \leqslant 2$ **c** $x < -1$

2 Solve:

a $x + 3 > 5$ **b** $m - 2 \leqslant 6$ **c** $n + 7 < 4$

1 Solve the following inequations and graph the solution on a number line.

a $x + 6 < 15$ **b** $y + 7 > 9$

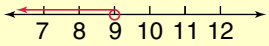
c $m + 6 < 4$ **d** $m + 10 > 5$

e $5 + p \leqslant 2$ **f** $15 \geqslant m + 6$

g $y - 3 > 2$ **h** $m - 6 < 1$

i $x - 5 \geqslant 5$ **j** $12 < m - 6$

k $3 > p - 2$ **l** $-1 \leqslant x - 1$



2 Solve the following inequations.

a $3m > 21$ **b** $6p \leqslant 42$

c $12m < 24$ **d** $5y \geqslant -42$

e $15 > 4x$ **f** $-20 < 10x$

g $\frac{x}{3} < 5$ **h** $\frac{y}{2} > 6$

i $\frac{m}{4} \geqslant 1$ **j** $\frac{x}{5} < -10$

k $3 > \frac{x}{2}$ **l** $-4 < \frac{x}{3}$

3 Solve the following inequations.

a $-3m < 24$ **b** $-4x > 16$

c $-5p \leqslant 20$ **d** $-6x < -12$

e $15 < -6x$ **f** $10 \geqslant -3x$

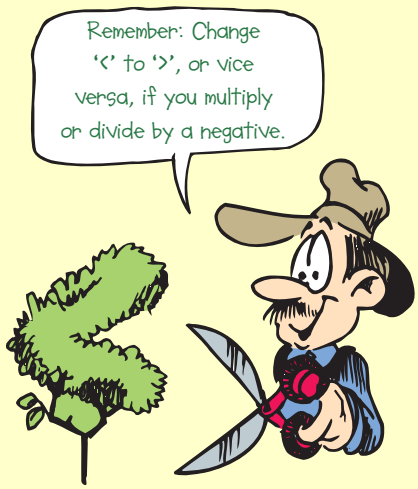
g $-x > 4$ **h** $-x < 3$

i $5 \geqslant -x$ **j** $-\frac{1}{2}x > 3$

k $-\frac{1}{5}x > 1$ **l** $-\frac{1}{4}x \geqslant 2$

m $\frac{x}{-2} < 1$ **n** $\frac{x}{-3} > 2$

o $-\frac{x}{4} > 3$



4 Solve:

a $2x + 5 > 11$ **b** $4m + 3 < 19$

c $2p + 1 \geqslant 7$ **d** $10 + 3p \leqslant 7$

e $4 + 5p \geqslant 10$ **f** $13 + 2x < 6$

g $3x - 8 \leqslant 4$ **h** $5p - 1 \geqslant 9$

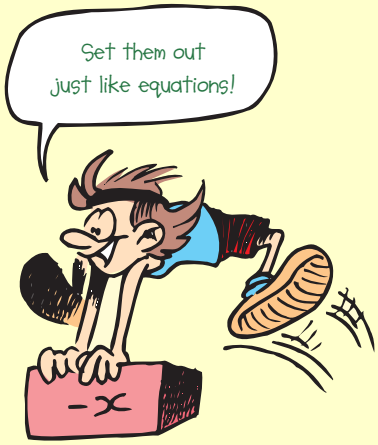
i $4y - 3 < 2$ **j** $12 > 5x - 3$

k $3 < 2x - 1$ **l** $3 \geqslant 8x - 9$

m $2(x + 3) < 14$ **n** $3(m + 2) > 15$

o $3(2x - 5) \leqslant 6$ **p** $4(x - 3) \geqslant 5$

q $4 < 2(2m - 3)$ **r** $2(5p - 4) > 22$



5 Solve:

a $5 - 3m > 11$ **b** $10 - 2y < 14$ **c** $12 \leqslant 6 - 4x$

d $10 - x \leqslant 12$ **e** $2 - m \geqslant -4$ **f** $17 - 2m \leqslant 1$

g $9 - 4p < 7$ **h** $2(1 - x) < 6$ **i** $3(4 - y) > 15$

j $3(4 - 2x) \geqslant 18$ **k** $2(3 - 5y) \leqslant -4$ **l** $2(3 - 2p) > 8$

m $4 \geqslant 3(1 - 2x)$ **n** $2(5 - 2a) \leqslant -5$ **o** $3(1 - 2x) > 2$

6 Find the solution of each of the following inequations.

a $5x + 6 > x + 18$ **b** $3x - 5 < x + 6$ **c** $3 - a \leqslant 5 - 2a$

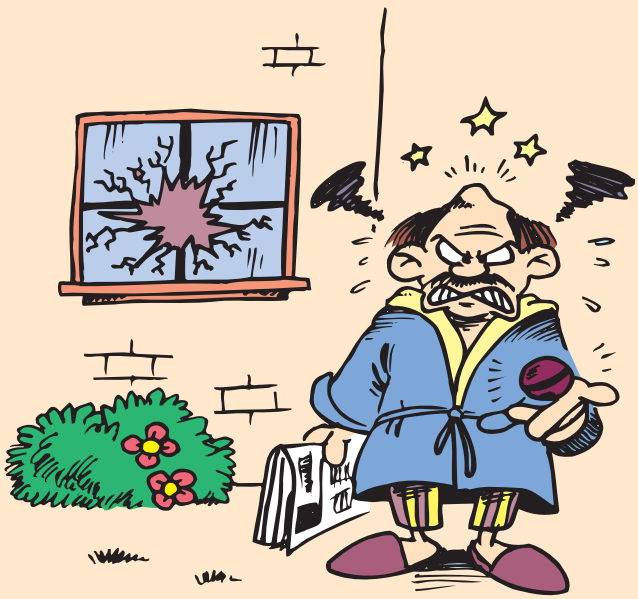
d $12 - b \geqslant 2b + 21$ **e** $\frac{x}{2} + 1 < 6$ **f** $\frac{p}{3} - 1 > 4$

g $5 - \frac{2y}{3} \leqslant 6$ **h** $\frac{p - 1}{4} < 2$ **i** $\frac{4 - x}{3} \geqslant 1$

fun spot 7:08

Fun Spot 7:08 | Who 'dunnit'?

Four boys playing cricket in the park were having a great game when suddenly one of them hit a mighty six, right through Mr Yorker's window. Mr Yorker stormed out and demanded to know who did it.



Their responses were:

Bob: 'Steve did it.'
Steve: 'Mark did it.'
Alan: 'I didn't do it.'
Mark: 'Steve lied when he said I did it.'

If only one of the boys is *not* telling the truth, who hit the ball through the window?

On the other hand, if only one of the four statements is true, who hit the ball?

Mathematical terms 7

equation

- A number sentence where one or more of the numbers is missing or unknown.
- The unknown number is represented by a pronumeral.

  eg $x + 5 = 8$, $\frac{3x + 1}{7} = \frac{x - 5}{2}$

expression

- An algebraic expression consists of one or more terms joined together by operation signs.

  eg $a + 5$, $x^2 - x + 4$, $\frac{3m - 1}{7}$

- An expression does *not* have an 'equals' sign like an equation.

grouping symbols

- The most common types are:

  parentheses ( )
  brackets [ ]
  braces { }
- Used to 'group' a number of terms together in an expression.

  eg $5(x + 3)$

inequality signs

- $>$ greater than, $<$ less than
- $\geqslant$ greater than or equal to, $\leqslant$ less than or equal to

  eg $x + 3 < 4$ means that $x + 3$ *is less than* 4

inequation

- An *equation* where the 'equals' sign has been replaced by an inequality sign.

  eg $4x - 1 > 5$ or $\frac{x}{3} \leqslant 4$

inverse operation

- The operation that will reverse or 'undo' a previous operation.

  eg addition is the inverse operation of subtraction; division is the inverse operation of multiplication

pronumeral

- A symbol used to represent a number.
- Usually a letter such as x.

solution

- Method of finding the answer to a problem OR
  the answer to a problem.
- The solution to an equation is the number or numbers that satisfy the equation or make it a true sentence.

  eg $x = 3$ is the solution to $x + 2 = 5$

solve

- Find the *solution* or answer to a problem or equation.

substitute

- To replace a pronumeral with a numeral.

  eg to substitute 3 for a in the expression $4a - 2$ would give:
  $4(3) - 2$
  $= 12 - 2$
  $= 10$

Diagnostic Test 7: | Equations and Inequations

diagnostic test 7

- Each part of this test has similar items which test a certain type of question.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
1 Solve: **a** $x+7=13$ **b** $y+10=19$ **c** $p-4=10$ **d** $m-7=3$	7:01
2 Solve: **a** $5x=30$ **b** $6x=48$ **c** $\frac{y}{3}=9$ **d** $\frac{y}{4}=5$	7:01
3 Solve: **a** $6+p=24$ **b** $30-x=10$ **c** $\frac{12}{x}=2$ **d** $12=24-t$	7:01
4 Check the given solution to see if it is correct or incorrect. **a** $a+3=5$ $a=12$ **b** $y-3=9$ $y=6$ **c** $6x=42$ $x=6$ **d** $\frac{x}{3}=6$ $x=18$	7:01
5 Solve: **a** $4p+3=31$ **b** $2m-7=17$ **c** $9p=76$ **d** $25=5-2m$	7:03
6 Solve: **a** $3x+5=2x+1$ **b** $5a-7=3a-1$ **c** $4b+7=b-8$	7:04
7 Solve: **a** $3(x+1)=9$ **b** $4(a-3)=24$ **c** $6(x-3)+4x=8$	7:05
8 Solve: **a** $\frac{y}{2}+1=7$ **b** $\frac{m}{2}-1=5$ **c** $4=13-\frac{p}{2}$ **d** $\frac{3m}{5}=6$	7:06
9 Solve: **a** $\frac{m+3}{4}=2$ **b** $\frac{m-6}{3}=1$ **c** $\frac{3p-7}{2}=7$ **d** $\frac{5+3x}{5}=5$	7:06
10 Solve: **a** $\frac{m}{5}+2=m$ **b** $2x+1=\frac{3x}{2}$ **c** $\frac{n+7}{4}=2n$ **d** $\frac{5a-2}{3}=3-a$	7:06
11 Graph the following on a number line. **a** $x>3$ **b** $x\leqslant -1$ **c** $x\geqslant 0$ **d** $x<5$	7:07
12 Solve: **a** $2x+5\geqslant 6$ **b** $\frac{2x-1}{3}<6$ **c** $3x-7>x+3$	7:08
13 Solve: **a** $5-3x>11$ **b** $-\frac{1}{3}x>21$ **c** $3-4a>2-a$	7:08

assignment 7A

Chapter 7 | Revision Assignment

1 Solve these one-step equations.

a $4x = 13$ b $\frac{a}{2} = 6$

c $m + 73 = 117$ d $n - 61 = 27$

2 Solve:

a $5m - 7 = 8$ b $3y + 7 = 4$

c $6m - 1 = 17$ d $4n + 10 = 2$

e $3p + 6 = 5$ f $2k - 7 = 10$

g $5y - 7 = 2$ h $9q + 2 = 5$

3 Solve for x:

a $5x + 3 = 4x + 7$ b $9x - 3 = 7x + 5$

c $8x + 6 = 5x - 3$ d $6x - 4 = 4x + 5$

e $3x + 7 = 5x - 4$ f $x - 5 = 6x + 3$

g $10 + 4x = 3x - 1$ h $8 - 3x = 7 + 2x$

i $12 - 5x = 10 - 3x$

4 Solve these equations which involve grouping symbols.

a $5(x + 7) = 30$ b $7(a - 3) = 21$

c $8(m - 1) = 4$ d $4(x + 3) = 3(x + 2)$

e $5(n - 2) = 3(n + 4)$

f $10(x - 7) = 7(x - 10)$

g $4(2a + 3) + 3(a - 3) = 5$

h $5(3n + 4) + 2(5 - 2n) = 7$

i $6(m + 4) - 5(m + 3) = 6$

j $7(4x + 3) - 3(8x - 5) = 0$

5 Solve:

a $\frac{x}{4} + 3 = 5$ b $\frac{m}{5} - 2 = 1$

c $\frac{2x}{3} - 4 = 2$ d $\frac{a + 7}{4} = 6$

e $\frac{y - 5}{3} = 1$ f $\frac{3p + 1}{5} = 2$

g $\frac{2m}{3} - 1 = m$ h $\frac{5m - 1}{4} = 2m$

i $2n + 5 = \frac{n - 1}{3}$

6 Solve and graph each solution on a number line.

a $m + 7 \geqslant 5$ b $2x - 1 < 7$

c $5n + 1 > 3$ d $3x + 7 \leqslant x + 10$

e $y - 5 > 3y - 8$ f $4n + 7 \geqslant 7n - 4$

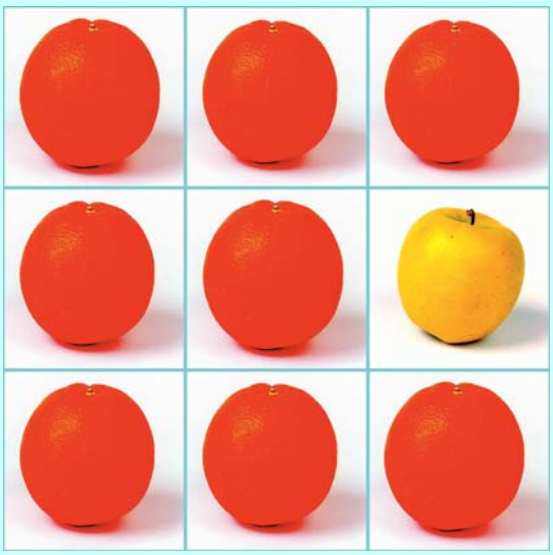
g $6 - 2x > 14$ h $-\frac{2x}{3} \geqslant 6$

i $10 - 3a \leqslant 7 - a$

Technology Applications Drag and Drops
1 Equations
3 Solving inequations

Technology Applications Activities
Flowcharts

Chapter Review Questions



- An apple costs 20c more than an orange. The cost of 8 oranges and 1 apple is $6.50. An equation that represents this information is $8x + (x + 20) = 650$.

Chapter 7 | Working Mathematically

assignment 7B

1 Use ID Card 7 on page xix to identify:

a 13	b 14	c 15	d 16	e 19
f 20	g 21	h 22	i 23	j 24

2 Use the Algebra Card on page xxii to:

a multiply column H by column J
b multiply column N by column A
c square column G
d multiply column O by column H

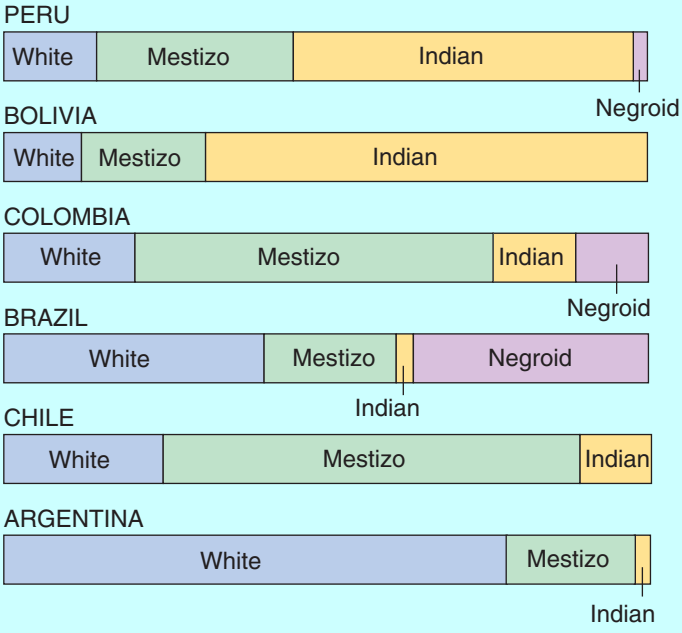
3 Ten students lined up at a tap to get a drink. Each person took one minute to drink.

a How long did it take before the last student finished drinking?
b What was the total time spent by all ten students in waiting and drinking.

4 Find the basic numeral for:

$(1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{5}) + (\frac{1}{5} - \frac{1}{6}) + (\frac{1}{6} - \frac{1}{7}) + (\frac{1}{7} - \frac{1}{8}) + (\frac{1}{8} - \frac{1}{9}) + (\frac{1}{9} - \frac{1}{10})$

5 Racial composition of South American Republics.



a Of the six countries which has:
  i the largest percentage of white population?
  ii the largest percentage of Indian population?
  iii the largest percentage of Mestizo population?
  iv the largest percentage of Negroid population?
b Which countries do not have the four racial types in their population?
c What percentage of Colombia's population is Mestizo?
d What percentage of Brazil's population is Negroid?
e Can you tell which country has the largest population?

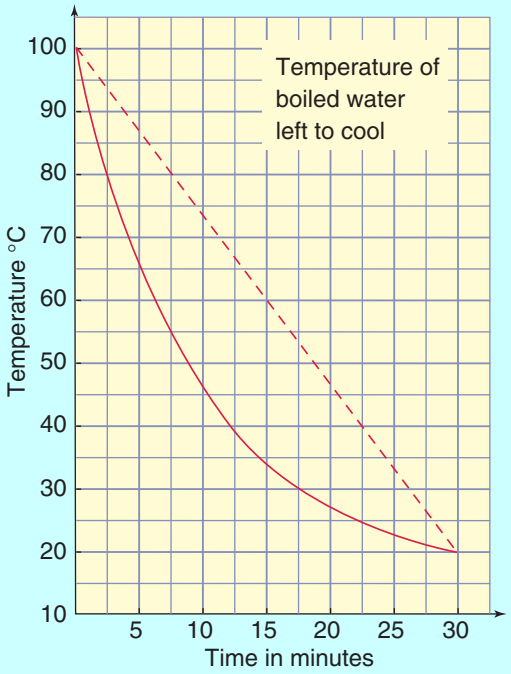
6 a How long does it take the water to reach a temperature of 80°C?

b What is the temperature of the water after 10 min?

c What is the temperature of the water after 20 min?

d From the graph it can be seen that the water doesn't cool at a constant rate.

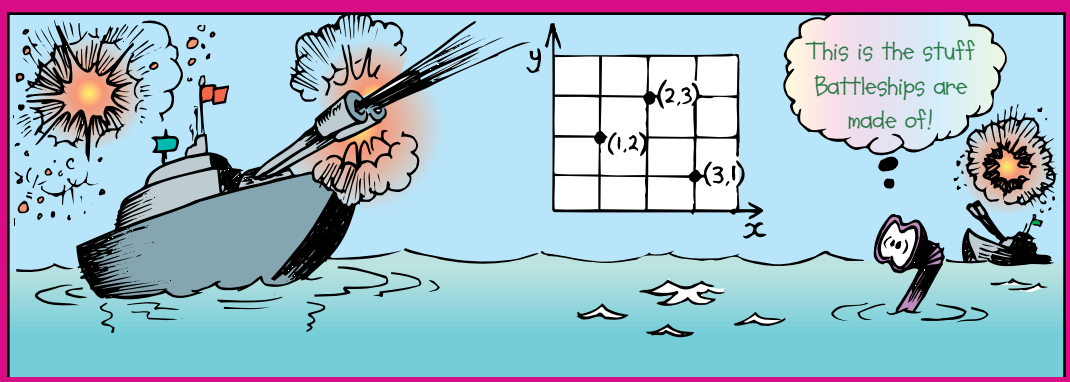
The dotted line represents a constant cooling rate. If the water had cooled at a constant rate, what would its temperature have been after 10 min?



• Many equations need to be solved during the design of an aeroplane.

8

Coordinate Geometry: Graphing Straight Lines



Chapter Contents

Learning Outcomes

Students will be able to:

- Graph and interpret linear relationships on the number plane.
- Use algebraic techniques to find intercepts and explore equations of lines on the number plane.
- Solve simultaneous equations graphically.
- Determine the gradient of an interval between two points.
- Determine the gradient and *y*-intercept of a line given its equation.
- Determine whether lines are parallel from their equation.
- Draw and interpret distance time graphs.

Areas of Interaction

Approaches to Learning, Homo Faber

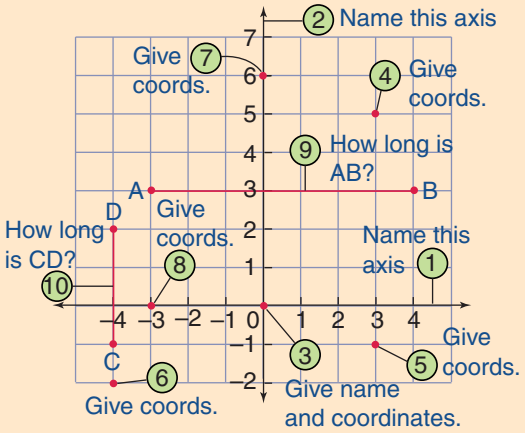
8:01 | Number Plane Review

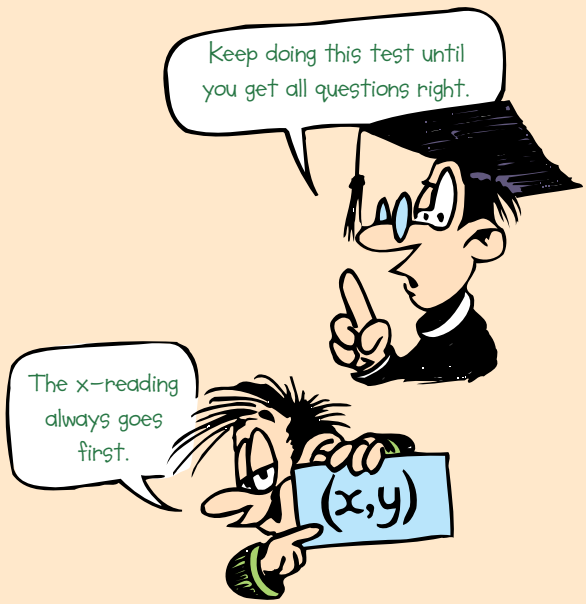
The French mathematician René Descartes first introduced the number plane. He realised that using two sets of lines to form a square grid allowed the position of a point in the plane to be recorded using a pair of numbers or coordinates.

The basic ideas behind his system should have been met in books 1 and 2. Check your knowledge by doing the Prep Quiz below.

prep quiz

8:01





Exercise 8:01

Foundation Worksheet 8:01

Co-ordinates

1 On the number plane shown give the coordinates of these points:
a A **b** M . . .

2 Write down the letter naming the points:
a (1, 2) **b** (2, 4) . . .

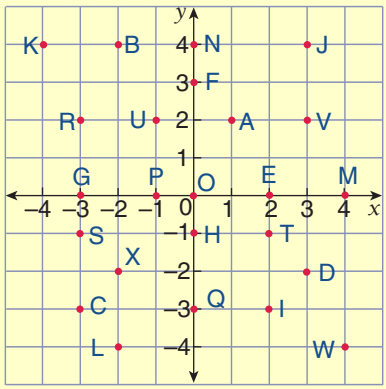
3 Write down the coordinates of the points A, B, C and D on the following lines.

1 Write down the letter naming the point with these coordinates.

a (3, 2)	**b** (−2, 4)	**c** (3, 4)
d (2, −1)	**e** (0, 3)	**f** (−2, −2)
g (0, 0)	**h** (−3, 0)	**i** (3, −2)

2 From the figure on the right write down the coordinates of these points.

a *A*	**b** *B*	**c** *C*	**d** *D*
e *E*	**f** *F*	**g** *G*	**h** *H*
i *I*	**j** *J*	**k** *K*	**l** *L*



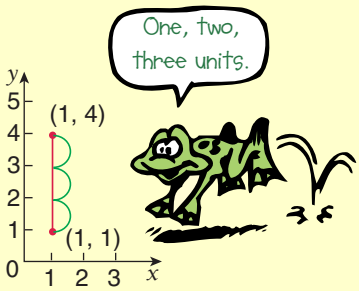
3 From the figure on the right write down the points:
a with a *y*-coordinate of 2
b with an *x*-coordinate of −3

4 Plot the points below on a number plane. What shapes are formed if the points are joined in the given order?

a (−3, 1), (3, 1), (3, −1), (−3, −1), (−3, 1)	**b** (−1, −1), (0, 4), (1, −1), (−1, −1)
c (2, 3), (6, 3), (5, 0), (1, 0), (2, 3)	**d** (1, 3), (3, 3), (4, 1), (−2, 1), (1, 3)
e (−2, 0), (−1, 2), (0, 0), (−1, −2), (−2, 0)	

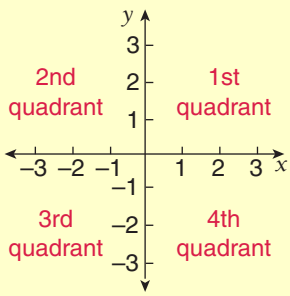
5 Find the number of units or distance between:

a (2, 0) and (5, 0)	**b** (0, 1) and (0, 4)
c (−1, 0) and (1, 0)	**d** (0, −3) and (0, 1)
e (−5, 0) and (−1, 0)	**f** (0, −4) and (0, −2)
g (3, 0) and (−1, 0)	**h** (2, 0) and (−2, 0)
i (0, 5) and (0, −2)	**j** (1, 2) and (4, 2)
k (−2, −1) and (1, −1)	**l** (4, 3) and (−2, 3)
m (3, 1) and (3, 4)	**n** (−2, −3) and (−2, 1)
o (1, 5) and (1, −3)	**p** (0, −2) and (3, −2)
q (4, −4) and (4, 0)	**r** (0, −3) and (0, −5)



6 In which of the quadrants does each point fall?

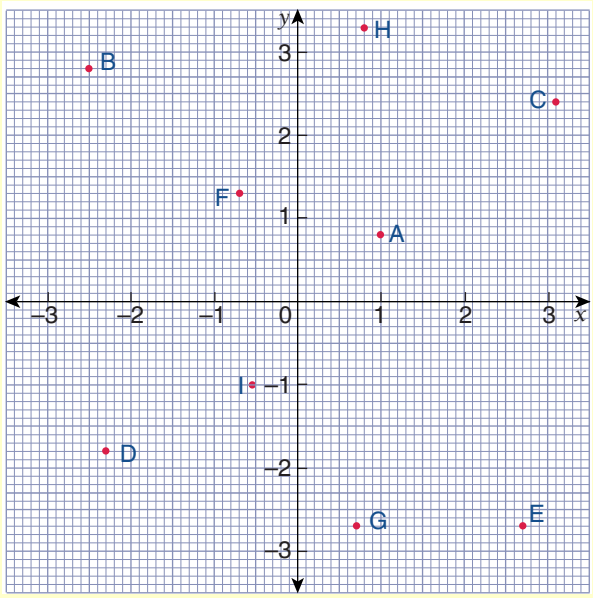
a (3, 2)	**b** (−1, 2)	**c** (−2, −2)
d (1, −2)	**e** (−3, −2)	**f** (1, 1)
g (−1, −2)	**h** (−3, 1)	**i** (3, −1)
j (−1, 1)	**k** (−2, −1)	**l** (1, −1)
m (2, 2)	**n** (2, −2)	**o** (−2, 2)
p (3, 1)	**q** (−4, −1)	**r** (4, 4)



7 The grid size of the number plane on the right allows the coordinates of the points to be given to 1 decimal place.

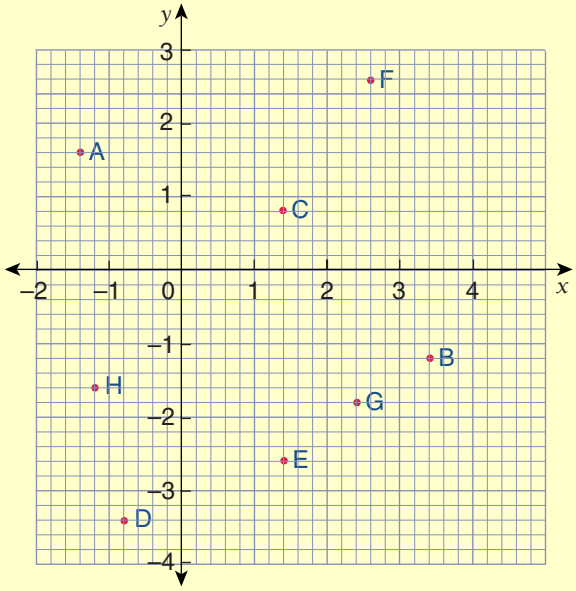
a Write down the coordinates of:
i A **ii** B **iii** C **iv** D

b Write down the letter naming the points:
i (−0·7, 1·3) **ii** (−0·5, −1·0)
iii (2·7, −2·7) **iv** (0·8, 3·3)

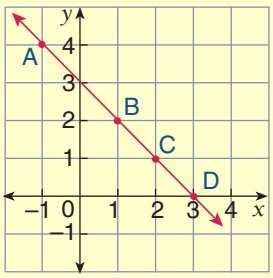


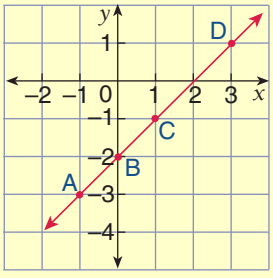
8 **a** Write down the letter naming the points:
i (−1·2, −1·6) **ii** (3·4, −1·2)
iii (1·4, −2·6) **iv** (2·4, −1·8)

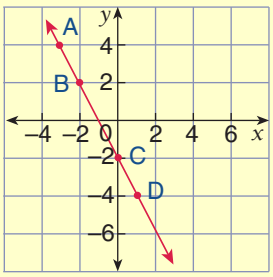
b Write down the coordinates of:
i C **ii** A **iii** E **iv** D



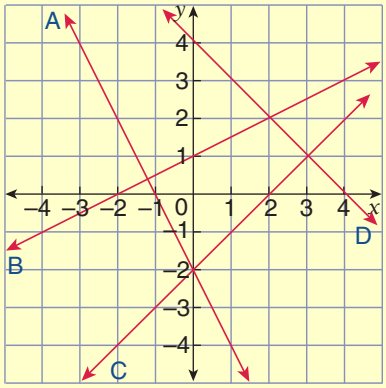
9 Write down the coordinates of the points A, B, C and D on each of the following lines.

a


b


c


10 **a** The x-coordinates for some of the points on the lines A to D are given. Find the missing y-coordinates.



Line A

x	−3	−2	0	1
y				

Line B

x	−4	−2	0	2
y				

Line C

x	−2	−1	2	3
y				

Line D

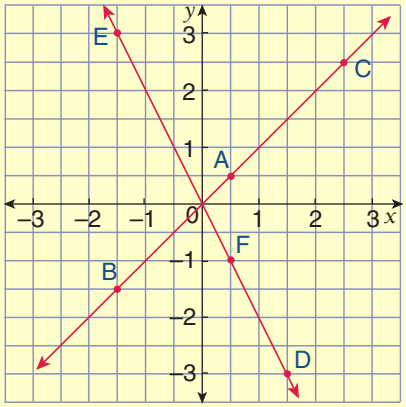
x	−1	0	2	4
y				

b If Line C was extended it would pass through the points $(5, a)$ and $(-4, b)$. What is the value of a and b?

11 **a** Write down the coordinates of three points that lie on the line AB between the points A and C.

b Write down the coordinates of three points on the line ED between F and D.

c If the line ED was extended it would pass through the points $(2, a)$, $(-2, b)$ and $(-2\frac{1}{2}, c)$. Find the values of a, b and c.



12 The points on a line can be described by a rule. Which of the lines A to F on the next page is best described by the following rule.

a The x-coordinate is equal to the y-coordinate.

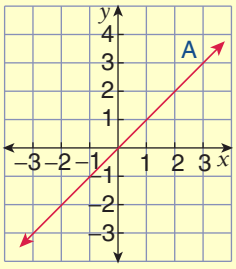
b The y-coordinate is double the x-coordinate.

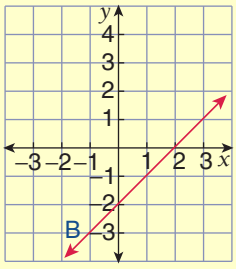
c The x-coordinate and the y-coordinate add up to 3.

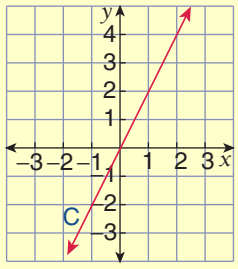
d The y-coordinate is 2 less than the x-coordinate.

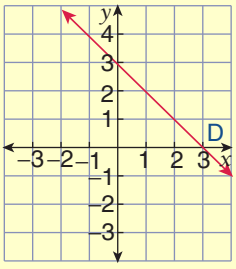
e Twice the x-coordinate plus the y-coordinate equals 2.

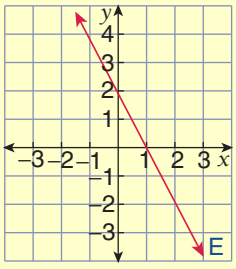
f Twice the x-coordinate plus the y-coordinate equals 0.

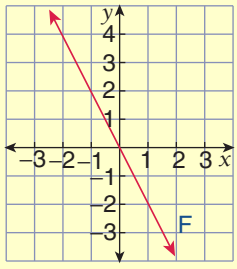










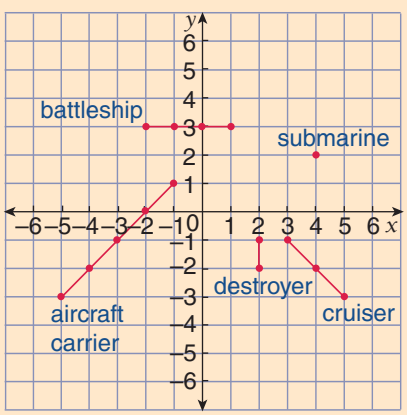


fun spot

8:01

Fun Spot 8:01 | Battleships: a number plane game

- This game is between two players who know how to mark points on a number plane. Each player has a navy of five ships as shown in the diagram. Each of these five ships is drawn onto a number plane, extending from −5 to +5 on each axis, without the opposing player seeing their positions.
- Players then take it in turns to call out a pair of coordinates, for example (−3, 2). This would be a 'miss' on the number plane shown, and so the opponent would call out 'miss'.
- Note that it takes five 'hits' to sink an aircraft carrier, four for a battleship, and so on.
- The first player to sink his/her opponent's navy is the winner.



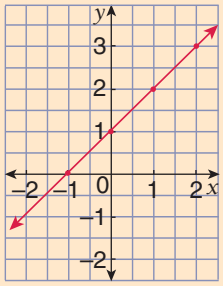
8:02 | Graphing Straight Lines

prep quiz 8:02

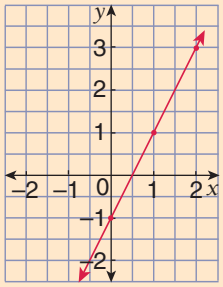
If $y = 2x + 1$ find y when x is: **1** 0 **2** 1 **3** −2

If $x + y = 3$ find y when x is: **4** 1 **5** −2

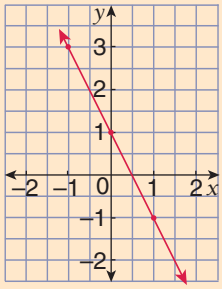
If $x - y = 3$ find y when x is: **6** 1 **7** −2



Graph A



Graph B



Graph C

The tables below give the coordinates of three points on a line. Which of the graphs above corresponds to each table.

8

x	−1	0	1
y	3	1	−1

9

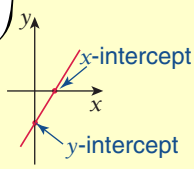
x	−1	0	1
y	0	1	2

10

x	0	1	2
y	−1	1	3

To graph a straight line we need:
- an equation to allow us to calculate the x- and y-coordinates for each point on the line
- a table to store at least two sets of coordinates
- a number plane on which to plot the points

Two important points on a line are:
- the x-intercept (where the line crosses the x-axis)
- the y-intercept (where the line crosses the y-axis)



worked examples

Draw the graph of each straight line. From the graph write down the line's x- and y-intercepts.

1 $x + y = 5$ **2** $y = 3x - 2$ **3** $4x + y = 2$

Solutions

1 $x + y = 5$

x	0	1	2
y	5	4	3

2 $y = 3x - 2$

x	0	1	2
y	−2	1	4

3 $4x + y = 2$

x	0	1	2
y	2	−2	−6

Note: We may choose any three values for x, but it is usually best to use 0, 1 and 2.

When $x = 0$,
$0 + y = 5$
$\therefore\ y = 5$

When $x = 1$,
$1 + y = 5$
$\therefore\ y = 4$

When $x = 2$,
$2 + y = 5$
$\therefore\ y = 3$

When $x = 0$,
$y = 3 \times 0 - 2$
$= -2$

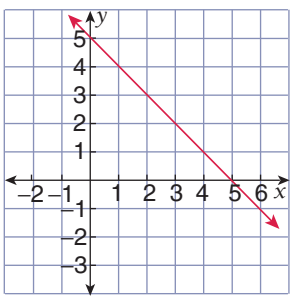
When $x = 1$,
$y = 3 \times 1 - 2$
$\therefore\ y = 1$

When $x = 2$,
$y = 3 \times 2 - 2$
$\therefore\ y = 4$

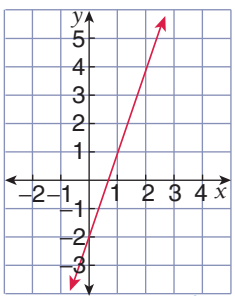
When $x = 0$,
$4 \times 0 + y = 2$
$0 + y = 2$
$\therefore\ y = 2$

When $x = 1$,
$4 \times 1 + y = 2$
$4 + y = 2$
$\therefore\ y = -2$

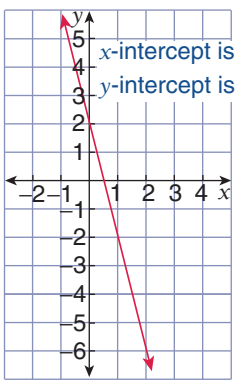
When $x = 2$,
$4 \times 2 + y = 2$
$8 + y = 2$
$\therefore\ y = -6$



x-intercept is 5
y-intercept is 5



x-intercept is $\frac{2}{3}$
y-intercept is -2



x-intercept is $\frac{1}{2}$
y-intercept is 2

Exercise 8:02

1 Complete the table for each equation.

a $y = 3x$

x	0	1	2
y			

b $y = x + 1$

x	0	1	2
y			

c $y = 5 - x$

x	0	1	2
y			

d $y = 6x - 7$

x	2	3	4
y			

e $y = 18 - 5x$

x	2	3	4
y			

f $y = 7x + 2$

x	2	3	4
y			

g $y = x - 7$

x	0	1	2
y			

h $y = 1 - x$

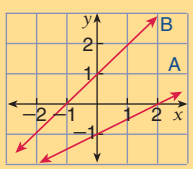
x	0	1	2
y			

Foundation Worksheet 8:02

Graphing lines

1 Complete the tables for the equations:
a $y = x$ **b** $y = x + 2$ **c** $x + y = 2$

2 Read off the x- and y-intercept of line:
a A **b** B



2 Draw the graph of each of the following lines. Use axes numbered from −3 to 7. In each case complete the table first.

a $y = x + 1$

x	0	1	2
y			

b $y = 2x$

x	0	1	2
y			

c $y = -x + 1$

x	0	1	2
y			

d $y = 3x + 1$

x	0	1	2
y			

e $y = 6 - 2x$

x	0	1	2
y			

f $y = x + 3$

x	0	1	2
y			

3 Graph the lines described by these equations.

a $x + y = 3$ **b** $x + y = 5$ **c** $x + y = 2$
d $y = x + 1$ **e** $y = x + 4$ **f** $y = x + 2$
g $y = 2x - 1$ **h** $y = 3x - 1$ **i** $y = 2x - 2$

4 On the same number plane draw the graphs of the equations.

a $y = 2x - 1$ **b** $y = 2x$ **c** $y = 2x + 3$

How are these lines alike?

5 Graph these equations on one number plane.

a $y = x$ **b** $y = 2x$ **c** $y = 3x$

What do these lines have in common?

6 On one number plane, graph these equations.

a $y = 2x - 1$ **b** $x + y = 2$ **c** $y = x$

What do these lines have in common?

■ A point lies on a line if its coordinates satisfy the equation of the line.

Example
Does the point (6, 7) lie on the line $y = 2x - 5$?

Solution
Substitute $x = 6$ and $y = 7$ into $y = 2x - 5$.
$y = 2x - 5$ becomes $7 = 2 \times 6 - 5$
$7 = 7$
which is true.
$\therefore$ (6, 7) lies on the line $y = 2x - 5$.

7 On which of the following lines does the point (6, 7) lie?

a $y = x + 1$ **b** $x + 2y = 20$ **c** $y = 3x - 4$
d $4x - 3y = 3$ **e** $x - y = 1$ **f** $y = x - 1$

8 **a** Does (4, 0) lie on the line $4x + 3y = 16$?
b Does the line $y = 2x - 3$ pass through the point (7, 11)?
c Which of the points (7, 2) and (7, −2) lie on the line $y = x - 9$?
d Which of the points (5, 0) and (0, 5) lie on the line $2y + x = 5$?

9 Draw the graph of each line and use it to find the x- and y-intercept.

a $2x + y = 2$ **b** $3x + y = 6$ **c** $2x + y = 4$

d $2x - y = 4$ **e** $3x - y = 3$ **f** $4x - y = 2$

10 Draw the graph of each equation.

a $y = \frac{3x}{2}$ **b** $y = \frac{x+1}{2}$ **c** $y = \frac{x-1}{2}$

d $3x + 2y = 7$ **e** $5x - 2y - 6 = 0$ **f** $2x - 3y - 5 = 0$

investigation 8:02

Investigation 8:02 | *x*- and *y*-intercepts

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Line 1 has equation $2y = 2 - x$
Line 2 has equation $2y = x$
Line 3 has equation $y = -2x - 4$

1 From the points A, B, C, D and E choose the x-intercept of:
a line 1 **b** line 2 **c** line 3

2 From the points A, B, C, D and E choose the y-intercept of:
a line 1 **b** line 2 **c** line 3

3 Write down the coordinates of the x-intercepts.
What do you notice?

4 Write down the coordinates of the y-intercepts.
What do you notice?

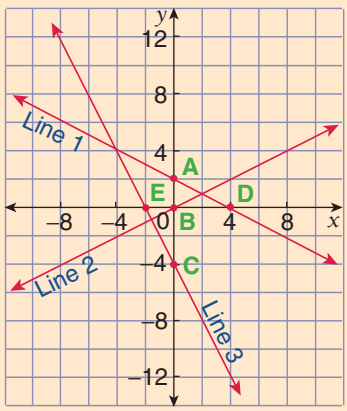
5 What can you say about the y-coordinate of every point on the x-axis.

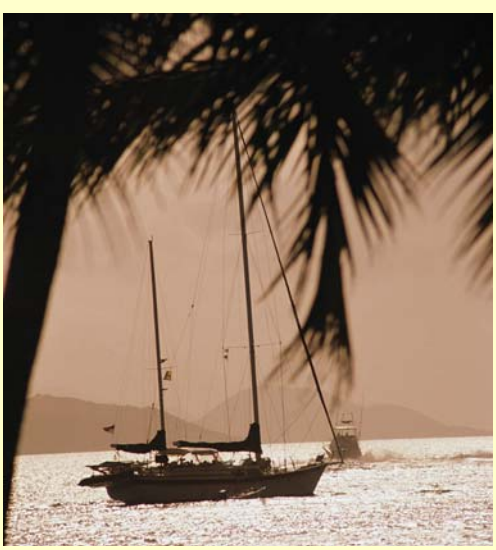
6 What can you say about the x-coordinate of every point on the y-axis.

7 How can you find the x-intercept from the line's equation?

8 How can you find the y-intercept from the line's equation?

9 Find the x- and y-intercepts of the lines in question **3** and **10** of Exercise 10:02.





- The intersection of lines is used in navigation to give a boat's position.

Assessment Grid for Investigation 8:02 | *x*- and *y*-intercepts

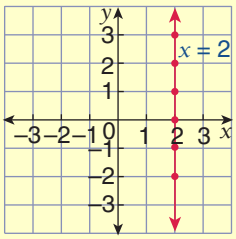
The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	A systematic approach has not been used. No patterns have been identified.	1	
			2	
	b	A systematic approach has been used and patterns have been identified with some success.	3	
			4	
	c	A systematic approach has been adopted and the patterns found have been described with some conclusions drawn.	5	
			6	
	d	The correct patterns have been recognised and justified clearly, and the method used for questions 7 and 8 are explained in detail.	7	
			8	
Criterion C Communication	a	No working out is shown and presentation is poor with little or no structure.	1	
			2	
	b	Working out is shown and explanations are given with some interpretation of the answers.	3	
			4	
	c	Results and their interpretation are well communicated and the work is well structured.	5	
			6	
Criterion D Reflection in Mathematics	a	An attempt has been made to explain the method and to check results in questions 5, 6, 7 and 8.	1	
			2	
	b	The method described in questions 7 and 8 is justified and results have been checked with some success.	3	
			4	
	c	A concise reasoned justification has been given for the method and processes used, particularly in questions 7 and 8, and the accuracy of the results has been checked thoroughly and applied to further examples.	5	
			6	

8:03 | Horizontal and Vertical Lines

The line shown on the graph on the right is vertical.

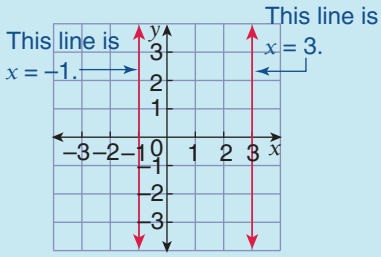
- Below, we have put the points on the line into a table.



x	2	2	2	2	2	2
y	−2	−1	0	1	2	3

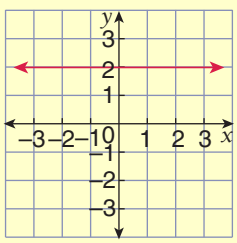
- There seems to be no connection between x and y. However, x is always 2. So the equation is $x = 2$.

Vertical lines have equations of the form $x = a$ where a is where the line cuts the x-axis.



The line on the right is horizontal.

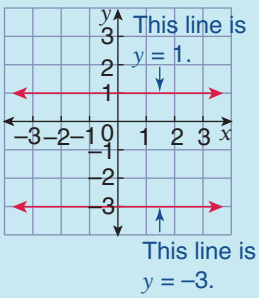
- Below, we have put the points on the line, into a table.



x	−2	−1	0	1	2	3
y	2	2	2	2	2	2

- There seems to be no connection between x and y. However, y is always 2. So the equation is $y = 2$.

Horizontal lines have equations of the form $y = b$ where b is where the line cuts the y-axis.



Exercise 8:03

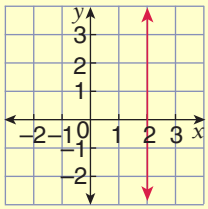
Foundation Worksheet 8:03

Horizontal and vertical lines

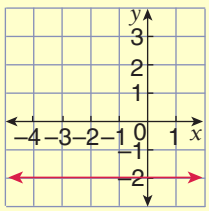
1 Write down the coordinates of the points A, B, C and D on each of the lines.
2 What do you notice about your answers in question 1?
3 On the number plane plot three points with:
a an x-coordinate of 3
b a y-coordinate of 1

1 Write down the equation of each line.

a

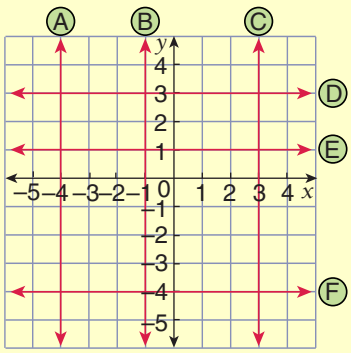


b

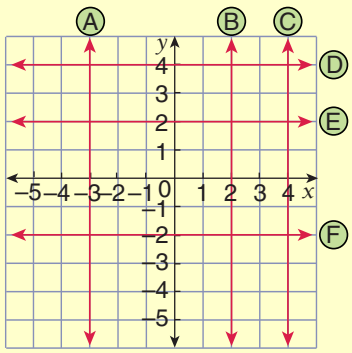


2 For each number plane write down the equations of the lines (A) to (F).

a



b

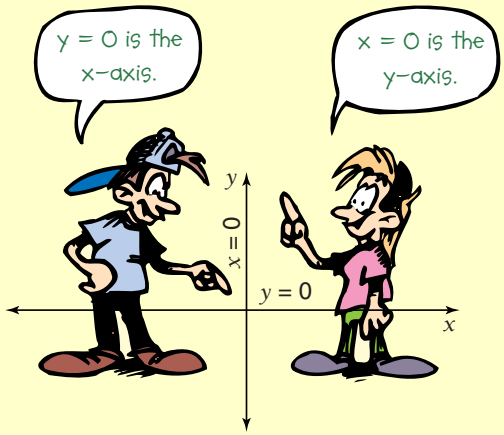


3 **a** What is the equation of the y-axis?
b What is the equation of the x-axis?

4 Using values from −5 to 5 on each axis, draw the graphs of the following straight lines. Use a new diagram for each part.

a $y = 4$, $x = 5$, $y = -1$, $x = 0$
b $x = 1$, $y = 0$, $x = 2$, $y = 3$
c $y = 4$, $x = 2$, $y = -2$, $x = -4$
d $x = 5$, $y = -5$, $x = 2$, $y = 2$
e $y = -2$, $y = 0$, $x = 0$, $x = 3$

Which of these encloses a square region?



5 Give the point where each pair of lines cross. (You can use the graph in question **2**.)

a $x = 3$ and $y = 1$
b $x = -4$ and $y = 3$
c $x = -1$ and $y = -4$
d $x = 2$ and $y = -2$
e $x = -3$ and $y = 2$
f $x = 4$ and $y = 4$
g $x = 0$ and $y = 3$
h $x = 2$ and $y = 0$
i $x = 0$ and $y = 0$

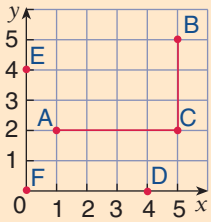
fun spot

8:03

Fun Spot 8:03 | What is the easiest job in a watch factory?

Answer each question and put the letter for that question in the box above the correct answer.

A The length of AC = ☐ units.
N The length of BC = ☐ units.
D The length of AB = ☐ units.
(Use Pythagoras' theorem.)



What are the coordinates of:
A D? **N** E? **S** F?
T Which quadrant is shown here?

What is the equation of:
A AC? **O** BC? **C** the x-axis?
D the y-axis?

If $y = 4x - 10$ what is the value of y when:
A $x = 5$? **E** $x = 2$? **F** $x = 0$? **G** $x = 1$?

Which of the equations $y = x + 3$, $y = 2x + 1$, $y = 3x$ has produced the table:

I

x	0	1	2	3
y	0	3	6	9

K

x	0	1	2	3
y	3	4	5	6

M

x	3	4	5	6
y	7	9	11	13

Solve:
N $2x = x + 8$ **O** $3y - 8 = 10$ **R** $11x = 5x + 12$
S $\frac{x-1}{3} = 2$ **T** $x^3 = 1$ **U** $y^3 = -1$

1st	$y = 6$	$x = 7$	$x = 1$	4	$x = 8$	5	10	$x = 2$	$x = 5$	$y = -1$	3	$x = 0$

$y = 2x + 1$	$y = 2$	$y = x + 3$	$y = 3x$	(0, 4)	−6	−10	(4, 0)	$y = 0$	−2	(0, 0)

8:04 | The Intersection of Two Straight Lines

prep quiz 8:04

1 What are the coordinates of the origin?
2 What is the equation of the x-axis?
3 What is the equation of the y-axis?
4 What point is the intersection of the two axes?
5 What do the sets {3, 4, 5, 6} and {0, 5, 10, 15} have in common?
6 Does (2, 1) lie on the line $y = 1$?
7 Does (1, 1) lie on the line $x + y = 2$?
8 Does (1, 1) lie on the line $y = 2x - 1$?
9 Is (1, 1) on both $x + y = 2$ and $y = 2x - 1$?
10 Where do the lines $x + y = 2$ and $y = 2x - 1$ cross?

The intersection of two lines is the point where the lines cross. Unless the two lines are parallel they will cross at only *one* point.

worked examples

In each case graph the pair of lines and find their point of intersection.

1 $\begin{cases} y = 2x + 1 \\ y = 4x - 3 \end{cases}$ 2 $\begin{cases} y = 0 \\ 2x + y = 4 \end{cases}$ 3 $\begin{cases} x + y = 5 \\ y = 2x - 1 \end{cases}$

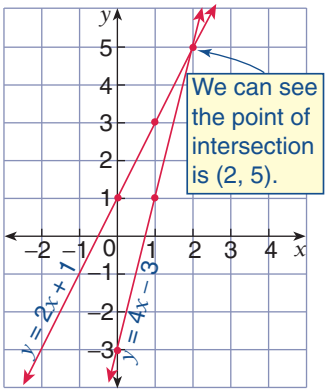
Solutions

1 $y = 2x + 1$

x	0	1	2
y	1	3	5

$y = 4x - 3$

x	0	1	2
y	−3	1	5

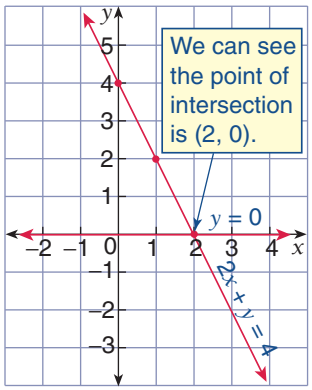


2 $y = 0$

x	0	1	2
y	0	0	0

$2x + y = 4$

x	0	1	2
y	4	2	0

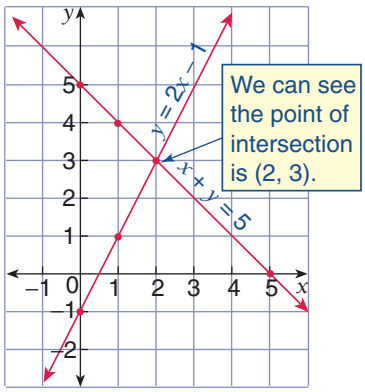


3 $x + y = 5$

x	0	1	2
y	5	4	3

$y = 2x - 1$

x	0	1	2
y	−1	1	3



Checking answers: The point of intersection must lie on both lines. Substitute the coordinates of the point for x and y in each equation.

The coordinates of the point of intersection will make *both* equations true. We say that the point of intersection *satisfies* both equations.

We also say that the two equations have been 'solved simultaneously'. That is, the one pair of x and y values that satisfies both equations has been found.

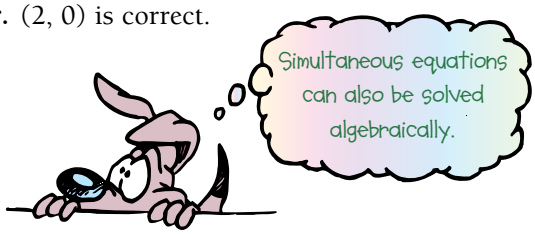
continued →→→

Checking the examples above:

1 (2, 5) is the point
$y = 2x + 1 \rightarrow 5 = (2 \times 2) + 1$ ... True
$y = 4x - 3 \rightarrow 5 = (4 \times 2) - 3$ ... True
$\therefore$ (2, 5) is correct.

2 (2, 0) is the point
$y = 0 \rightarrow 0 = 0$ ... True
$2x + y = 4 \rightarrow 2(2) + 0 = 4$ ... True
$\therefore$ (2, 0) is correct.

3 (2, 3) is the point
$x + y = 5 \rightarrow 2 + 3 = 5$ ... True
$y = 2x - 1 \rightarrow 3 = 2(2) - 1$ ... True
$\therefore$ (2, 3) is correct.



Exercise 8:04

Foundation Worksheet 8:04

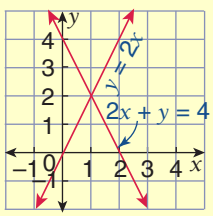
Intersecting lines

1 Use the graph to find the point of intersection of the lines:
a A and F **b** C and H ...

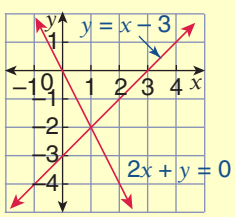
2 Graph the following pairs of lines to find their point of intersection.
a $y = x$ and $x + y = 2$ **b** ...

1 What is the point of intersection of the lines in each of the following?

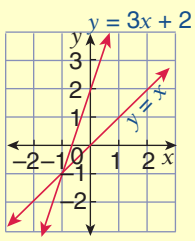
a



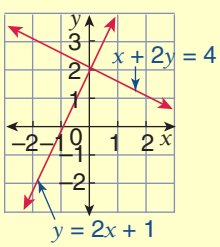
b



c



d



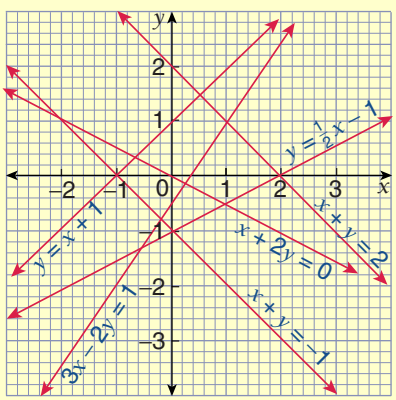
2 Find the point of intersection of the lines:

a $x + y = 2$ and $3x - 2y = 1$

b $x + y = 2$ and $y = \frac{1}{2}x - 1$

c $x + y = -1$ and $y = \frac{1}{2}x - 1$

d $x + 2y = -1$ and $x + 2y = 0$



3 Use the graph in question **2** to estimate the point of intersection of the lines:

a $y = x + 1$ and $x + y = 2$

b $y = x + 1$ and $x + 2y = 0$

c $3x - 2y = 1$ and $x + y = -1$

d $y = \frac{1}{2}x - 1$ and $x + 2y = 0$

4 Draw tables for each pair of lines. Graph them on a number plane and write down their point of intersection. Check each of your answers by substituting the coordinates in the original equations.

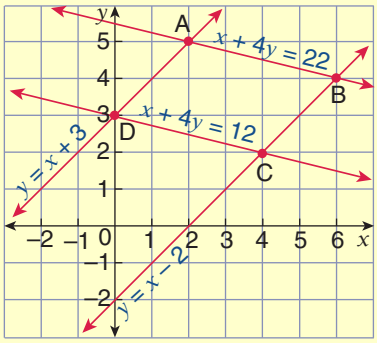
a $\begin{cases} y = x \\ y = 3x \end{cases}$ **b** $\begin{cases} y = -x + 2 \\ y = 2x - 1 \end{cases}$ **c** $\begin{cases} y = -x + 1 \\ y = x + 3 \end{cases}$ **d** $\begin{cases} y = -x \\ y = x + 2 \end{cases}$

e $\begin{cases} y = x + 3 \\ y = 2 \end{cases}$ **f** $\begin{cases} x = 3 \\ y = 2x \end{cases}$ **g** $\begin{cases} y = 1 \\ y = x - 3 \end{cases}$ **h** $\begin{cases} 2x + y = 4 \\ x = 1 \end{cases}$

i $\begin{cases} 2x + y = 1 \\ y = 2x + 1 \end{cases}$ **j** $\begin{cases} x - 2y = 4 \\ x + y = -2 \end{cases}$ **k** $\begin{cases} x + y = 4 \\ 2x - y = 5 \end{cases}$ **l** $\begin{cases} 2x + y = 4 \\ y = x - 2 \end{cases}$

5 **a** Find the point of intersection of the lines $x + y = -2$ and $x = -1$.
b Find the point where the line $y = x + 2$ crosses the line $y = 2x$.
c Where does the line $x + 2y = 4$ cut the y-axis?
d Where does the line $x + 2y = 4$ intersect the x-axis?
e Where do the lines $y = 3x - 1$ and $2x + y = -6$ intersect?
(*Note:* Before drawing your axes, look at the tables to see what values of y you will need.)
f Show that (1, 3) is the point of intersection of $3x + y = 6$ and $x + y = 4$.
g Find the coordinates of the point where $x = -2$ and $y = 2x + 1$ cross.

6 On the number plane are drawn four straight lines.



a Give the coordinates of the point of intersection of:

i $\begin{cases} y = x + 3 \\ x + 4y = 22 \end{cases}$ **ii** $\begin{cases} y = x + 3 \\ x + 4y = 12 \end{cases}$

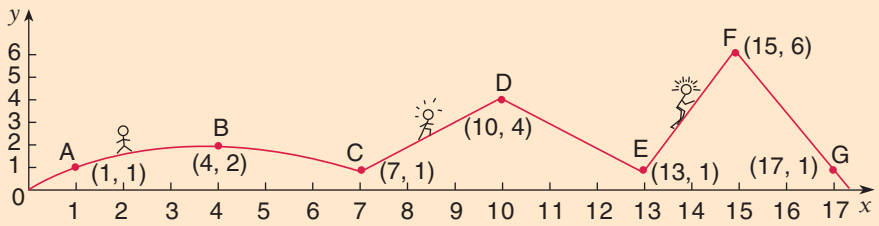
iii $\begin{cases} y = x - 2 \\ x + 4y = 12 \end{cases}$ **iv** $\begin{cases} y = x - 2 \\ x + 4y = 22 \end{cases}$

b What kind of figure is *ABCD*?
c Do the lines $y = x + 3$ and $y = x - 2$ intersect? Why not?

- Grid lines on maps are used to give a point's position.

8:05 | The Gradient of an Interval

prep quiz 8:05



1 Which is steepest, AB or EF?

2 **3** and **4** If I travel from left to right, between which 3 pairs of letters am I travelling upwards?

5 **6** and **7** Between which 3 pairs of letters am I travelling downwards?

Say whether the hill is sloping up, down, or not at all, at the points

8 A **9** G **10** F

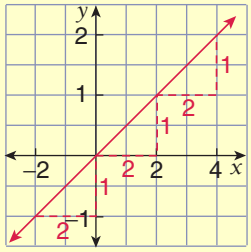
The **gradient** or **slope** of a line is a measure of *how steep* it is.

Negative gradient | Steep (positive gradient) | Steeper

- If we move from left to right the line going down is said to have a *negative* gradient (or slope). The line going up is said to have a *positive* gradient (or slope).
- If the line is horizontal (not going up or down) its gradient is *zero*.
- We find the gradient of a line by comparing its rise (change in y) with its run (change in x).

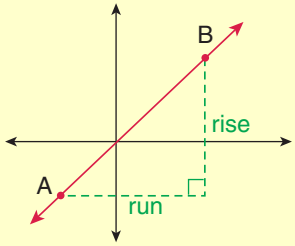
$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

- So, a gradient of $\frac{1}{2}$ means that for every run of 2 there is a rise of 1 (or for every 2 you go across, you go up 1).



Finding the gradient of a line

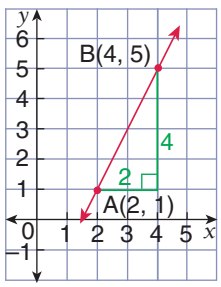
1. Select any two points on the line.
2. Join the points and form a right-angled triangle by drawing a vertical line from the higher point and a horizontal side from the lower point.
3. Find the change in the y-coordinates (rise) and the change in the x-coordinates.
4. Use the formula above to find the gradient.



worked examples

Use the points A and B to find the gradient of the line AB in each case.

1



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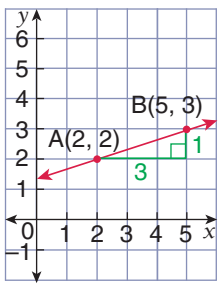
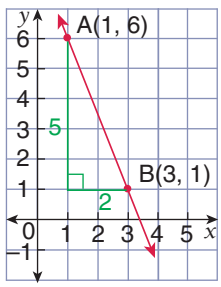


3



Solutions

1 Gradient

$$= \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{\text{up } 4}{\text{across } 2}$$

$$= \frac{4}{2}$$

$$= 2$$

2 $m = \dfrac{\text{change in } y}{\text{change in } x}$

$$= \frac{\text{up } 1}{\text{across } 3}$$

$$= \frac{1}{3}$$

3 $m = \dfrac{\text{change in } y}{\text{change in } x}$

$$= \frac{\text{down } 5}{\text{across } 2}$$

$$= \frac{-5}{2}$$

$$= -2\tfrac{1}{2}$$

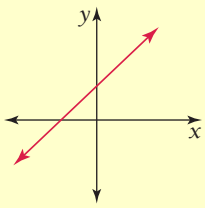
■ m is used for 'gradient'

- Architectural design often requires an understanding of gradients (slopes).

Exercise 8:05

1 For each of the following state if the line has a positive or negative gradient.

a



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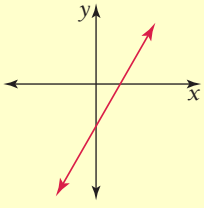
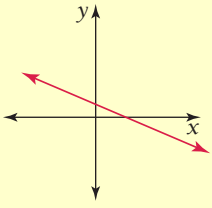


c



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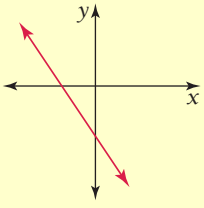
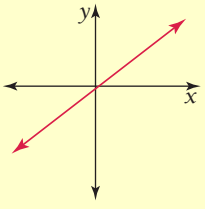


e



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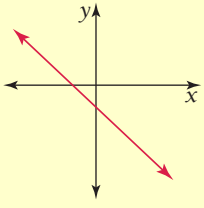
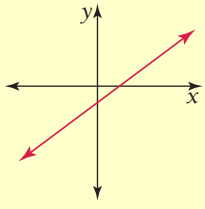
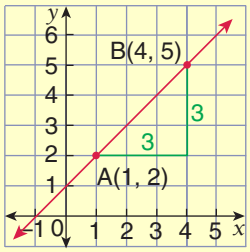


g



2 Find the gradient of the line AB.

a



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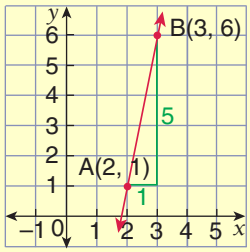

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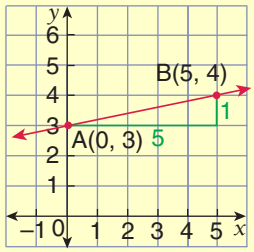

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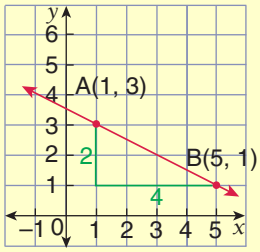

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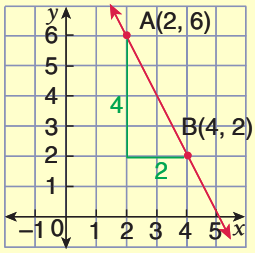

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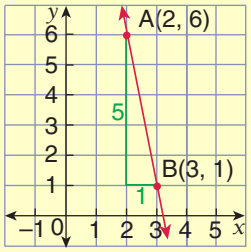
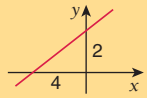


Foundation Worksheet 8:05

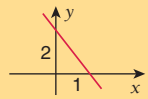
Gradients

1 Find the gradient of the line from the graph.

a

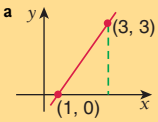


b

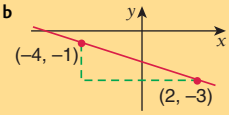


2 Find the lengths of the missing sides on each triangle and then find the gradient.

a

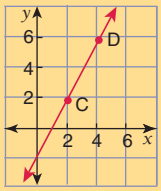


b

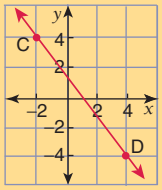


2 Find the slope of CD in each of the following.

a

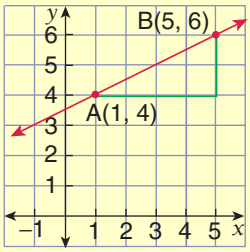


b

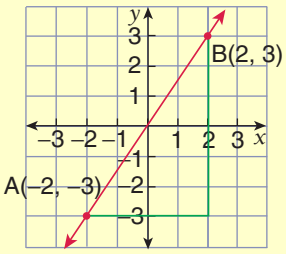


3 Find the gradient of the line AB.

a



b



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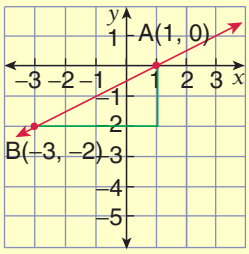
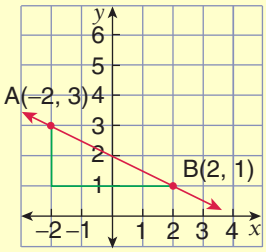


d



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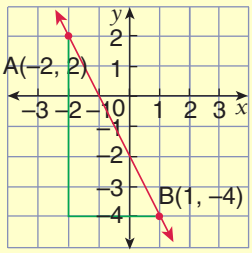

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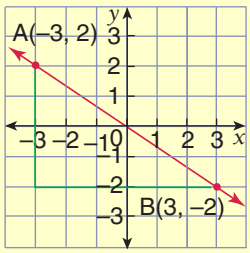
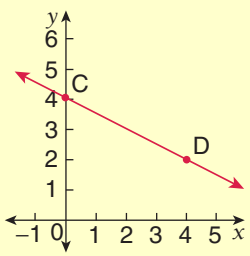
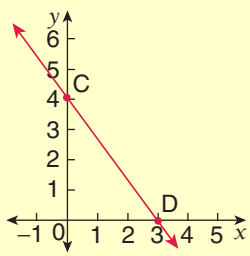


4 Use the points C and D to find the slope of the line.

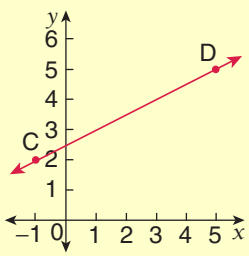
a



b



c



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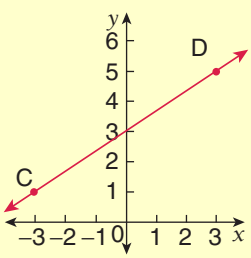
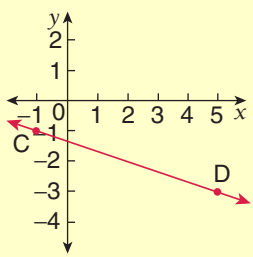
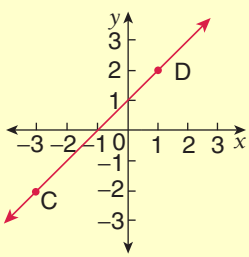


e



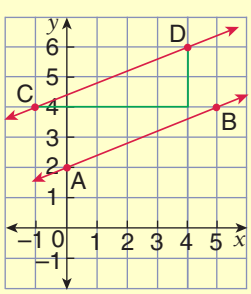
f



5 Find the slope of the line that passes through:

a (1, 0) and (3, 4)
b (−2, 1) and (0, 3)
c (1, −3) and (3, −2)
d (−1, 4) and (1, 2)
e (−2, −2) and (0, −5)
f (0, 4) and (2, −2)

6 **a** Find the gradient of the line AB.
b Find the gradient of the line CD.
c Are the lines AB and CD parallel?



Lines with the same gradient are parallel.

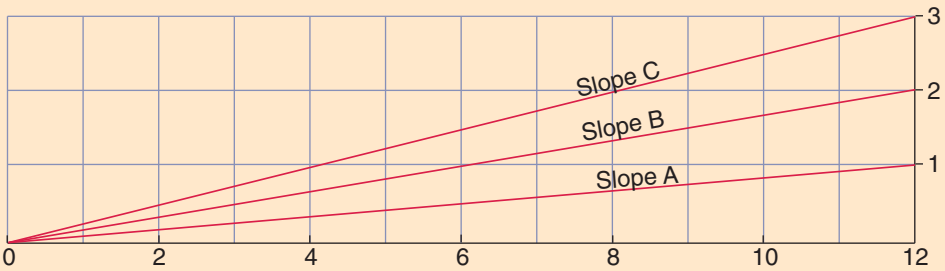
investigation 8:05

Investigation 8:05 | Gradients and building

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

- Engineers, architects and builders need to understand and calculate gradients when designing and constructing ramps, driveways and roads.
- They usually refer to a gradient in ratio form.
  eg A driveway can have a maximum gradient of 1 in 4 (1 : 4). This means a run of 4 units will produce a rise (or fall) of 1 unit. (We would say the gradient is $\frac{1}{4}$.)

1 The graph below shows three slopes. Which of the slopes A, B or C is:
a 1 in 4? **b** 1 in 6? **c** 1 in 12?



2 A slope is to be 1 in 4 (1 : 4). What would the run have to be if the rise is:
a 2 m? **b** 1·4 m? **c** 3·2 m?

3 Use the graph above (or ratio) to find the rise for:
a a 1 in 6 slope if the run is 9 m? **b** a 1 in 12 slope if the run is 6 m?

4 A builder has to construct a wheelchair ramp from the front of the house to the front of the property, a run of 10 m. He calculates the fall to be 0·8 m. Building regulations state that a ramp has to have a maximum gradient of 1 in 14. Is it possible for the builder to build a straight ramp. If not, how can he build it and satisfy the regulations?

Assessment Grid for Investigation 8:05 | Gradients and building

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	An organised approach has not been used and the student has had difficulty following instructions.	1	
			2	
	b	A systematic approach has been used. The student has attempted to describe any pattern between the ratios and steepness of the lines.	3	
			4	
	c	A systematic approach has been used and the patterns identified have been described with some conclusions drawn.	5	
			6	
	d	The correct patterns have been recognised and justified clearly through question 4, and used to make further predictions.	7	
			8	
Criterion C Communication	a	Little working out is shown and only oral explanations have been given.	1	
			2	
	b	Presentation is good and explanations illustrate some interpretation of the results.	3	
			4	
	c	All the working out shows clear progression and communicates interpretations using correct symbols and/or diagrams.	5	
			6	
Criterion D Reflection in Mathematics	a	Little understanding of the method is demonstrated and the findings have not been evaluated.	1	
			2	
	b	An explanation of the method is given and results have been evaluated against diagrams and information provided with some success.	3	
			4	
	c	A thorough, reasoned justification has been given for the method used, the accuracy of all results has been checked thoroughly, and possible improvements and applications have been discussed.	5	
			6	

8:06 | The Gradient–Intercept Form of a Straight Line: $y = mx + c$

prep quiz 8:06

If $x = 0$, what is the value of:

1 $2x$ **2** $mx + c$

If $x = 0$, what is the value of y when:

3 $y = 3x + 2$ **4** $y = 4x - 1$

What is the gradient of:

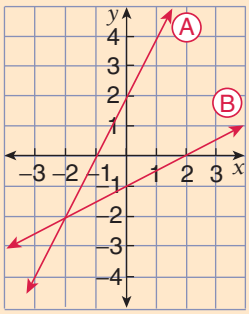
5 line A **6** line B

What are the coordinates of the y-intercept of:

7 line A **8** line B

9 Does every point on the y-axis have an x-coordinate of 0?

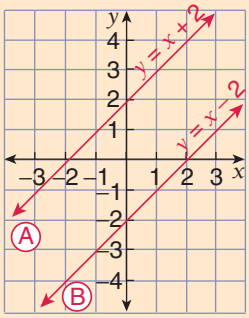
10 Can the y-intercept of a line be found by putting $x = 0$.

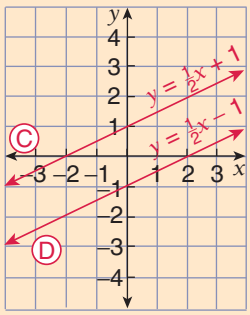


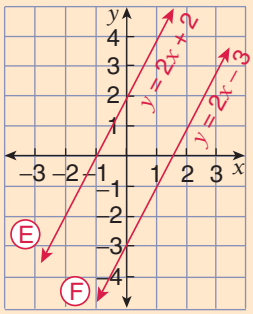
- The equation of a line can be written in several ways. For instance, $x - y - 4 = 0$, $y = x - 4$ and $x - y = 4$ are different ways of writing the same equation.
- When the equation is written in the form $x - y - 4 = 0$, it is said to be in general form.
- The form $y = x - 4$ is a particularly useful way of writing the equation of a line. It allows us to get information about the line directly from the equation.

investigation 8:06

Investigation 8:06 | What does $y = mx + c$ tell us?







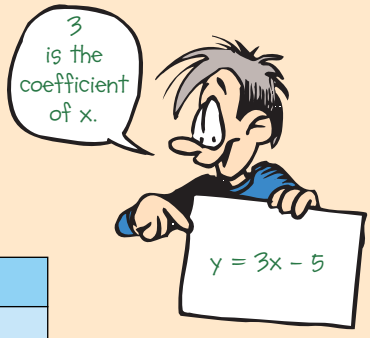
Please use the Assessment Grid on page 190 to help you understand what is required for this Investigation.

1 Use the graphs above to complete the following table.

Line	*Equation*	*Gradient*	*y-intercept*
A	$y = x + 2$		
B	$y = x - 2$		
C	$y = \frac{1}{2}x + 1$		
D	$y = \frac{1}{2}x - 1$		
E	$y = 2x + 2$		
F	$y = 2x - 3$		

2 In an equation like $y = 3x - 5$

- the number 3, in front of x, tells us how many of x we have and is called the coefficient of x.
- similarly, the number in front of y is called the coefficient of y. In this case the coefficient of y is 1.
- the number -5 is called the constant.



Complete the table below.

Line	*Equation*	*Coefficient of x*	*Constant*
A	$y = x + 2$		
B	$y = x - 2$		
C	$y = \frac{1}{2}x + 1$		
D	$y = \frac{1}{2}x - 1$		
E	$y = 2x + 2$		
F	$y = 2x - 3$		

3 From the tables in questions **1** and **2**, how are the gradient and y-intercept of a line related to its equation.

4 The equations $2x + y - 3 = 0$ and $y = -2x + 3$ are the same line. The gradient of the line is -2 and the y-intercept is 3. Which form of the equation gives this information directly?

5 What does the form $y = mx + c$ tell us about a line?

- **When an equation of a line is written in the form $y = mx + c$:**
  m gives the gradient of the line
  c gives the y-intercept of the line
- **Clearly, lines with the same gradient are parallel.**
  (See the pairs of lines in Investigation 8:06.)

Assessment Grid for Investigation 8:06 | What does $y = mx + c$ tell us?

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	The student has not made a connection between the results obtained and the graphs given.	1	
			2	
	b	The student has recognised some of the patterns formed and has attempted to describe the patterns.	3	
			4	
	c	The connections between the results and the graphs have been described using words or symbols with some conclusions drawn.	5	
			6	
	d	All correct connections between the equations and lines, and the lines themselves are identified and justified clearly.	7	
			8	
Criterion C Communication	a	No working out is shown and only oral explanations can be given.	1	
			2	
	b	Written explanations are given and include some interpretation of the results obtained.	3	
			4	
	c	Work is well structured and explanations provide a full interpretation of the results.	5	
			6	
Criterion D Reflection in Mathematics	a	There has been some attempt to check results and to describe the process involved in finding them.	1	
			2	
	b	Results have been checked with some success using the graphs, and the processes of working out the results are described.	3	
			4	
	c	A thorough justification has been given for the results obtained; the accuracy of all results has been checked thoroughly and verified through further examples.	5	
			6	

worked examples

1 Write down the gradient and y-intercept of these lines.

a $y = 3x - 5$

Here $m = 3$, $c = -5$.
The gradient is 3,
the y-intercept is -5.

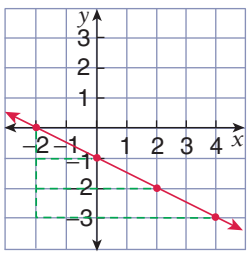
b $y = -2x$

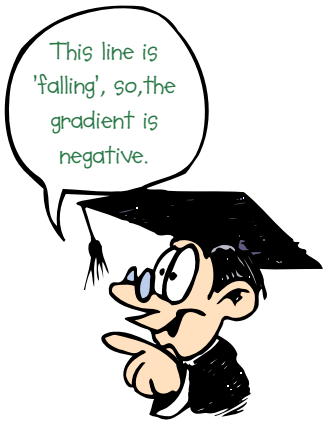
Here $m = -2$, $c = 0$.
The gradient is -2,
the y-intercept is 0.

c $y = 4 - 3x$

Here $m = -3$, $c = 4$.
The gradient is -3,
the y-intercept is 4.

2 Find the gradient and y-intercept from the graph and write down the equation of the line.





From the graph:

For every run of 2 there is a fall of 1,

So gradient $= -\frac{1}{2}$

y-intercept $= -1$

$\therefore$ Equation of the line is $y = -\frac{1}{2}x - 1$

3 By changing the equation into the form $y = mx + c$, find the gradient and y-intercept of the following.

a $2y = 4x + 3$

$2y = 4x + 3$
$\div 2 \quad \div 2 \quad \div 2$
$y = 2x + 1\frac{1}{2}$

$\therefore$ gradient $= 2$
y-intercept $= 1\frac{1}{2}$

b $3x + y = 1$

$3x + y = 1$
$-3x \qquad -3x$
$y = 1 - 3x$

$\therefore$ gradient $= -3$
y-intercept $= 1$

c $2x - 3y + 6 = 0$

$2x - 3y + 6 = 0$
$+3y \qquad +3y$
$2x + 6 = 3y$
$\div 3 \quad \div 3 \quad \div 3$
$\frac{2}{3}x + 2 = y$

$\therefore$ gradient $= \frac{2}{3}$
y-intercept $= 2$

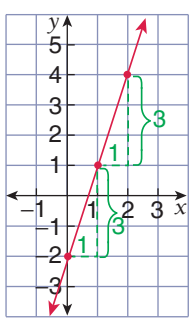
4 Use the y-intercept and gradient to graph the lines:

a $y = 3x - 2$

Start at the y-intercept of -2.

Now gradient $= \dfrac{\text{rise}}{\text{run}} = 3 = \dfrac{3}{1}$

$\therefore$ For every run of 1 there is a rise of 3.



continued ➜➜➜

b $4x + y = 2$

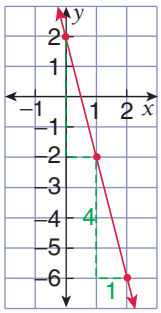
First, rearrange the equation.

$4x + y = 2$

$-4x \quad -4x$

to $y = -4x + 2$.

Then graph the line using the y-intercept of 2 and gradient of -4.



Exercise 8:06

Foundation Worksheet 8:06

Gradient–intercept form

1 For each line find from the graph its:
a y-intercept **b** gradient

2 What is $y = mx + c$ when:
a $m = 4$ and $c = 3$ **b** $m = -3$ and $c = -1$

3 Find the equation of a line with:
a a gradient of 2 and y-intercept of 2

1 What are the gradient and y-intercept of each of the following lines?

a $y = 2x + 3$ **b** $y = 5x + 1$ **c** $y = 3x + 2$

d $y = 1x + 6$ **e** $y = 4x + 0$ **f** $y = x$

g $y = x - 2$ **h** $y = 5x - 1$ **i** $y = 6x - 4$ **j** $y = -2x + 3$

k $y = -x - 2$ **l** $y = -3x + 1$ **m** $y = \frac{1}{2}x + 4$ **n** $y = \frac{3}{4}x - 2$

o $y = \frac{x}{3} + 5$ **p** $y = 4 - 3x$ **q** $y = -2 + 4x$ **r** $y = 3 - \frac{1}{2}x$

2 Find the equation of the line which has:

a a gradient of 4 and a y-intercept of 9

b a gradient of -2 and a y-intercept of 3

c a gradient of 7 and a y-intercept of -1

d a gradient of -5 and a y-intercept of -2

e a gradient of $\frac{1}{2}$ and a y-intercept of 5

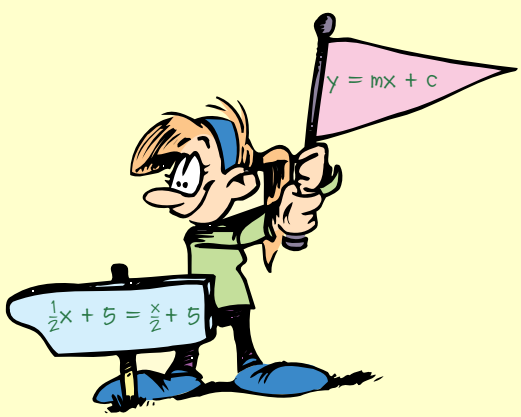
f a gradient of $\frac{2}{3}$ and a y-intercept of -4

g a y-intercept of 1 and a gradient of 3

h a y-intercept of -3 and a gradient of 2

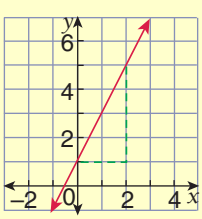
i a y-intercept of $\frac{1}{2}$ and a gradient of 5

j a gradient of 1 and a y-intercept of $1\frac{1}{2}$

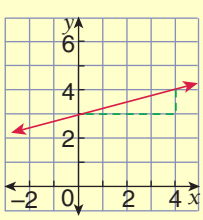


3 For each diagram find the gradient and the y-intercept and use these to write down the equation of each line.

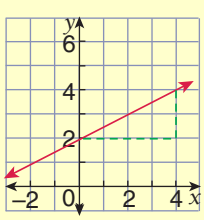
a



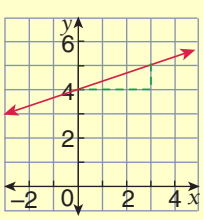
b



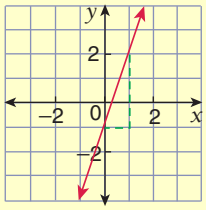
c



d



e



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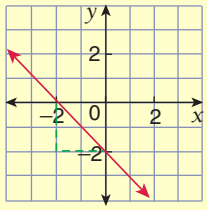

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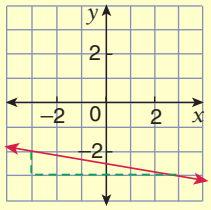
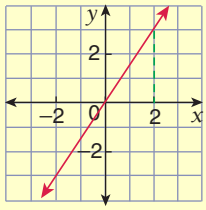


h



4 Match the graphs A to G with the equations below.

a $y = x$

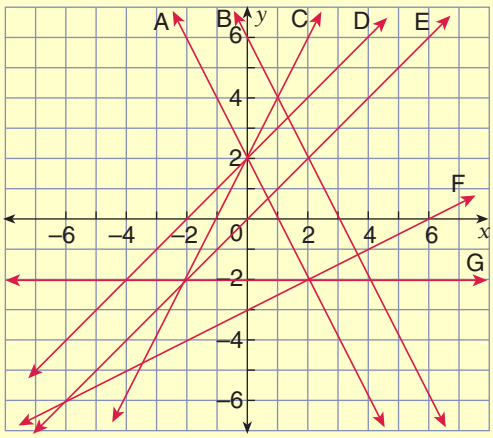
b $y = 6 - 2x$

c $y = x + 2$

d $y = -2$

e $y = \frac{1}{2}x - 3$

f $y = -2x + 2$



5 By first changing the equation into the form $y = mx + c$, find the gradient and y-intercept of the following.

a $2y = 4x + 10$

$\div 2 \quad \div 2$

$y =$ .....

$\therefore$ gradient is ...
y-intercept is ...

b $y + 3 = 3x$

$- 3 \quad - 3$

$y =$ .....

$\therefore$ gradient is ...
y-intercept is ...

c $\frac{y}{2} = x + 2$

$\times 2 \quad \times 2$

$y =$ .....

$\therefore$ gradient is ...
y-intercept is ...

d $3y = 6x - 3$

e $y - 8 = 5x$

f $\frac{y}{3} = 2x - 4$

g $4y = 12x - 8$

h $y + 4 = -2x$

i $y - 2x = 7$

j $x + y = 4$

k $2y = x + 3$

l $3x + y = 5$

m $6x + 2y = 4$

$-6x \quad -6x$

$2y =$ .....

$\div 2 \quad \div 2$

$y =$ .....

$\therefore$ gradient is ...
y-intercept is ...

n $3x + 2y = 6$

$- 3x \quad - 3x$

$2y =$ .....

$\div 2 \quad \div 2$

$y =$ .....

$\therefore$ gradient is ...
y-intercept is ...

o $2x - 5y + 1 = 0$

$+ 5y \quad + 5y$

..... $= 5y$

$\div 5 \quad \div 5$

..... $= y$

$\therefore$ gradient is ...
y-intercept is ...

6 Lines with the same gradient are parallel. Are the following pairs of lines parallel or not?

a $y = 3x + 2$ and $y = 3x - 1$ **b** $y = 5x - 2$ and $y = 2x - 5$

c $y = x + 7$ and $y = x + 1$ **d** $y = x - 3$ and $y = 1x + 2$

7 **a** Which of the following lines are parallel to $y = 2x + 3$?

$y = 3x + 2$ $2x - y + 6 = 0$ $2y = x + 3$ $y = 2x - 3$

b Two of the following lines are parallel. Which are they?

$y = x - 3$ $x + y = 3$ $y = 3x$ $3y = x$ $y = -x + 8$

■ If two lines are parallel, they have the same gradient.

8 Draw the graph of each line on a separate number plane by following these steps. (Look at example 4.)

- Mark the y-intercept (the value of b) on the y-axis.
- 'Count' the gradient (the value of m) from this point, and mark a second point.
- Draw a line through this second point and the y-intercept.

a $y = 2x + 1$ **b** $y = x + 2$ **c** $y = 3x - 1$

d $y = -2x - 1$ **e** $y = -x + 1$ **f** $y = -3x - 3$

g $y = \frac{1}{2}x + 2$ **h** $y = \frac{1}{3}x - 1$ **i** $y = \frac{3}{2}x$

j $y - -\frac{1}{2}x + 1$ **k** $y = -\frac{1}{3}x$ **l** $y = -\frac{3}{2}x - 1$

9 **a** On the same number plane graph lines with a y-intercept of 1 and a gradient of 0, 1, 2 and -1.

b On the same number plane graph lines with a slope of -1 and y-intercepts of -2, 0, 1 and 2.

10 The equation of each line has been given in general form. Find the gradient of each line by rearranging the equation into gradient–intercept form.

a $2x + y + 6 = 0$ **b** $4x - 2y + 5 = 0$ **c** $x + 2y + 1 = 0$

d $4x - 3y + 6 = 0$ **e** $4x - y + 3 = 0$ **f** $x + 3y - 6 = 0$

- At Hannah's Ice Cream Parlour, single cones cost \$2 and double cones cost \$3. In a day Hannah sells 300 cones and has sales totalling \$780. How many double cones did she sell?

8:07 | Distance–Time Graphs

- A distance–time graph (or travel graph) is a special type of line graph used to describe one or more trips or journeys.
- The vertical axis represents distance from a certain point, while the horizontal axis represents time.
- The formulae that connect distance travelled (D), time taken (T) and average speed (S) are given below.

$$D = S \times T \qquad S = \frac{D}{T} \qquad T = \frac{D}{S}$$

prep quiz 8:07

A car is travelling at an average speed of 100 km/h. How far will it travel in:

1 2 h? **2** $4\frac{1}{2}$ h? **3** 15 min?

How long will it take to travel 200 km at an average speed of:

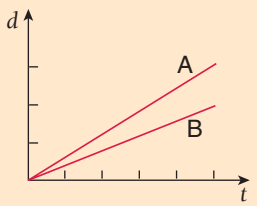
4 50 km/h? **5** 20 km/h?

6 A car travels 350 km in 5 h. What is its average speed?

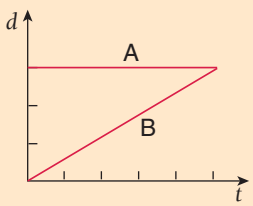
7 A plane flies 1500 km in 3 h. What is its average speed?

For each of the following graphs, which is the steepest, A or B?

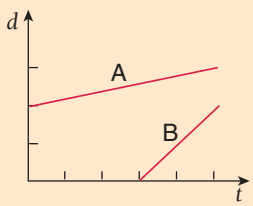
8



9

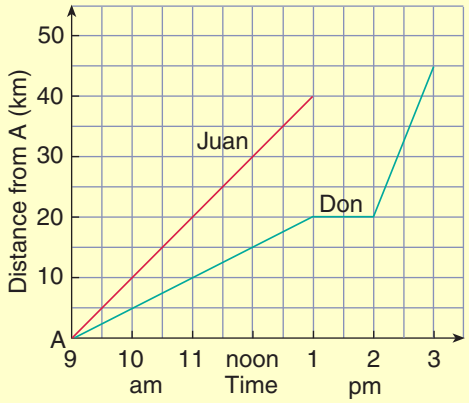


10



- How would the distance–time graph of a roller-blader compare to that of a walker?

Don and Juan are travelling along the same straight road. The graph gives the distance of Don and Juan from town *A* at different times.



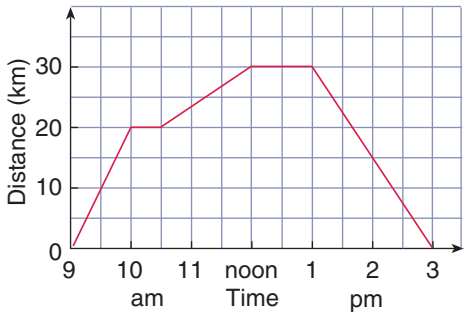
- From our knowledge of line graphs we can work out what distances correspond to what times and vice versa. For example, at 1:00 pm Juan is 40 km from *A*.
- Don is 35 km from *A* at 2:30 pm. Furthermore, the straight lines give us information about the speeds. From 9:00 am to 1:00 pm Juan travelled a greater distance than Don. Because Juan travelled a greater distance in the same time he must have travelled at a greater speed.
- From 9:00 am to 1:00 pm the line representing Juan's trip is steeper than the line representing Don's trip. From this we can see that the steeper the line the greater the speed. From the graph we can also see that Juan's trip finishes at 1:00 pm when he is 40 km from *A*. However, Don's trip continues.
- From 1:00 pm to 2:00 pm we notice that Don's distance from *A* has not changed. He has remained the same distance from *A* and so would have stopped.
- At 2:00 pm Don's journey continues. He travelled at a faster speed from 2:00 pm to 3:00 pm than he did from 9:00 am to 1:00 pm.
- A change in speed is indicated by a change in steepness. The graph shows us that Juan's speed is unchanged.

Summary

- **A change in steepness means a change in speed.**
- **The steeper the line, the faster the journey. The flatter the line, the slower the journey.**
- **A horizontal line indicates that the person or object is stationary.**

worked examples

Example 1



This travel graph gives the distance of a boy from home.

a When did the boy leave home? When did he return?

b How far was he from home at 1:00 pm?

c At what time was he 15 km from home?

d At what times did he rest?

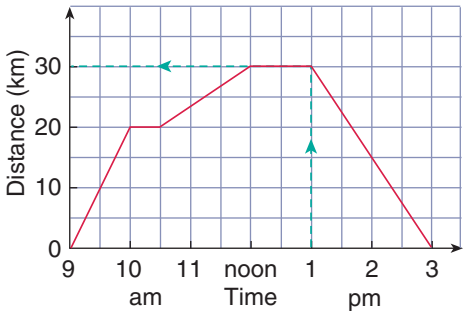
e When was he travelling most quickly?

Solution 1

a When he is at home the boy's distance from home is 0 km. Now, the boy is 0 km from home at 9:00 am and 3:00 pm

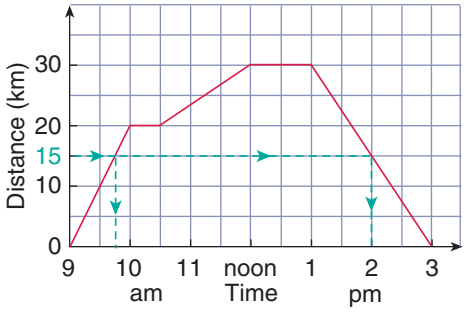
∴ He leaves home at 9:00 am.
He returns home at 3:00 pm.

b



At 1:00 pm the boy is 30 km from home.

c

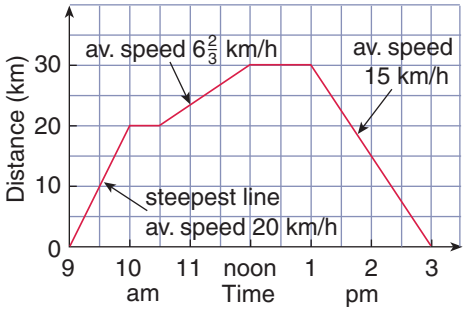


The boy was 15 km from home at 9:45 am and 2:00 pm.

d When the boy is resting he is stationary.
The horizontal sections of the graph indicate that he is stationary or resting.

∴ He is resting from 10:00 am until 10:30 am and from noon to 1:00 pm.

e



From the graph, the steepest line is for the part of the journey from 9:00 am to 10:00 am.

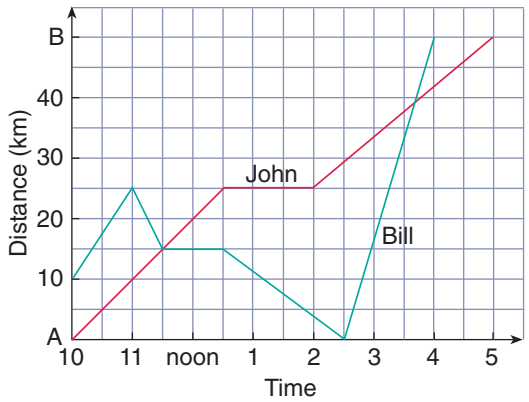
As this is the steepest line on the graph it must represent the greatest speed of the journey.

We could also find the highest speed of the journey by calculating the average speed for each part of the journey.

continued →→→

Example 2

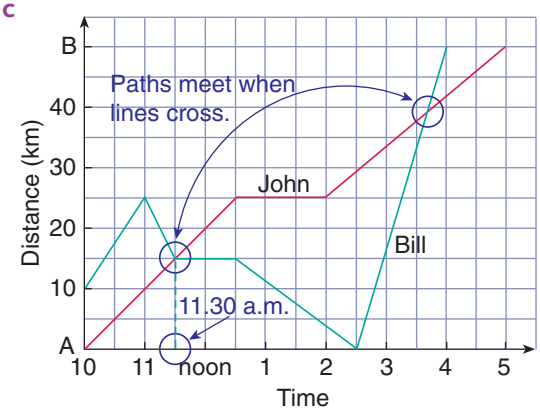
This travel graph shows the journeys of John and Bill between town A and town B. (They travel on the same road.)



a How far from A is Bill when he commences his journey?
b How far is John from B at 2:30 pm?
c When do John and Bill first meet?
d Who reaches town B first?
e At what time does Bill stop to rest?
f How far does John travel?
g How far apart are John and Bill when Bill is at town A?
h How far does Bill travel?

Solution 2

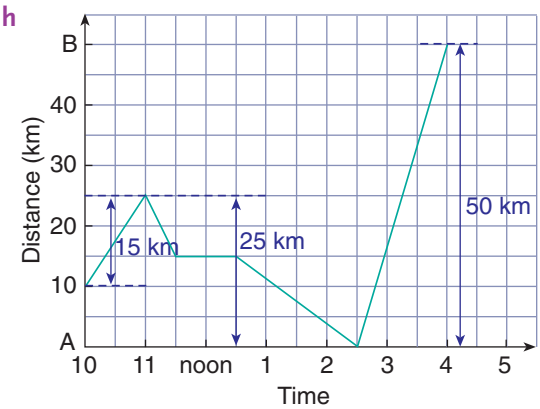
a Bill commences his journey at 10:00 am. At that time he is 10 km from town A.
b At 2:30 pm John is 20 km from B (because he is 30 km from A).
c



John and Bill first meet at 11:30 am.

d Bill reaches town B at 4:00 pm.
John reaches town B at 5:00 pm.
∴ Bill reaches town B first.
e The horizontal section indicates a rest.
∴ Bill stops at 11:30 am.
f John travels from town A to town B without backtracking.
∴ John travels 50 km.
g Bill is at town A at 2:30 pm. At that time John is about 30 km from A.
∴ They are about 30 km apart when Bill is at A.

h



Bill's journey involves backtracking.
He moves towards B, then returns to A and then moves to B.
Distance travelled (10:00 am–11:00 am)
= 25 − 10 = 15 km
Distance travelled (11:00 am–2:30 pm)
= 25 − 0 = 25 km
Distance travelled (2:30 pm–4:00 pm)
= 50 − 0 = 50 km
Total distance travelled = (15 + 25 + 50) km
= 90 km.

Example 3

Use the following information to construct a travel graph.

A motorist left home at 9:00 am to drive to a country town. He drove 180 km in 2 hours. After resting for half an hour he drove 200 km in $2\frac{1}{2}$ hours. He then spent $1\frac{1}{2}$ hours in the town before starting his return journey. On the return journey he drove 180 km in 2 hours, rested for half an hour and then reached home taking another 2 hours.

Solution 3

Before the graph is begun we must tabulate the distances and times and then decide on a suitable scale.

Left home at 9:00 am

(Drove 180 km in 2 h.)

∴ 180 km from home at 11:00 am.

(Rested for 1 h.)

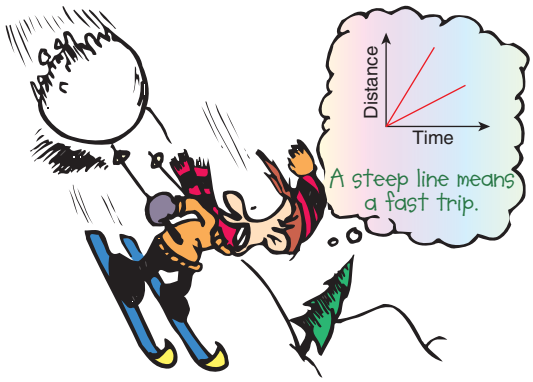
∴ 180 km from home at 12:00 pm.

(Drove 200 km in 2 h.)

∴ 380 km from home at 2:00 pm.

(Spent 2 h in town.)

∴ 380 km from home at 4:00 pm.



Now return journey:

(Drove 180 km in $1\frac{1}{2}$ h.)

∴ 200 km from home at 5:30 pm.

(Rested for $\frac{1}{2}$ h.)

∴ 200 km from home at 6:00 pm.

(Reached home after another 2 h.)

∴ 0 km from home at 8:00 pm.

Now, distance ranges from 0 to 380 km.

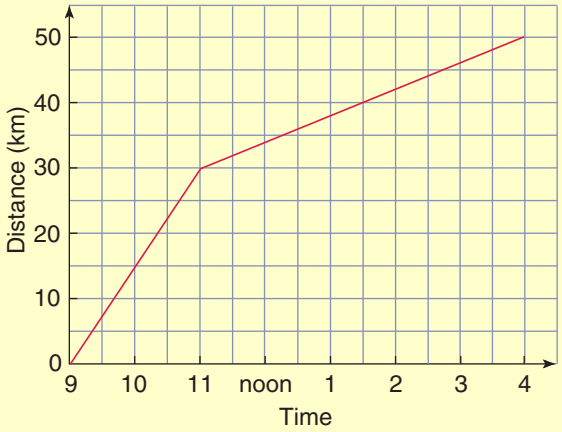
∴ A suitable scale is 1 cm : 40 km.

Time ranges from 9:00 am to 8:00 pm.

∴ A suitable scale is 1 cm : 2 h.

Exercise 8:07

1

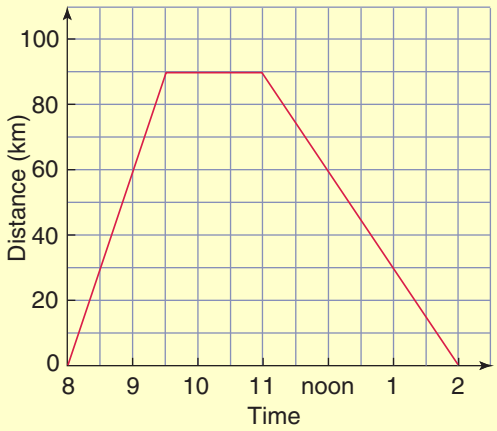


The graph shows the trip of a boy who travels from his home.

- **a** How far from home is he at 11:00 am?
- **b** When is he 40 km from home?
- **c** How far from home is he at the end of his trip?
- **d** How far does he travel from 11:00 am to 4:00 pm?
- **e** Does his speed change during the journey? If so, at what time?

2 The graph shows the journey of a man who drives to a friend's place and then rides a bike back home. The graph shows his distance from home between the times of 8:00 am and 2:00 pm.

- **a** What time does he leave home?
- **b** At what time does he return home?
- **c** How long does it take to reach his friend's place?
- **d** How far from his friend's place is he at 9:00 am?
- **e** How far does he travel altogether?



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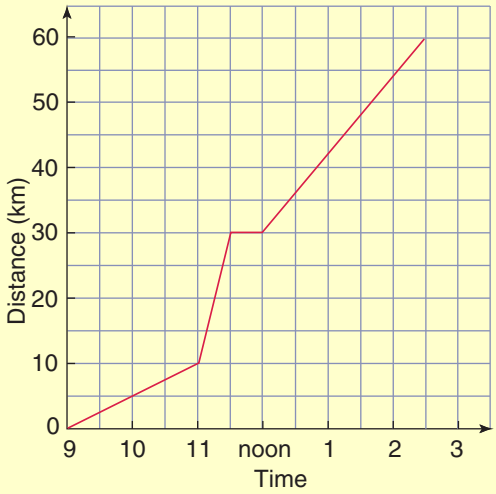


The graph gives the distance of a woman from her home at different times.

- **a** How far from home is she at 10:00 am?
- **b** At what time does she stop to rest?
- **c** What is the total distance travelled?
- **d** It is known that the woman completes her journey by riding in a car, walking, and riding a bike (not necessarily in that order). Between which times was she riding in the car?
- **e** What is her average speed from 9:00 am to 11:00 am?

4 The travel graph shows the distance of a cyclist from his home between the times of 10:00 am and 4:00 pm.

a How far does the cyclist travel in the first 2 hours?

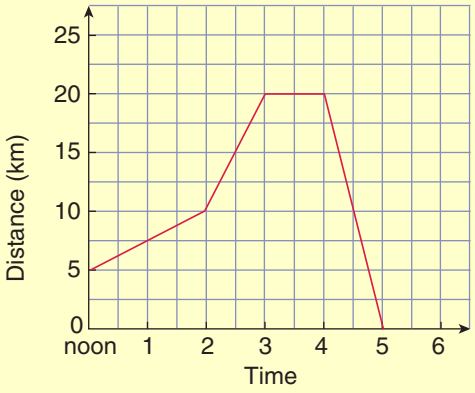
b How far from home is he when he stops to rest?

c At what time does he commence the return journey?

d At 3:00 pm his speed changes. Does it increase or decrease? How can you tell without calculating the actual speeds?

e How far does he travel?

5 The graph shows the distance of a man from town A, between the times of noon and 6:00 pm.



a How far from A is the man when he starts his trip?

b What is the furthest distance from A that he reaches?

c How far does he travel on his trip?

d At 2:00 pm his speed changes. Does it increase or decrease?

e Between what times is his speed greatest?

6 The graph shows the journeys of two motorists, Conway and Bartley. They are travelling on the same road and in the same direction leaving town A at 9:00 am.

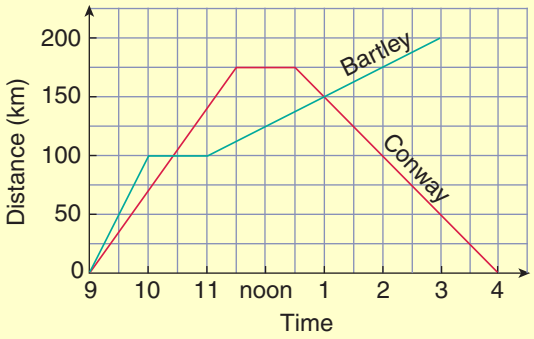
a Who travels the fastest in the first hour?

b How many times do they pass each other?

c At what time do they pass the second time?

d How far apart are they at 3:00 pm?

e How far does each man travel altogether?



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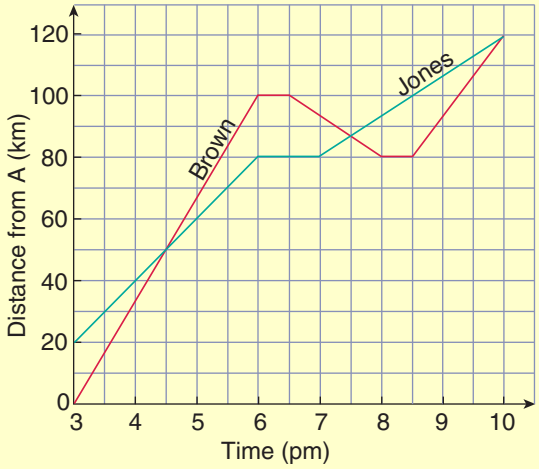
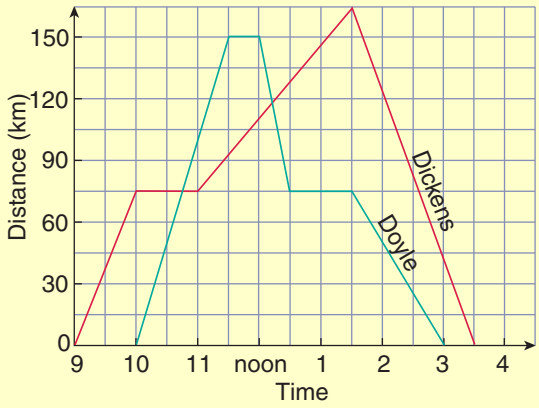


Two men set out to travel from town A to town B. They travel along the same road and in the same direction.

- **a** State when and where each man begins his trip.
- **b** At what times do their paths cross?
- **c** Both of them stop at 6 pm. How far apart are they then?
- **d** At what time does Brown begin to move back towards A?
- **e** How far does each man travel?

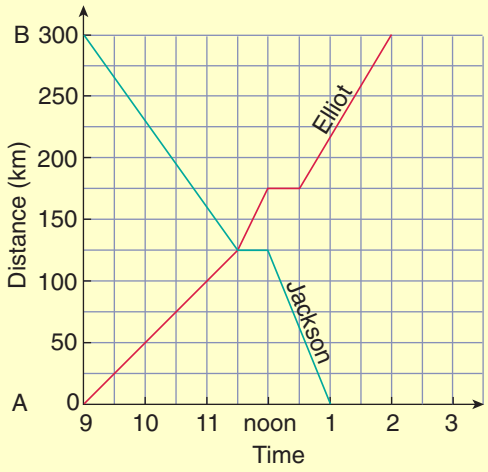
8 The graph shows the journeys of two motorists, and gives their distances from their home town.

- **a** How far is Dickens from the town when Doyle starts his journey?
- **b** At approximately what times do their paths cross?
- **c** At what time does Doyle begin his return journey?
- **d** How far is Doyle from town when Dickens begins his return journey?
- **e** Who returns to town first? What time elapses before the other motorist arrives?



9 Two towns A and B are 300 km apart. One motorist travels from A to B while the other travels from B to A. Their journeys are shown on the graph.

- **a** Who completes the trip in the shortest time?
- **b** What is Jackson's average speed for the trip?
- **c** At 11:30 am does Elliot increase or decrease speed?
- **d** Which period of Jackson's trip is the slower: from 10 to 11 am or from noon to 1 pm?
- **e** How far from town B is Elliot when he stops to rest?



10 Noel cycled to his friend's place. He started at 8:00 am and covered 15 km in the first hour. After resting for half an hour he then covered the next 20 km to his friend's place in 2 hours.

a At what time did he reach his friend's place?

b How far was it from his home to his friend's place?

c Using a scale of 1 cm : 5 km on the distance axis and 1 cm : $\frac{1}{2}$ h on the time axis, make a travel graph for Noel's trip.

11 A family leaves Kuala Lumpur by car at 10:00 am. They drive 160 km in 2 hours and then stop for lunch. After 1 hour they continue on, driving 200 km in the next 2 hours. The speed is then reduced and 90 km is covered in $1\frac{1}{2}$ hours. After stopping to rest for half an hour they then complete the last 60 km of their trip in 45 minutes.

a When did they complete their journey?

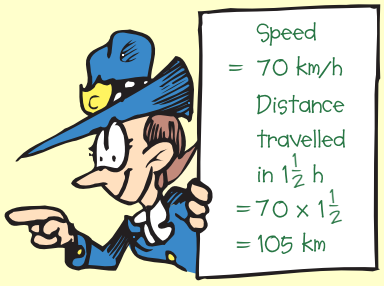
b How far did they drive?

c At what time did they stop for lunch?

d When did they make their second stop?

e Make a travel graph to illustrate this journey.

12 A group of people go on a bus tour. The bus leaves their town at 9:00 am. The bus travels at an average speed of 80 km/h and reaches town B in 1 hour. After spending 30 minutes for morning tea, the bus continues on to town C which it reaches after $1\frac{1}{2}$ hours. Its average speed for this part of the journey is 70 km/h.



The bus spends 1 hour at town C before returning to town A. The trip from town C to town A is completed without any stops and it takes 3 hours.

By dividing this journey into its separate parts, make a table comparing distance from home with time, and use it to complete a travel graph for this journey.

fun spot

8:07

Fun Spot 8:07 | Can you count around corners?

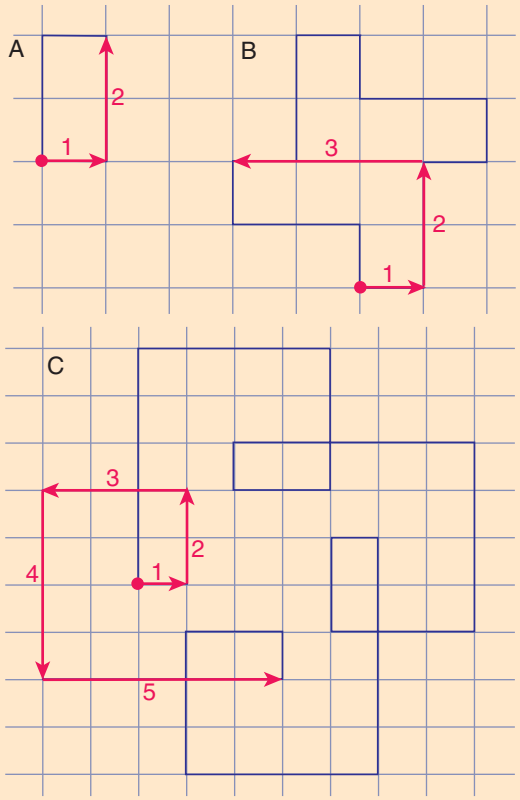
Each of these patterns has been formed by counting squares on grid paper and at the same time making a left-hand turn for each new number.

Figure A has been formed by counting to 2, and then starting again.

Figure B has been formed by counting to 3, repeatedly until the line arrives back at its starting point.

For figure C a count to 5 has been repeated.

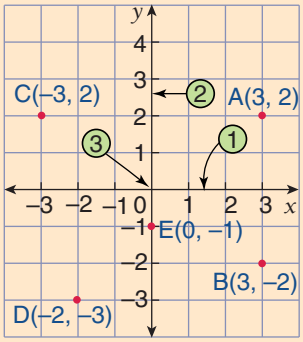
Try these yourself on grid paper and then try counting to some other numbers and see what patterns you can form.



- A garden design inspired by a square grid.

Mathematical terms 8

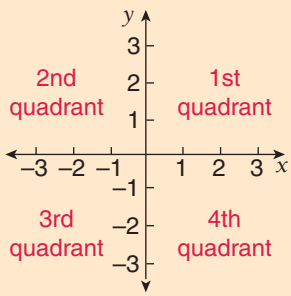
number plane
- A rectangular grid that allows the position of points in a plane to be identified by an ordered pair of numbers.



plot
- To mark the position of a point on the number plane.

graph (a line)
- All the points on a line
  or
- to plot the points that lie on a line.

quadrants
- The four quarters that the number plane is divided into by the *x*- and *y*-axes.



x-axis
- The horizontal number line in a number plane. See (1) under **number plane**.

y-axis
- The vertical number line in a number plane. See (2) under **number plane**.

origin
- The point where the *x*-axis and *y*-axis intersect, (0, 0). See (3) under **number plane**.

coordinates
- A pair of numbers that gives the position of a point in a number plane relative to the origin. See (3, 2), (3, −2), (−3, 2), (−2, −3) and (0, −1) under **number plane**.

x-coordinate
- The first of the coordinates. It tells how far right (or left) the point is from the origin.

y-coordinate
- The second of the coordinates. It tells how far the point is above (or below) the origin.

x-intercept
- The point where a line crosses the *x*-axis.

y-intercept
- The point where a line crosses the *y*-axis.

coefficient
- The number that multiplies a pronumeral in an equation.
  eg $3x - 5y = 6$
  coefficient of x is 3
  coefficient of y is -5

constant
- The number part of an equation or expression.
  eg $3x - 5y = 6$
  The constant is 6.

interval
- The part of a line between two given points.

gradient–intercept form
- A way of writing the equation of a line.
  eg $y = 2x - 5$, $y = \frac{1}{2}x + 2$
  When an equation is rearranged and written in the form $y = mx + b$ then m is the gradient and b is the *y*-intercept.

speed
- Relative rate of motion or action.
- Defined as: $\frac{\text{distance}}{\text{time}}$
- Measuring in units such as km/h or m/s.

travel graph
- A line graph where distance travelled is plotted against time taken.
- The gradient (or slope) of the line is an indication of the speed of the motion.

diagnostic test 8

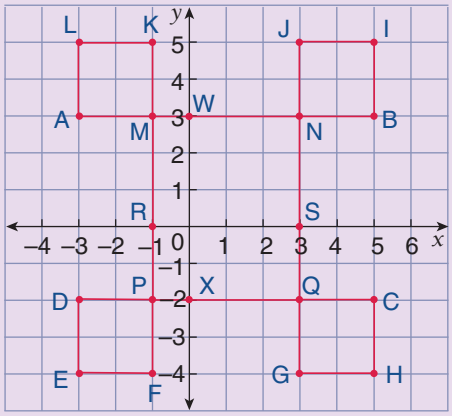
Diagnostic Test 8: | Coordinate Geometry: Graphing Straight Lines

- These questions reflect the important skills introduced in this chapter.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

Section

1 Write down the letter naming the point with these coordinates. — 8:01

a (3, 5) **b** (−1, −2)
c (3, −2) **d** (−3, 5)



2 Write down the coordinates of these points. — 8:01

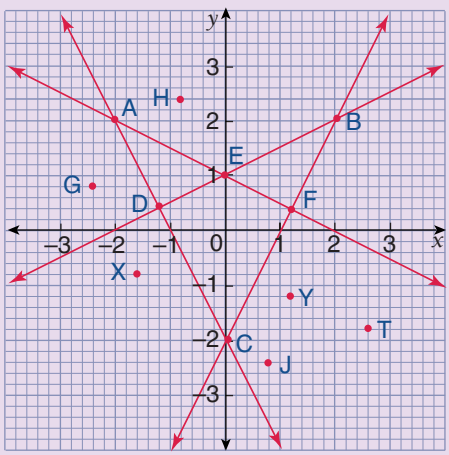
a A **b** D **c** Q **d** W

3 Find the distance between: — 8:01

a (0, 3) and (5, 3)
b (3, 3) and (3, −2)
c (−1, −4) and (−1, 3)
d (−3, 5) and (5, 5)

4 Write down the letter naming the point with the coordinates: — 8:01

a (1·2, 0·4) **b** (−1·2, 0·4)
c (−2·4, 0·8) **d** (0·8, −2·4)



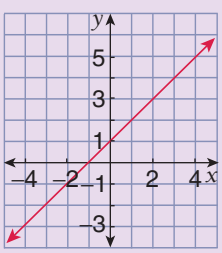
5 Write down the coordinates of these points. — 8:01

a X **b** T **c** J **d** H

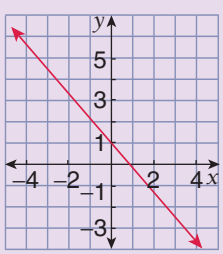
6 Choose the equation that describes each of the lines below. — 8:01

A $y = 1 - x$ **B** $y = x + 1$ **C** $y = 1 - 2x$ **D** $y = x - 1$

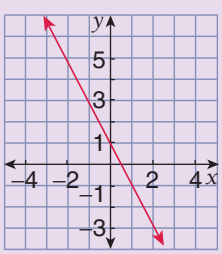
a

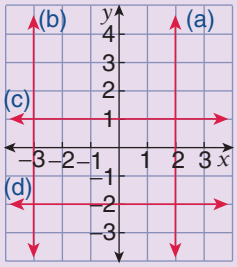
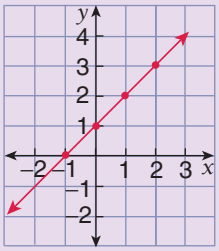
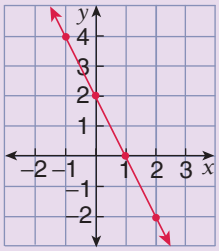
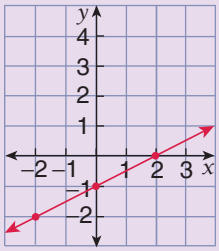


b



c

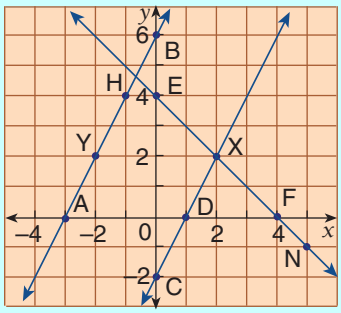


	Section
7 Graph the lines: **a** $y = 2x + 1$ **b** $y = 2 - x$ **c** $y = 3x$	8:02
8 **a** Does the point (3, 2) lie on the line $x + y = 5$? **b** Does the point $(-1, 3)$ lie on the line $y = x + 2$? **c** Does the point $(2, -2)$ lie on the line $y = x - 4$?	8:02
9 State the x-intercept of the lines: **a** $y = x - 3$ **b** $x + y = 4$ **c** $x + 2y = 4$	8:02
10 State the y-intercept of the lines: **a** $y = x + 4$ **b** $x + y = -1$ **c** $x - y = 5$	8:02
11 Write down the equation of each line in this diagram. 	8:03
12 Graph each pair of lines and find their point of intersection. **a** $y = 2x - 1$, $y = x + 1$ **b** $y = -x + 2$, $y = -1$ **c** $y = 3x - 2$, $y = -2x + 3$	8:04
13 What is the gradient of each line? **a**  **b**  **c** 	8:05
14 Write down the equation of the line which has: **a** a gradient of 3 and a y-intercept of 2 **b** a gradient of $\frac{1}{2}$ and a y-intercept of -3 **c** a y-intercept of 3 and a gradient of -1 **d** a y-intercept of $-\frac{1}{2}$ and a gradient of $\frac{1}{3}$	8:06
15 What is the gradient and y-intercept of the lines: **a** $y = 2x + 3$? **b** $y = \frac{1}{2}x - 3$? **c** $y = 3 - 2x$? **d** $y = 4 - x$?	8:06
16 Write down the equation of the lines in question **13**.	8:06

assignment 8

Chapter 8 | Revision Assignment

1 Write down the coordinates of:
a D **b** E **c** Y



2 Which letter is naming the point:
a $(4, 0)$? **b** $(-1, 4)$? **c** $(5, -1)$?

3 If the lines AB, CX and EF were extended which line would pass through the point:
a $(-4, -2)$? **b** $(6, -2)$? **c** $(-3, 7)$?

4 **a** Give the coordinates of the point of intersection of the lines CX and EF.
b Estimate the coordinates of the point of intersection of the lines AB and EF.

5 **a** What is the x-intercept of the line:
i AB? **ii** EF?
b What is the y-intercept of the line:
i CX? **ii** EF?

6 **a** Where would the line $x = 3$ intersect the line:
i EF? **ii** CX?
b Where would the line $y = -1$ intersect the line:
i EF? **ii** AB?

7 If the line $y = x + 2$ was graphed on this number plane where would it intersect the following lines?
a EF **b** AB **c** CX

8 Which of the equations $y = 2x + 6$, $y = 2x - 2$ or $x + y = 4$ describes the line:
a AB? **b** EF?

9 Find the x- and y-intercepts of the line $x + y = 2$. By lining these points up with a ruler estimate where $x + y = 2$ would intersect the lines AB and EF.

10 Estimate where the line YO would intersect the line CX. Check your answer by substituting the coordinates of the point into the equation of YO which is $y = -x$.

Technology Applications Drag and Drops

1 Number plane review
2 Graphing lines
3 x- and y-intercepts
4 Horizontal and vertical lines
5 The gradient–intercept form of a straight line

Technology Applications Activities

Plotting points in four quadrants

Chapter Review Questions

- The grid system for locating points on a sphere is based on circles.

Chapter 8 | Working Mathematically

assignment 8

1 Use ID Card 4 on page xvi to identify:

a 3 b 4 c 5 d 9 e 10
f 11 g 12 h 17 i 20 j 24

2 Use ID Card 5 on page xviii to identify:

a 8 b 9 c 10 d 11 e 12
f 13 g 14 h 15 i 18 j 24

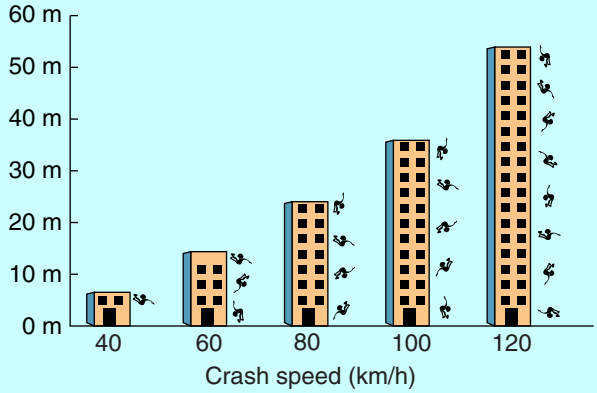
3 Of 50 table tennis players, 8 can play with the left hand but 3 can play with both the left hand and the right hand.

a How many can only play left-handed?
b How many can only play with the right hand?
c How many can play right-handed?

4 **Speeding**

Speeding has a major influence on the *risk* of being involved in a crash and the *severity* of a crash. If you do have a crash, the energy involved is not just proportional to the speed. Doubling the speed of impact multiplies by four the damage you do to the vehicle and anything else (or anyone) that gets in the way. It all goes back to Newton's laws of motion.

The force of impact on the human body in a crash can be compared with that of falling from a high building.



Crashing at 100 km/h is the same as falling from a 12-storey building.

a In a road accident, what effect on damage done is brought about by doubling the speed?
b Use the information in the graph above to draw your own graph using a curve of best fit. Do not truncate the horizontal axis as has been done here.
c What is the equivalent falling height for a pedestrian hit by a car going at:
i 40 km/h? ii 80 km/h? iii 120 km/h?
iv 110 km/h? v 20 km/h?
d What speed is equivalent to a fall from a height of:
i 50 m? ii 30 m? iii 10 m? iv 40 m?

5 How many numbers greater than 4000 can be formed from each of the set of cards below.

a

1	2	3	4

b

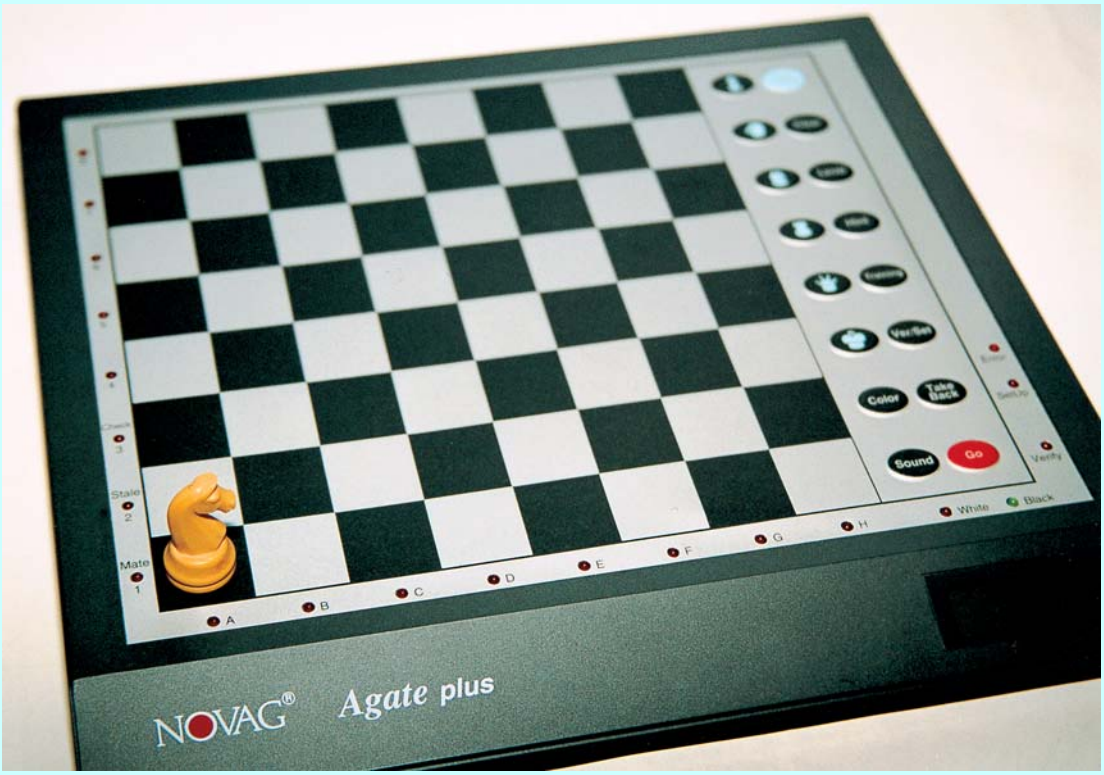
1	2	3	4	5

6 A 4 × 4 grid is drawn and the numbers 1, 2, 3 and 4 are placed in the grid so that every number occurs only once in each row and only once in each column.

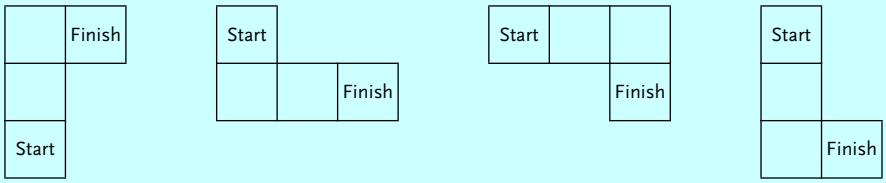
a Find the missing numbers in the following.

1	2	3	
	4		2
4			

b Now place the numbers in a 4 × 4 grid, following the rules above, so that the sums of the diagonals are 16 and 4.



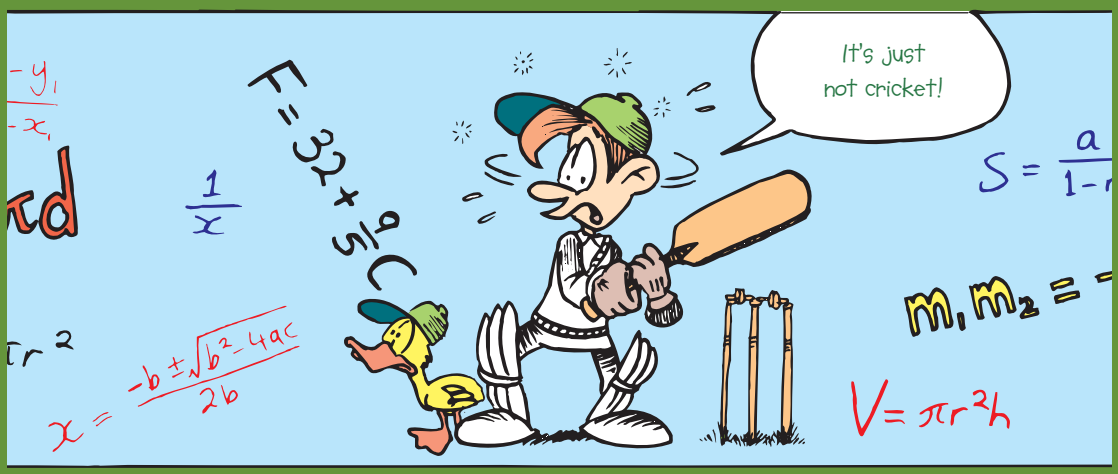
- Chess is played on an 8 by 8 square grid. Each square is named using a letter and a number which refer to the row and column that the square is in. The Knight pictured is standing on the square A1.
- In Chess a Knight can move 3 squares from its starting position to its finishing position. The squares must form an 'L' shape in any direction. Some possible moves are shown below.



- Which squares can the knight in the photo move to? If the Knight was standing on the square C1 what squares could it move to? Give a sequence of squares showing how the Knight could move from A1 to B1 to C1 to . . . H1.

9

Formulae and Problem Solving



Chapter Contents

Learning Outcomes

Students will be able to:

- Translate written problems into numerical and algebraic expressions.
- Substitute values into formulae.
- Solve equations resulting from substitution into formulae.
- Solve written problems by translating them into algebraic expressions.

Areas of Interaction

Approaches to Learning, Homo Faber

9:01 | Generalised Arithmetic

prep quiz 9:01

Find:

1. the sum of 7 and 5
2. the difference between 9 and 2
3. the number 8 less than 25
4. the quotient of 48 and 6
5. the product of 7 and 3
6. 12 more than 8
7. the average of 41 and 47
8. the total of 13 and 21
9. the number of times 23 can be taken from 138
10. the number 8 less than the product of 4 and 5

In mathematics the method of solving a problem is sometimes hard to express in words. In cases like this, pronumerals are often used. The result could be a simple formula.

- Some numbers in a pattern are known. How can we find the others?
  For example: 9, 8, 7, 6, . . . or 3, 5, 7, 9, . . .
  Patterns like these can be written in a table of values, where n represents the position of the number in the pattern, and T the actual number (or term).

n	1	2	3	4	5
T	9	8	7	6	

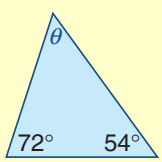
Here we can see that:
$T = 10 - n$
So an algebraic expression that represents this pattern would be:
$10 - n$

n	1	2	3	4	5
T	3	5	7	9	

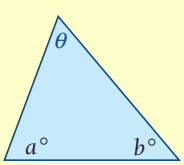
Here we can see that:
$T = 2n + 1$
So an algebraic expression that represents this pattern would be:
$2n + 1$

- Two angles of a triangle are known. How can we find the third?

A Consider a numerical example.
$\theta = 180° - (72 + 54)°$
$= 54°$



B Show the general result.
$\theta = 180° - (a + b)°$
or $180° - a° - b°$



$180° - (a + b)°$ is called the *general case*.

worked examples

1. The sum of 8 and 12 $= 8 + 12$
   so the sum of x and y $= x + y$
2. The cost of 6 books at 30c each $= 6 \times 30c$
   so the cost of x books at 30c each $= x \times 30$
   $= 30x$ cents

3 The average of 9 and 13 $= \dfrac{9+13}{2}$

so the average of a and b $= \dfrac{a+b}{2}$

4 The change from \$10 after buying 3 books at \$2 each $= 10 - (2 \times 3)$ dollars

so the change from \$10 after buying x books at \$2 each $= 10 - 2 \times x$

$= 10 - 2x$ dollars

■ The aim of 'generalised arithmetic' is to write an algebraic expression that shows the steps to be taken, no matter which numbers are involved.

Exercise 9:01

Foundation Worksheet 9:01

Generalised arithmetic

1 Write expressions for:
- **a** the sum of $3a$ and $2b$
- **b** the average of m and n

2 a Find the cost of x books at 75c each.
- **b** Find the age of Bill, who is 25 years old, in another y years.

1 Match each table of values with the correct algebraic expression from the given list to complete the statement $T = \ldots$

a

n	1	2	3	4
T	3	4	5	6

b

n	1	2	3	4
T	−2	−1	0	1

c

n	1	2	3	4
T	3	6	9	16

d

n	1	2	3	4
T	5	8	11	18

e

n	1	2	3	4
T	1	4	9	16

f

n	1	2	3	4
T	1	3	5	7

List

A $3n$
B n^2
C $n + 2$
D $3n + 2$
E $2n - 1$
F $n - 3$

2 Write down an algebraic expression that represents each pattern of numbers, using n to represent the position of each number in the pattern.

a 2, 4, 6, 8, ...
b 4, 5, 6, 7, ...
c 7, 6, 5, 4, ...
d 5, 7, 9, 11, ...
e $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, ...
f −3, −1, 1, 3, ...

■ Use a table of values if you need to.

For questions 3 to 8 write expressions for each.

3 **a** the sum of 5 and 7
b the sum of 5 and y
c the sum of x and y

4 **a** the product of 3 and 7
b the product of a and 7
c the product of a and b

5 **a** the difference between 8 and 3
b the difference between 8 and p
c the difference between q and p

6 **a** the average of 8 and 12
b the average of 8 and x
c the average of w and x

7 **a** the cost of 5 books at 75c each
b the cost of a books at 75c each
c the cost of a books at b cents each

8 **a** dividing 30 cm into 5 equal lengths
b dividing 30 cm into t equal lengths
c dividing A cm into t equal lengths

9 **a** If Steve is 15 years old, how old will he be in 6 years?
b How old will Steve be in y years?

10 **a** If a car travels at 60 km/h for 3 hours, how far does it travel?
b If the same car travels for h hours, how far does it travel?

11 **a** If three lengths of rope, each 2 m long, are cut from a piece of rope 10 m long, what length is left?
b If two lengths of rope, each x m long, are cut from a piece of rope X m long, what length is left?

12 Hien and Bob have \$1 between them. If Hien has x cents how much has Bob?

13 **a** What is the next even number after 6?
b What is the next even number after y, if y is an even number?
c What is the largest odd number less than y if y is even?

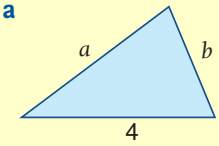
■ Try it with numbers first.

14 **a** How far will a person walk at m km/h in h hours?
b What is the average speed of a car that travels k km in h hours?
c How long will it take to travel k km at m km/h?

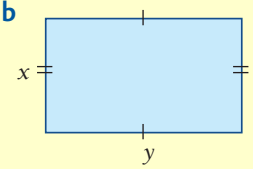
15 **a** A TV set is bought for \$$P$. If it is sold for \$$R$, what is the profit?
b If a gain of \$$G$ is to be made what should the selling price of the TV be?

16 Write an expression for the perimeter of each figure below.
All measurements are in centimetres.

a



b



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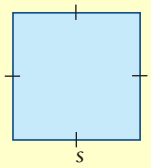

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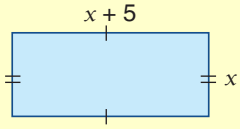
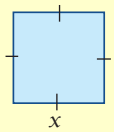
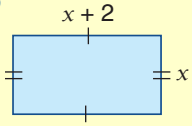


17 Write an expression for the area of each figure.

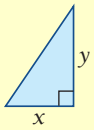
a



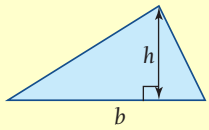
b



c



d



18 If N is the number of matches in a matchbox, then if there are 4 boxes plus 5 loose matches this could be represented by:

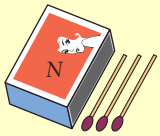
the expression $4N + 5$ or the diagram

The expression $2(N + 3) + 3$ would mean there were two extra matches in each of two boxes, plus three loose ones. The diagram would look like this:

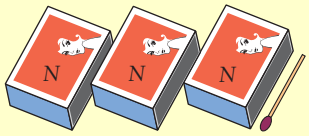
$2(N + 2) + 3 =$

Write down expressions for the following diagrams:

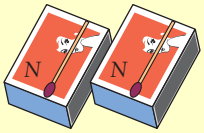
a



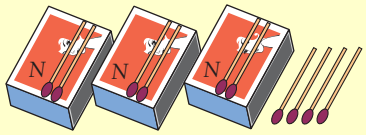
b



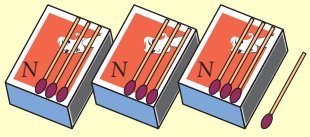
c



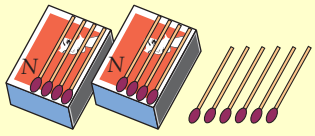
d



e



f



How could a diagram be drawn for an expression like $N - 3$?

19 Translate these algebraic expressions into words. There may be more than one way of doing this. For example, $5 - 2a$ may be written as:

'Subtract the product of two and a, from five.' OR 'Five take away two times a.'

a $3m + 4$ **b** $2y - 5$ **c** $3 - 4x$

d $\frac{a}{2} + 1$ **e** $\frac{a+1}{2}$ **f** $4 - \frac{a}{2}$

g $2(m + 1)$ **h** $4(3 - a)$ **i** $9(a + b)$

j $5(a + 3) + 4$ **k** $2(x - 1) - 3$ **l** $10 - 3(a + 5)$

fun spot 9:01

Fun Spot 9:01 | How much logic do you have?

See if you can solve the three problems below.

1 What is the next letter in this sequence? O, T, T, F, F, S, S, ?

2 A man passing a beggar in the street exclaimed, 'I am that beggar's father!' But the beggar was not the man's son. How can this be?

3 Two guards are guarding two sacks. One guard always tells the truth, but the other guard always lies, but you do not know which guard is which. One of the sacks is full of gold; the other is full of peanuts. You are permitted to take one of the sacks but you are not sure which one contains the gold. You are also allowed to ask one of the guards just one question.

What question should you ask to ensure you get the sack of gold?

9:02 | Substitution

prep quiz 9:02

Write the following expressions in their simplest form.

1 $L + L + B + B$ **2** $L \times B$ **3** $\pi \times r \times r$ **4** $2 \times r + 2 \times w$

Find the value of:

5 $2 + 4 \times 3$ **6** $3 \times 4 + 2 \times 5$ **7** 4×5^2

8 $3(6 - 10)$ **9** $\frac{1}{2} \times 6 - 4$ **10** $\frac{8 - 2}{2}$

A value for an algebraic expression can be obtained if a number is substituted for each of the pronumerals.

worked examples

Example 1

Find the value of the following expressions given that $a = 10$, $b = 4$, $x = 5$, $y = 3$.

a ab **b** $3a + 2b$ **c** $x^2 + y^2$ **d** $\frac{1}{2}ab^2$ **e** $\frac{1}{a} + \frac{1}{b}$

Solution 1

a ab
$= a \times b$
$= 10 \times 4$
$= 40$

b $3a + 2b$
$= 3 \times a + 2 \times b$
$= 3 \times 10 + 2 \times 4$
$= 30 + 8$
$= 38$

c $x^2 + y^2$
$= x \times x + y \times y$
$= 5 \times 5 + 3 \times 3$
$= 25 + 9$
$= 34$

d $\frac{1}{2}ab^2$
$= \frac{1}{2} \times a \times b \times b$
$= \frac{1}{2} \times 10 \times 4 \times 4$
$= \frac{1}{2} \times 160$
$= 80$

e $\frac{1}{a} + \frac{1}{b}$
$= \frac{1}{10} + \frac{1}{4}$
$= \frac{2}{20} + \frac{5}{20}$
$= \frac{7}{20}$

Example 2

a Find the value of $mx + c$ if $m = 2$, $x = 6$ and $c = 1$.

b Find the value of $\frac{h}{2}(a + b)$ if $h = 10$, $a = 3$ and $b = 6$.

c Evaluate πr^2 if $\pi = 3{\cdot}1$ and $r = 10$.

Solution 2

a $mx + c$
$= 2(6) + 1$
$= 12 + 1$
$= 13$

b $\frac{h}{2}(a + b)$
$= \frac{10}{2}(3 + 6)$
$= 5 \times 9$
$= 45$

c πr^2
$= 3{\cdot}1 \times 10^2$
$= 3{\cdot}1 \times 100$
$= 310$

Exercise 9:02

Foundation Worksheet 9:02

Substitution

1 Find the value of:
a $5m$, if $m = 7$
b $2a + 3b$, if $a = 3$, $b = 4$

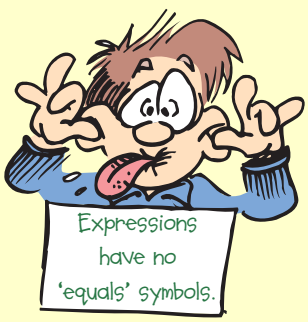
2 If $x = 3$, $y = 5$, $z = -2$, find the value of:
a xy **b** $x^2 + z^2$ **c** xyz

1 Evaluate the following expressions, if $a = 6$ and $b = 10$.

a $3a$ **b** $4b$
c $2a + 5$ **d** $3b - 6$
e $4(a + 2)$ **f** $3(12 - b)$
g $\frac{a}{2}$ **h** $\frac{b}{5}$
i $\frac{1}{2}b$ **j** $\frac{1}{3}a$ **k** $\frac{1}{2}a + 4$ **l** $10 + \frac{1}{4}b$
m $\frac{a + 10}{4}$ **n** $\frac{b - 5}{5}$ **o** $\frac{1}{2}(a + b)$ **p** $3(a + b)$
q a^2 **r** b^2 **s** $4a^2$ **t** $3b^3$
u $\frac{5b}{4}$ **v** $\frac{2}{3}a$ **w** $\frac{a}{3} + \frac{b}{10}$ **x** $4(3a + 2)$

2 Evaluate the following expressions if $x = 3$, $y = 4$ and $z = 8$.

a $x + y$ **b** $3x + 2y$ **c** $2x - y$
d x^2y **e** $(x + y)^2$ **f** $x^2 + y^2$
g $z(x + y)$ **h** $xz - 10$ **i** $xy - xz$
j $\frac{x + y}{2}$ **k** $\frac{x}{2} + \frac{y}{4}$ **l** $z^2 - z$
m $xy^2 + x^2y$ **n** $xz - yz$ **o** $4(x - 2y)$
p $x^2(y^2 - z^2)$ **q** $x - 3y$ **r** $y^2 - z^2$
s $z - xy$ **t** $\frac{x}{y} + \frac{y}{x}$ **u** $\frac{x + y}{xy}$
v $\frac{x + y}{x - y}$ **w** $\frac{1}{2}xyz^2$ **x** $\sqrt{3xy}$



3 **a** Find the value of $mx + c$ if:
i $m = 3$, $x = 10$, $c = 1$ **ii** $m = 5$, $x = 4$, $c = -3$ **iii** $m = -2$, $x = 7$, $c = -4$

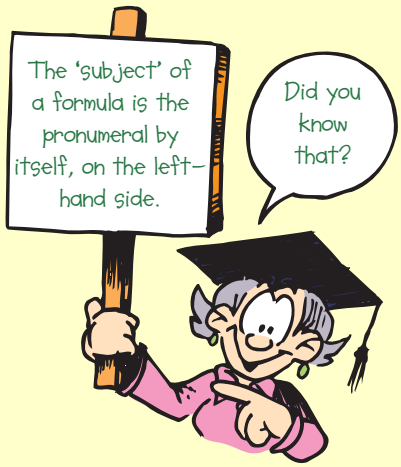
b Find the value of $\frac{h}{2}(a + b)$ if:
i $h = 4$, $a = 7$, $b = 5$ **ii** $h = 7$, $a = 4$, $b = 6$ **iii** $h = 3{\cdot}4$, $a = 9{\cdot}2$, $b = 3{\cdot}7$

c Find the value of πr^2 if:
i $\pi = 3{\cdot}1$, $r = 8$ **ii** $\pi = 3{\cdot}14$, $r = 2{\cdot}5$ **iii** $\pi = 3{\cdot}142$, $r = 100$

9:03 | Formulae: Evaluating the Subject

Formulae are special types of equations. A formula represents a relationship between physical quantities. For instance, the formula $A = L \times B$ represents the connection between the area of a rectangle and its length and breadth.

A formula is different from an equation in that it will always have more than one pronumeral. However, to find the value of a pronumeral in a formula we must be told the values of every other pronumeral in the formula.



prep quiz

9:03

Write the following algebraic expressions in their simplest form.

1 $2 \times a \times b$ **2** $3 \times a \times b \times b$ **3** $3 \times a + 2 \times b$

Write the following expressions in expanded form.

4 $4xy^2$ **5** $(x + y)^2$ **6** $x^2 + y^2$

True or false?

7 $\frac{2a}{3} = \frac{2}{3}a$.

If $a = 6$ and $b = 10$, evaluate:

8 $\frac{1}{2}ab^2$ **9** $\frac{a+b}{2}$ **10** $3(a - b)$

worked examples

1 Given that $I = \frac{Prn}{100}$, find I when $P = 500$, $r = 12$ and $n = 4$.

2 If $V = \frac{1}{3}Ah$, find V when $A = 15$ and $h = 4$.

3 Given that $a = 4$ and $b = 3$, find c when $c = \sqrt{a^2 + b^2}$.

4 If $K = \frac{1}{2}mv^2$, find K when $m = 5$ and $v = 6$.

Solutions

1 $P = 500$, $r = 12$ and $n = 4$

$$I = \frac{Prn}{100}$$
$$= \frac{^{5}\cancel{500} \times 12 \times 4}{\cancel{100}_{1}}$$
$$= 5 \times 12 \times 4$$
$$\therefore I = 240$$

2 $A = 15$ and $h = 4$

$$V = \frac{1}{3}Ah$$
$$= \frac{1}{3} \times 15 \times 4$$
$$= \frac{1}{\cancel{3}_{1}} \times \cancel{60}^{20}$$
$$V = 20$$

3 $a = 4$ and $b = 3$

$$c = \sqrt{a^2 + b^2}$$
$$= \sqrt{4^2 + 3^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$c = 5$$

4 $m = 5$ and $v = 6$

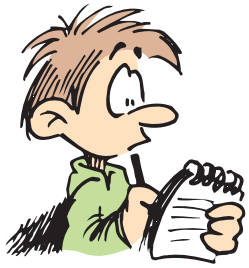
$K = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 5 \times 6^2$

$= \frac{1}{2} \times 5 \times 36$

$= \frac{1}{\cancel{2}_1} \times \cancel{180}^{90}$

$K = 90$



Remember:
- Replace the pronumerals with the given numerals.
- To find the value of one of the pronumerals you must be given the value of every other pronumeral.

Exercise 9:03

1 Match each formula with its title taken from the list beneath the table.

a $A = LB$	**b** $A = bh$	**c** $A = \frac{1}{2}bh$
d $A = s^2$	**e** $A = \frac{1}{2}xy$	**f** $A = \pi r^2$
g $A = \frac{1}{2}h(a + b)$	**h** $C = \pi d$	**i** $V = lbh$
j $V = Ah$	**k** $P = 2L + 2B$	**l** $c^2 = a^2 + b^2$
m $\alpha + \beta + \gamma = 180°$	**n** $\alpha + \beta + \gamma + \delta = 360°$	**o** $F + V = E + 2$

A Euler's theorem
B Area of a circle
C Pythagoras' theorem
D Volume of a prism
E Area of a rhombus
F Area of a square
G Angle sum of a triangle
H Circumference of a circle
I Area of a rectangle
J Area of a parallelogram
K Perimeter of a rectangle
L Angle sum of a quadrilateral
M Area of a trapezium
N Area of a triangle
O Volume of a rectangular prism

2 **a** If $A = LB$, find A when $L = 3{\cdot}6$ and $B = 2{\cdot}4$.

b If $A = bh$, find A when $b = 15$ and $h = 3{\cdot}6$.

c Given that $A = \frac{1}{2}bh$, find A when $b = 15$ and $h = 3{\cdot}6$.

d Given that $A = s^2$, find A when $s = 3{\cdot}5$.

e If $A = \frac{1}{2}xy$, find A when $x = 8$ and $y = 11$.

f If $A = \pi r^2$, find A correct to 1 decimal place if $r = 3{\cdot}4$.

g Given that $A = \frac{1}{2}h(a + b)$, find A when $h = 8$, $a = 11$ and $b = 9$.

h Given $C = \pi d$, find C correct to 1 decimal place if $d = 6{\cdot}8$.

i If $V = lbh$, find V if $l = 8{\cdot}1$, $b = 7{\cdot}2$ and $h = 4{\cdot}5$.

j If $V = Ah$, find V if $A = 341{\cdot}6$ and $h = 6{\cdot}15$.

k Given that $P = 2L + 2B$, find P when $L = 3{\cdot}75$ and $B = 2{\cdot}45$.

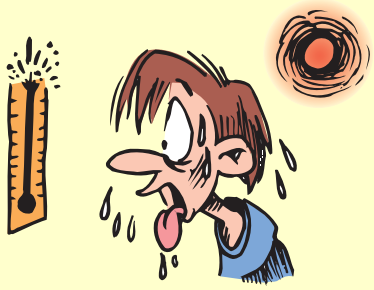
l Given that $c = \sqrt{a^2 + b^2}$, find c when $a = 5$ and $b = 12$.

m If $\alpha = 180° - \beta - \gamma$, find α when $\beta = 30°$ and $\gamma = 45°$.

n If $\alpha = 360° - \beta - \gamma - \delta$, find α when $\beta = 37°$, $\gamma = 51°$ and $\delta = 132°$.

o Given that $E = F + V - 2$, find the number of edges (E) when the number of faces (F) is 7 and the number of vertices (V) is 10.

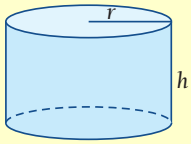
3 **a** The formula $C = \frac{5}{9}(F - 32)$ converts degrees Fahrenheit (F) to degrees Celsius (C). Find C when:
i $F = 212$ **ii** $F = 32$ **iii** $F = 104$



b The kinetic energy (E) of a particle is given by the formula $E = \frac{1}{2}mv^2$ where m and v are the mass and velocity of the particle respectively. Find E if $m = 6$ and $v = 2{\cdot}5$.

c The surface area (S) of a cylinder is given by the formula $S = 2\pi r^2 + 2\pi rh$. Evaluate S when $\pi = 3{\cdot}14$, $r = 1{\cdot}50$ and $h = 3{\cdot}25$. Give the answer correct to 3 significant figures.

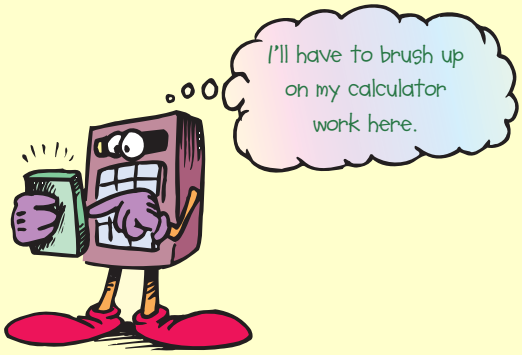
d The volume (V) of a cylinder is given by the formula $V = \pi r^2 h$. Find the volume correct to the nearest cubic centimetre if the radius (r) is 8 cm and the height (h) is 10 cm.



4 **a** If $P = RI^2$, evaluate P when $R = 2{\cdot}5$ and $I = 0{\cdot}6$.

b It is known that $F = \frac{mp}{r^2}$. If $r = 0{\cdot}2$, $m = 3{\cdot}6$ and $p = 9{\cdot}2$, find the value of F.

c $T = 2m\sqrt{\frac{L}{g}}$. Find T if $m = 3{\cdot}6$, $L = 2{\cdot}5$ and $g = 0{\cdot}1$.



d Given the formula $P = \frac{nkT}{V}$, find P when $n = 2{\cdot}4$, $k = 7{\cdot}6$, $T = 6$ and $V = 0{\cdot}15$.

e If $S = \frac{n}{2}[2a + (n-1)d]$, find S when $n = 20$, $a = -4$ and $d = 0{\cdot}4$.

f Given the formula $S = ut + \frac{1}{2}at^2$, find S when $u = 8$, $t = 4$ and $a = -10$.

g If $I = \frac{Prn}{100}$, find I when $P = 450$, $r = 12$ and $n = 3{\cdot}5$.

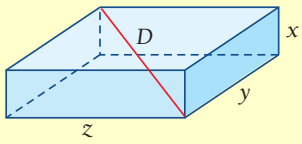
h If $S = \frac{a(r^n - 1)}{r - 1}$, find S when $a = 5{\cdot}2$, $r = 1{\cdot}4$ and $n = 2$.

i If $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$, find x when $b = 3$, $a = 2$ and $c = -10$. Give the answer correct to 2 decimal places.

5 The distance D from the top corner of a rectangular box to the opposite bottom corner is given by the formula:

$D = \sqrt{x^2 + y^2 + z^2}$

where x, y and z are the dimensions of the box.
Find D if the dimensions x, y and z are:



a 3 cm, 4 cm, 5 cm **b** 7 cm, 11 cm, 15 cm
c 2·3 cm, 4·2 cm, 4·9 cm **d** 11·6 cm, 13·2 cm, 17·8 cm

Answer correct to 1 decimal place.

Investigation 9:03 | Spreadsheet formulae

investigation 9:03

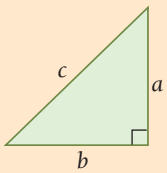
Technology Applications Activities

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

In spreadsheet programs such as Excel a formula can be inserted in the 'formula bar' which will operate on selected 'cells'.

The example below shows a simple example which uses Pythagoras' theorem to find the hypotenuse of a right triangle.

For this triangle, $c = \sqrt{a^2 + b^2}$.



You can see this formula in the formula bar written as:

=SQRT(A2^2+B2^2)

■ SQRT means 'square root'.
^ means 'to the power of'.

This means that the number in cell C2 will be equal to the square root of (cell A2 squared plus cell B2 squared).

If the numbers in cells A2, B2 are changed, cell C2 will also change automatically.

Formula bar

C2		=SQRT(A2^2+B2^2)			
	A	B	C	D	E	F
1	a	b	c			
2	3	6	6.71			
3						
4						
5						

- Try other applications (you may have to ask your teacher for the formulae):
  - Finding the perimeter of a rectangle
  - Finding the length of a rectangle given the perimeter and the width
  - Finding the new price of items reduced by 15% in a sale
  - Finding the new wage for workers whose wage has increased by 8%
- Try some examples of your own.
- Comment on the application of such a spreadsheet.

(Note: In Excel you can alter the number of decimal places by going to NUMBER in the FORMAT option.)

- Spreadsheets can be very useful mathematical tools. Can you set up spreadsheets to automatically calculate the formulae in Exercise 9:03 question 1 (p 219).

Assessment Grid for Investigation 9:03 | Spreadsheet formulae

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (C, D) for this investigation				Achieved ✓
Criterion C Communication	a	No explanation or working out is given.	1	
			2	
	b	Explanations are given at every step to explain how the formulae have been applied.	3	
			4	
	c	Discussion demonstrates not only the ability to manipulate formulae, but also the ability to apply the formulae.	5	
			6	
Criterion D Reflection in Mathematics	a	The student has attempted to explain the processes involved in manipulating formulae and has attempted to check results.	1	
			2	
	b	The explanation of the processes and the checking of the results have been carried out with some success.	3	
			4	
	c	The reliability and accuracy of the results is discussed fully, and the student has used original examples to demonstrate further applications of spreadsheets in a practical way.	5	
			6	

9:04 | Formulae: Equations Arising from Substitution

Consider the following formulae.

$$S = \pi rl \qquad T = 2\pi\sqrt{\frac{l}{g}} \qquad D = \frac{M}{V} \qquad S = ut + \tfrac{1}{2}at^2$$

- The pronumeral that is on the left-hand side of each of these formulae is called the **subject** of the formula. In all the exercises so far, you have been asked to find the subject of the formula.
- We often know the value of the subject and are asked to find the value of one of the other pronumerals.
- To find the value of this pronumeral we will need to solve an equation.

prep quiz 9:04

Solve:

1 $x + 152 = 315$ **2** $y - 73 = 149$ **3** $7x = 343$

4 $\frac{x}{10} = 37$ **5** $110 = 14 + x$ **6** $96 = y - 87$

7 $112 = 4p$ **8** $18 = \frac{m}{3}$ **9** $2a + 1 = 11$

10 $50 = 16 + 17t$

worked examples

1 Given that $V = \frac{AH}{3}$, find H when $V = 12$ and $A = 5$.

2 $A = \frac{1}{2}h(x + y)$. Find the value of x correct to 1 decimal place if $A = 11$, $h = 3{\cdot}6$ and $y = 4{\cdot}5$.

3 If $S = \frac{a}{1 - r}$, find r when $s = 10$ and $a = 1{\cdot}5$.

Solutions

1 $V = 12$ and $A = 5$

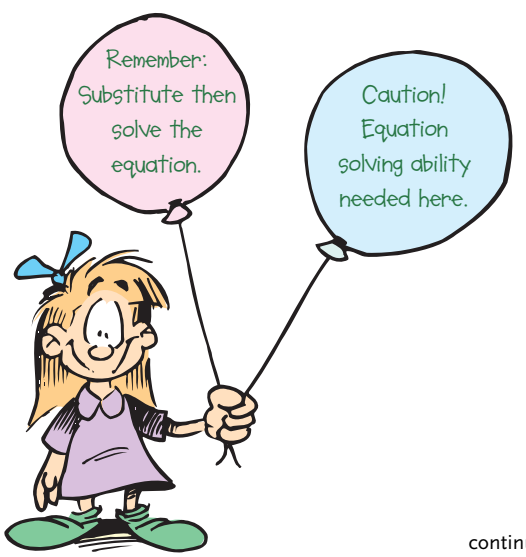
$$V = \frac{AH}{3}$$

$$\therefore 12 = \frac{5H}{3}$$

$$36 = 5H$$

$$\therefore H = \frac{36}{5}$$

$$= 7\tfrac{1}{5}$$



continued ➜➜➜

2 $A = 11$, $h = 3{\cdot}6$ and $y = 4{\cdot}5$

$$A = \tfrac{1}{2}h(x + y)$$

$$\therefore\ 11 = \frac{1}{\cancel{2}_{1}} \times \cancel{3{\cdot}6}^{1{\cdot}8}\ (x + 4{\cdot}5)$$

$$11 = 1{\cdot}8(x + 4{\cdot}5)$$

$$= 1{\cdot}8x + 1{\cdot}8 \times 4{\cdot}5$$

$$11 = 1{\cdot}8x + 8{\cdot}1$$

$$11 - 8{\cdot}1 = 1{\cdot}8x$$

$$2{\cdot}9 = 1{\cdot}8x$$

$$\frac{2{\cdot}9}{1{\cdot}8} = x$$

$\therefore = 1{\cdot}6$ (correct to 1 decimal place)

3 $S = 10$ and $a = 1{\cdot}5$

$$S = \frac{a}{1 - r}$$

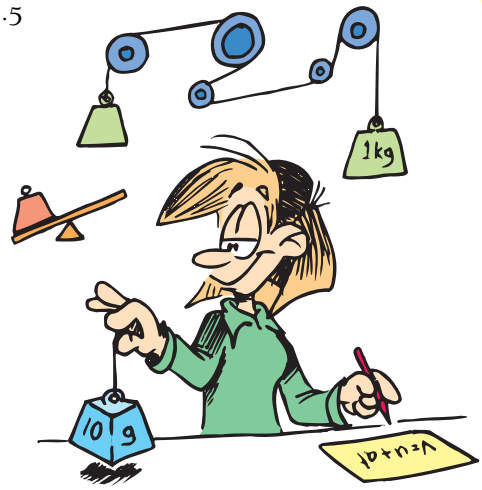
$$\therefore\ 10 = \frac{1{\cdot}5}{1 - r}$$

$$10(1 - r) = 1{\cdot}5$$

$$10 - 10r = 1{\cdot}5$$

$$-10r = -8{\cdot}5$$

$$\therefore\ r = 0{\cdot}85$$



Exercise 9:04

1 If $v = u + at$, find:

a u when $v = 25$, $a = 3$, $t = 5$
b a when $v = 40$, $u = 10$, $t = 2$
c t when $v = 16$, $u = 4$, $a = 5$
d u when $v = 27{\cdot}2$, $a = 4$, $t = 3{\cdot}5$
e a when $v = 15$, $u = 5$, $t = \frac{1}{4}$
f t when $v = 26$, $u = -16$, $a = 7$
g u when $v = 12{\cdot}68$, $a = 4{\cdot}2$, $t = 1{\cdot}5$
h a when $v = 16$, $u = -4$, $t = 0{\cdot}75$

2 Given that $V = \dfrac{AH}{3}$, find:

a H when $V = 5$ and $A = 3$
b H when $V = 12$ and $A = 9$
c H when $V = 16$ and $A = 10$
d H when $V = 15{\cdot}5$ and $A = 3{\cdot}1$
e A when $V = 6$ and $H = 5$
f A when $V = 15$ and $H = 12$
g A when $V = 7$ and $H = 0{\cdot}7$
h A when $V = 3{\cdot}6$ and $H = 0{\cdot}4$

3 For the formula $K = \frac{1}{2}mv^2$, find:

a m when $K = 60$, $v = 2$
b m when $K = 15$, $v = 3$
c m when $K = 3{\cdot}2$, $v = 4$
d m when $K = 7$, $v = \frac{1}{2}$
e v when $K = 25$, $m = 2$
f v when $K = 36$, $m = 8$
g v when $K = 216$, $m = 3$
h v when $K = \frac{3}{8}$, $m = 3$

4 $A = \frac{1}{2}h(x + y)$. Find:

a x when $A = 10$, $h = 4$, $y = 3$
b x when $A = 20$, $h = 8$, $y = 2$
c y when $A = 6$, $h = 3$, $x = 4$
d y when $A = 45$, $h = 6$, $x = 3$
e h when $A = 3$, $x = 4$, $y = 6$
f h when $A = 18$, $x = 5$, $y = 13$
g h when $A = 7{\cdot}8$, $x = 1{\cdot}7$, $y = 0{\cdot}9$
h h when $A = 5{\cdot}16$, $x = 4{\cdot}6$, $y = 8{\cdot}3$

5 **a** If $A = LB$, find L when $A = 9{\cdot}6$ and $B = 2{\cdot}4$.

b If $V = RI$, find I when $V = 15$ and $R = 0{\cdot}6$.

c Given that $V = AH$, find H correct to 1 decimal place, if $V = 12{\cdot}6$ and $A = 4{\cdot}1$.

d It is known that $V = LBH$. Find B correct to 2 significant figures, if $V = 60$, $L = 3{\cdot}4$ and $H = 2{\cdot}6$.

e Calculate the value of r in the formula $A = 2\pi rh$ if $A = 75$, $\pi = 3{\cdot}14$ and $h = 7{\cdot}6$. (Give your answer correct to 1 decimal place.)

6 **a** If $S = \dfrac{a}{1-r}$, find a when $S = 5{\cdot}2$ and $r = 0{\cdot}3$.

b For the formula $S = ut + \frac{1}{2}at^2$ evaluate a, given that $S = 15$, $u = 2$ and $t = 5$.

c $X = \dfrac{x + ky}{1 + k}$. Find y when $X = 10$, $k = 2$ and $x = 3$.

d Given that $T = \dfrac{ab}{6} + C$, find b when $T = 15$, $a = 5$, $C = 4$.

e It is known that $F = 32 + \frac{9}{5}C$. Find C if $F = 212$.

7 In each of the following, give your answer correct to 2 decimal places.

a If $A = LB$, find B given that $A = 15$ and $L = 4{\cdot}8$.

b Given that $A = \pi r^2$, find r when $A = 10$ and $\pi = 3{\cdot}14$.

c If $T = a + (n - 1)d$, find d given $T = 19{\cdot}6$, $a = 3{\cdot}6$, $n = 12$.

d $P = RI^2$. Find R if $P = 100$ and $I = 3{\cdot}6$.

e $F = 32 + \dfrac{9C}{5}$. Find C if $F = 100$.

8 **a** The area of a circle can be found using the formula $A = \dfrac{\pi d^2}{4}$. Use this formula to find the area of a circle with a diameter of 12 cm.

b Use the formula $A = \pi r^2$ to find the area of a circle with a radius of 6 cm. Is your answer the same as in part **a**.

- Formulae are used in many occupations when solving everyday problems.

9:05 | Translating Problems into Equations

prep quiz 9:05

What is the sum of:

1 3 and 4? **2** x and 3? **3** x and y?

What is the product of:

4 3 and 4? **5** x and 3? **6** x and y?

7 I have \$50. I spend \$$x$. How much do I have left?

8 There are x books and each one costs \$5. What is the total cost of the books?

9 There are x cars with y people in each car. How many people are there altogether?

10 A man is x years old. How old will he be in 5 years time?

Consider the following simple problem.

'I think of a number. If I add 7 to the number the result is 22. What is the number?'

- This problem can be solved by forming an equation. If the missing number is represented by the pronumeral x, then the equation $x + 7 = 22$ represents the information given in the problem. Solving the equation then yields the answer to the original problem.
- Although this problem is quite simple it illustrates how 'word sentences' or problems are translated into number sentences. When solving problems we have to be able to translate the words into equations.

worked examples

Translate the following into number sentences.
In all cases use the 'x' to represent the unknown number.

1 I multiply a number by 2 and the result is 50.
2 If I add 6 to a number the answer is 11.
3 I subtract a number from 6 and the answer is 2.
4 A certain number is multiplied by 3 then 6 is added and the result is 17.

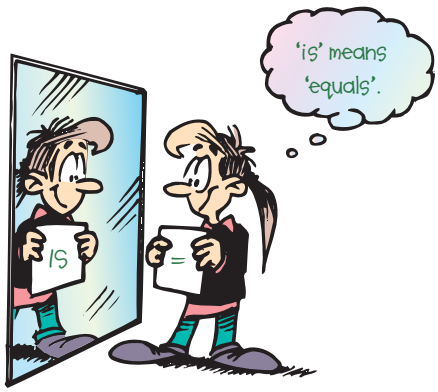
■ We often use 'x' to represent an unknown number.

Solutions

1 I multiply a number by 2 and the result is 50.
$2 \times x$ ↓↓ $= 50$
The equation is $2x = 50$.

2 If I add 6 to a number the answer is 11.
$6 + x$ ↓↓ $= 11$
The equation is $6 + x = 11$.

3 I subtract a number from 6 and the answer is 2.
$6 - x$ ↓↓ $= 2$
The equation is $6 - x = 2$.



4 A certain number is multiplied by 3, then 6 is added and the result is 17.

$x \times 3$ $+6$ $= 17$

The equation is $3x + 6 = 17$.

Exercise 9:05

Foundation Worksheet 9:05

Translating problems into equations

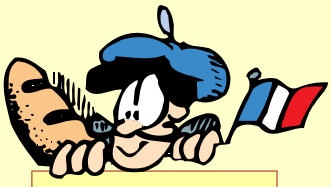
1 Write equations for:
- **a** the sum of a and 5 equals 8
- **b** the product of 7 and x equals 42

2 Write equations for the following and solve.
- **a** If I subtract 9 from 3 times a number the result is 27.

For questions 1 to 5 form an equation and then solve it.

1
- **a** I multiply a number by 8 and the result is 32.
- **b** If I multiply a number by 4 the answer is 100.
- **c** When a certain number is multiplied by 7, the result is 42.
- **d** Multiplying a certain number by 12 gives an answer of 78.
- **e** If a certain number is multiplied by 6 the result is 15·6.

2
- **a** If I add 6 to a number the result is 10.
- **b** If 5 is added to a number the answer is 22.
- **c** When 22 is added to a number the result is 50.
- **d** Adding 17 to a number gives an answer of 12.
- **e** What must be added to 12 to give 38?



'I subtract 3 from a certain number' translates to $x - 3$.

'I subtract a certain number from 3' translates to $3 - x$.

Also $x - 3$ is not the same as $3 - x$.

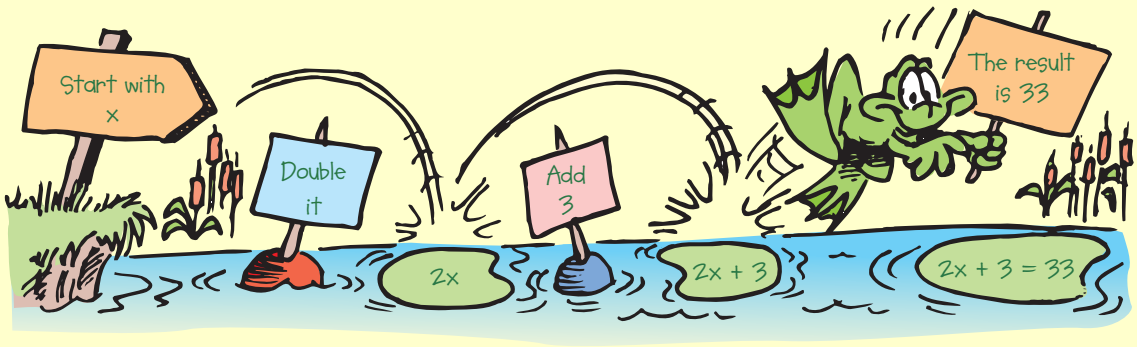
3
- **a** If I subtract 3 from a certain number the result is 10.
- **b** 5 is subtracted from a number and the answer is 22.
- **c** When 25 is subtracted from a certain number the answer is 10.
- **d** Subtracting 17 from a certain number gives an answer of 18.
- **e** If a number is subtracted from 6, the result is 4.

4
- **a** If a number is divided by 5, the answer is 4.
- **b** Dividing a certain number by 8 gives an answer of 7.
- **c** One-tenth of a certain number is 8.
- **d** What number does 6 go into 7 times?
- **e** If 10 is divided by a certain number the result is 2.

5
- **a** A number is multiplied by 2, then 6 is added and the result is 14.
- **b** Three times a certain number is added to 5 and the result is 20.
- **c** A certain number is multiplied by 5, then 8 is subtracted and the result is 22.
- **d** If 5 is added to a certain number and the result is multiplied by 4 the answer is 56.
- **e** When 5 is subtracted from half of a number the result is 3.

6 For each of the following problems form an equation and then solve it.

a I think of a number, double it, add 3 and the result is 33. What is the number?



b I think of a number and multiply it by 4. If I then subtract 3, the answer is 25. Find the number.

c I think of a number, add 3 and then double that result. If the answer is 22, find the number.

d I think of a number. After dividing it by 4 and subtracting 7 the result is 1. What is the number?

e I think of a number. If I add 4 and then divide by 3 the result is 8. Find the number.

7 Solve each of the following problems by first forming an equation.

a If 5 is added to 3 times a certain number the result is 38. What is the number?

b If I subtract 6 from 5 times a certain number the result is 29. What is the number?

c If 5 is subtracted from a certain number and that result is then halved, the answer is 6. What is the number?

d A number is doubled and then 5 is added. When this is divided by 3 the result is 7. What is the number?

Reading mathematics 9:05 | Read carefully (and think)!

Answer each problem below.

1 Which weighs more: 3 kg of lead or 3 kg of feathers?

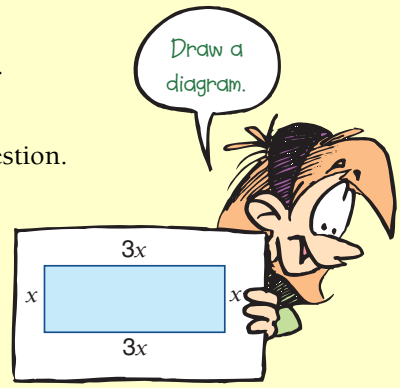
2 A company has 5943 employees. All but 742 of them attended the company picnic. How many did not attend?

3 There are 27 students in Mr Thomas's class, of which 18 were boys. How many were not girls?

4 Mrs Green left for work at 7:45 am, half an hour earlier than normal. She arrived at work at 8:25 am, worked for 8 hours and then went home. The trip home took 10 minutes longer than the trip to work. What time did Mrs Green usually leave for work in the morning.

9:06 | Further Problem Solving Using Equations

To use equations to solve problems we must be able to analyse a written problem, translate it into an equation and then solve it.



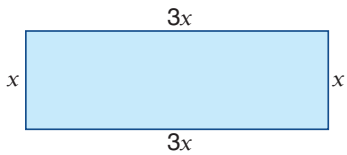
Approach

- Read the problem carefully, examining the wording of the question.
- Establish what is to be found and what information is given.
- Ask yourself whether any other information can be assumed, eg that a pack of cards mentioned is a standard pack.
- Try to connect the given information to form an equation. This will often require a knowledge of a formula or the meaning of mathematical terms.

worked examples

Example 1

A rectangle is three times longer than it is wide. If it has a perimeter of 192 m, what are its dimensions?



Solution 1

Let the width be x metres.

$\therefore$ The length $= 3 \times x$ metres
$= 3x$ metres

Now perimeter means the sum of the lengths of the sides (or the distance around the outside of the figure).

$\therefore 3x + x + 3x + x = 192$
$\therefore 8x = 192$
$\therefore x = 24$
$\therefore$ The width $= 24$ m
and the length $= 3 \times 24$ m
$= 72$ m

■ The first line of each solution should give a pronumeral to the thing you wish to find.

Example 2

My father was 28 years old when I was born. If he is now three times as old as I am, what are our present ages?

Solution 2

Let my present age be x years.

$\therefore$ My father's present age is $3 \times x$ years.
When I was born my father was 28.
$\therefore$ The difference in our ages is 28 years.
$\therefore$ Father's age – my age always equals 28 years.
$\therefore 3x - x = 28$
$2x = 28$
$x = 14$
$\therefore$ I am 14 years old and my father is 42 years old (ie 3×14 years).

continued ➜➜➜

Example 3

Car A left Sydney for Melbourne at 6:00 am and travelled at an average speed of 80 km/h. At 7:30 am car B left Sydney for Melbourne. If car B travels at an average speed of 100 km/h, at what time will it catch car A?

Solution 3

Car B will catch car A when both have travelled the same distance
and distance travelled = average speed × time.

Now *let car B catch up to car A t hours after it starts.*

$\therefore$ Car B has been travelling for t hours.

$\therefore$ Car A has been travelling for $(t + 1\frac{1}{2})$ hours (since it started at 6 am).

$\therefore$ Distance travelled by car A $= 80 \times (t + 1\frac{1}{2})$

Distance travelled by car B $= 100 \times t$

$$\therefore 80(t + 1\tfrac{1}{2}) = 100t$$

$$\therefore 80t + 80 \times 1\tfrac{1}{2} = 100t$$

$$\therefore 120 = 20t$$

$$\therefore t = 6$$

$\therefore$ Car B catches car A 6 hours after it starts, ie at 1:30 pm.

CAR A
Speed = 80 km/h
Starts at 6:00 am
Travels for $(t + 1\frac{1}{2})$ hours
Distance travelled = $80(t + 1\frac{1}{2})$

CAR B
Speed = 100 km/h
Starts at 7:30 am
Travels for t hours
Distance travelled = $100t$

Exercise 9:06

1 **a** My father is three times as old as I am. If he is 26 years older than I, what are our ages?

b Two men have $560 between them. If one man has six times as much money as the other, how much has each man?

c Joan has $7 less than Anne. Together they have $43. How much does each girl have?

d Prize money of $500 is divided between Alan and Jim so that Alan receives $170 more than Jim. How much does each receive?

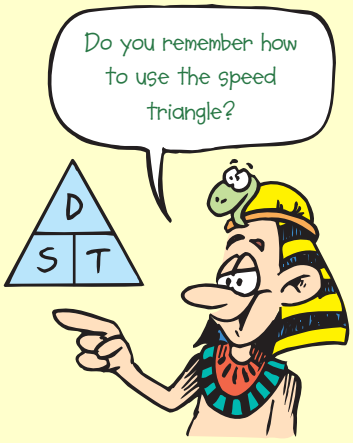
e If a father is five times as old as his son at present, how old is he if he was 32 years old when his son was born?

2 **a** In a class of 32 students, it is known that there are 6 more boys than girls. How many girls are there in the class?

b A rectangle is 6 cm longer than it is wide. Find its dimensions if its perimeter is 64 cm.

c If a quarter of the weight of a roast is lost in roasting, what weight of roast should be bought in order to have 3 kg of roasted meat?

d A town B is between towns A and C. B is five times as far from C as it is from A. The distance from A to C is 144 km. How far is it from A to B?



3 **a** Six kilograms of an inferior tea is mixed with 3 kilograms of tea that costs \$2 a kilogram more. The total price of the mixture is \$24. What was the price of the inferior tea?

b Two bike riders X and Y both start at 2 pm riding towards each other from 40 km apart. X rides at 30 km/h, Y at 20 km/h. If they meet after t hours, find when and where they meet.

c A man is twice as old as his son. If 9 years ago the sum of their ages was 66 years, find their present ages.

d A man notices that a tank is half full. After emptying 600 litres from the tank, he observes that it is now one-third full. How much does the tank hold when it is full?

4 **a** Fred is 25 years older than Bill and, in 5 years, he will be twice as old as Bill. Find their present ages.

b A bank teller notices that he has 50 coins all of which are 5c or 10c pieces. He finds that the value of the coins is \$4.20. How many of each must he have?

c A tennis player has won 36 out of 54 matches. His sponsor says that he must win 60% of his total number of matches to qualify for a bonus. If there are 26 matches remaining on the tour, how many more must he win to collect his bonus?

d One tank holds 300 litres more than another. If the smaller is two-thirds full, it holds as much as the larger when it is half full. What is the capacity of each?

e A certain journey took 40 min to complete. If half the distance was travelled at an average speed of 100 km/h and the other half at an average speed of 60 km/h, what was the length of the journey?

■ 60 km/h (A) 100 km/h (B)

$$\text{Time for A} + \text{Time for B} = 40 \text{ min}$$

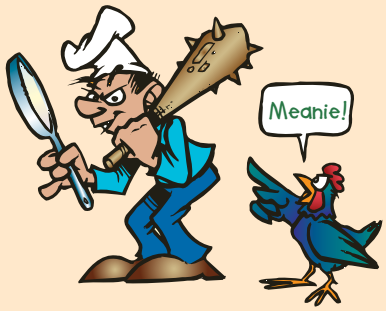
$$40 \text{ min} = \frac{2}{3} \text{ h}$$

fun spot

9:06

Fun Spot 9:06 | Why are cooks cruel?

Work out the answer to each part and put the letter for that part in the box that is above the correct answer.



Simplify:

E $9a - 5a$ **E** $10x - x$

E $4 \times 2y$ **E** $18m \div 2$

E $-3a + 9a$ **E** $(-3a)^2$

E $-a - 5a$ **A** $9x + x$

A $7x + a + x$ **A** $20m \div m$

A $-8 \times \frac{1}{2}x$ **A** $5x^2 - x - x$

I $ax \times 5a$ **I** $\frac{3}{a} \times \frac{a}{3}$

U $25x^2 \div 5x - 3x$ **T** $6a^2 - a^2 - 5$ **T** $16a^2 \div (-2a^2)$ **T** $8x^2 \div 4x \times 2x$

Write an expression for:

T one more than x **B** the product of a and b

B the average of x and a **B** the difference between x and a

Find the value of $x^2 - 4x + 4$ if:

S $x = 7$ **S** $x = 10$ **S** $x = 9$ **C** $x = -1$

Solve these equations:

H $x + 8 = 5$ **H** $x - 9 = 4$ **F** $\frac{-13}{x} = 1$ **C** $\frac{x}{3} = 1$

$l = 6$, $b = 8$, $h = 10$, $a = 5$, $b = 9$ and $s = 7$, find A if:

R $A = lb$ **M** $A = \frac{1}{2}bh$ **N** $A = S^2$ **D** $A = \frac{1}{2}h(a + b)$

Evaluate:

Y $\frac{a}{2}$ if $a = \frac{1}{2}$ **G** $\frac{1}{M}$ if $M = \frac{1}{2}$ **G** $90° - \theta$ if $\theta = 17°$ **H** $3{\cdot}14r^2$ if $r = 10$

R $\frac{8}{1 - r}$ if $r = 0{\cdot}5$ **P** $\frac{9C}{5}$ if $C = 20$ **W** $ut + \frac{1}{2}at^2$ if $u = 12$, $a = 9{\cdot}8$ and $t = 10$

ab	$4a$	9	20	$2x$	64	$9x$	-8	314	$8y$	$\frac{1}{4}$	$x - a$	$9m$	$10x$	$4x^2$	$6a$	$73°$	2	25

610	$x = 13$	$5a^2x$	36	$x = 3$	$A = 48$	$-6a$	$-4x$	$A = 40$	$5x^2 - 2x$	$A = 49$	$A = 70$	$\frac{x+a}{2}$	$8x + a$	$5a^2 - 5$	$x + 1$	$9a^2$	16	$x = -13$	1	49	$x = -3$

Mathematical terms 9

equation

- A number sentence where one or more of the numbers is missing or unknown.
- The unknown number is represented by a pronumeral.

  eg $x + 5 = 8$, $\frac{3x+1}{7} = \frac{x-5}{2}$

evaluate

- To find the value of a required expression.
  eg find the value of $3a + 7$ if $a = 3$

formula (plural: formulae)

- A formula represents a relationship between physical quantities.
- It will always have more than one pronumeral.
  eg $A = L \times B$ represents the relationship between the area (A) of a rectangle and its length (L) and breadth (B).

pronumeral

- A symbol used to represent a number — usually a letter such as x.

subject

- The subject of a formula is the pronumeral by itself, on the left-hand side.
  eg in the formula $v = u + at$ the subject is v.

substitution

- The replacing of a pronumeral with a numeral in a formula or expression.
  eg to substitute 3 for a in the expression $4a - 2$ would give:
  $4(3) - 2$
  $= 12 - 2$
  $= 10$

- Formulae are used extensively in physics and other sciences.

diagnostic test

9

Diagnostic Test 9: | Formulae and Problem Solving

- Each part of this test has similar items that test a certain question type.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

Section

1 For each table of values, find the expression in x that completes the rule $y = \ldots$ — 9:01

a

x	0	1	2	3
y	0	4	8	12

b

x	5	6	7	8
y	12	14	16	18

c

x	1	2	3	4
y	1	4	9	16

2 Write an algebraic expression for each of the following. — 9:01

a the sum of x and y **b** the average of 5 and m

c the cost of b books at p dollars each

d \$2 was shared between Sue and Jenny. If Sue received x cents, how many cents did Jenny receive?

3 If $m = 2$, $x = 6$, $c = 1$, $h = 10$, $a = 3$, $b = 6$, $\pi = 3{\cdot}1$ and $r = 10$, evaluate: — 9:02

a $mx + c$ **b** $\frac{h}{2}(a + b)$ **c** πr^2

4 If $v = u + at$, find v when: — 9:03

a $u = 6$, $a = 3$, $t = 4$ **b** $u = 6{\cdot}8$, $a = 9{\cdot}8$, $t = 3$

c $u = 5$, $a = 4{\cdot}6$, $t = 0{\cdot}6$

5 If $C = 2\pi r^2$, find C when: — 9:03

a $\pi = 3{\cdot}14$, $r = 5$ **b** $\pi = 3{\cdot}142$, $r = 1{\cdot}6$ **c** $\pi = \frac{22}{7}$, $r = 0{\cdot}75$

6 If $M = 2m + 3n$, find m when: — 9:04

a $M = 12$, $n = 1$ **b** $M = 17{\cdot}5$, $n = 0{\cdot}5$ **c** $M = 29$, $n = 3{\cdot}75$

7 If $V = \frac{Ah}{3}$, find h when: — 9:04

a $V = 15$, $A = 2$ **b** $V = 14{\cdot}8$, $A = 4$ **c** $V = 12$, $A = 1{\cdot}8$

8 Form an equation from the given data for each of these. (In each case let a represent the unknown number.) — 9:05

a I think of a number, multiply it by 2, add 7 and the result is 10.

b I think of a number, divide it by 3, subtract 4 and the result is 4.

c I think of a number, add 6, then multiply by 3 and the result is 32.

9 Form an equation for each question. (Let the unknown quantity be x.) — 9:06

a The sum of a brother's and sister's ages is 57 years. If the brother is 5 years older than his sister, find their ages.

b A rectangle is three times longer than it is wide. If its perimeter is 48 cm, find its length and width.

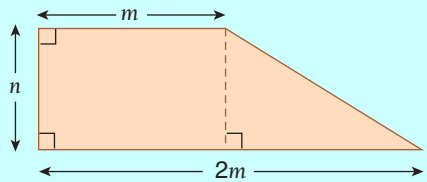
c A father is presently three times as old as his son. In 10 years he will be twice as old as his son. Find their present ages.

Chapter 9 | Revision Assignment

assignment 9A

1 Write down an algebraic expression for each of the following.

- **a** the product of $3a$ and $4b$
- **b** the total cost of 'x' books at \$7 each and 'y' books at \$9 each.
- **c** the time it will take a car to travel 'd' kilometres at 's' km/h
- **d** the area of this figure



• Formulae have an important role in engineering.

2 **a** If $a = 5$ and $b = 6$, evaluate:

i $a + b$ **ii** $ab - 10$

b If $x = 1{\cdot}6$ and $y = 2{\cdot}7$, evaluate:

i $2x + 3y$ **ii** $x^2 - x$

c Given that $m = 9$ and $n = 4{\cdot}5$, evaluate:

i $\frac{mn}{2}$ **ii** $\frac{m+n}{4}$

3 **a** If $v = u + at$, find v when:

i $u = 7$, $a = 3$, $t = 5$

ii $u = 9$, $a = 7$, $t = \frac{1}{2}$

iii $u = -10$, $a = 9{\cdot}8$, $t = 3{\cdot}7$

iv $u = 7{\cdot}25$, $a = -9{\cdot}8$, $t = 7{\cdot}5$

b Given that $P = 2L + 2B$, find B when:

i $P = 10$, $L = 2$

ii $P = 7$, $L = 1{\cdot}5$

iii $P = 77$, $L = 14$

iv $P = 68{\cdot}4$, $L = 14{\cdot}2$

v $P = 42$, $L = 6{\cdot}85$

c If $D = \frac{M}{V}$ find:

i D when $M = 16$, $V = 4$

ii D when $M = 20$, $V = 1{\cdot}5$

iii M when $D = 2$, $V = 5$

iv M when $D = 1{\cdot}12$, $V = 3{\cdot}6$

v V when $D = 2$, $M = 6$

vi V when $D = 1{\cdot}5$, $M = 0{\cdot}5$

4 **a** Given that $S = ut + \frac{1}{2}at^2$, find S when $u = 7$, $t = 3$, $a = 10$.

b If $C = 2mr$, find m when $C = 17{\cdot}6$ and $r = 1{\cdot}1$.

c If $P = \frac{1}{a} + \frac{1}{b}$, find P when $a = 0{\cdot}4$ and $b = 0{\cdot}625$.

5 Write an equation for each of the following and then solve it.

- **a** A number is multiplied by 3, then 7 is added and the result is 15.
- **b** Nine is subtracted from a number and the result is multiplied by five to equal thirty.
- **c** Eight times a certain number plus ten is equal to twelve times the same number minus seven.

Technology Applications Activities

Substituting and transposing formulae
Substitution

1 Substitution
2 Formulae
3 Equations from formulae

Technology Applications Drag and Drops

Chapter Review Questions

assignment 9B

Chapter 9 | Working Mathematically

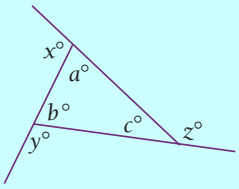
1 Use ID Card 5 on page xvii to identify:

a 5 b 12 c 14 d 16 e 17
f 20 g 21 h 22 i 23 j 24

2 Use ID Card 6 on page xviii to identify:

a 4 b 12 c 13 d 14 e 15
f 17 g 21 h 22 i 23 j 24

3 If the exterior angles $x°$, $y°$ and $z°$ of a triangle are in the ratio 4 : 5 : 6, what is the ratio of the interior angles $a°$, $b°$ and $c°$?

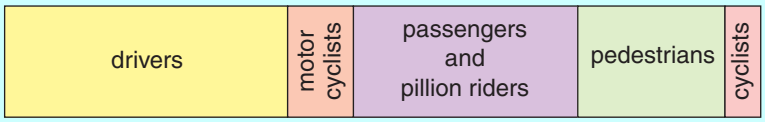


4 The average of five numbers is 11. A sixth number is added and the new average is 12. What is the sixth number?

5 A bottle containing 20 identical tablets has a mass of 180 grams. When 5 tablets are removed the mass is 165 grams. What is the mass of the bottle?

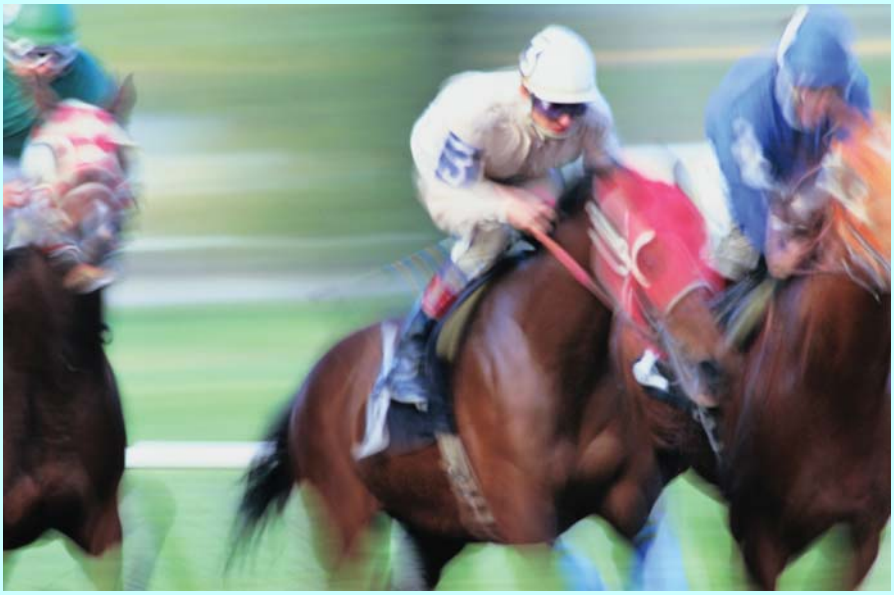
6

Road traffic fatalities by type of road user



Measure the length of this bar graph. This length represents 100% of road traffic fatalities.

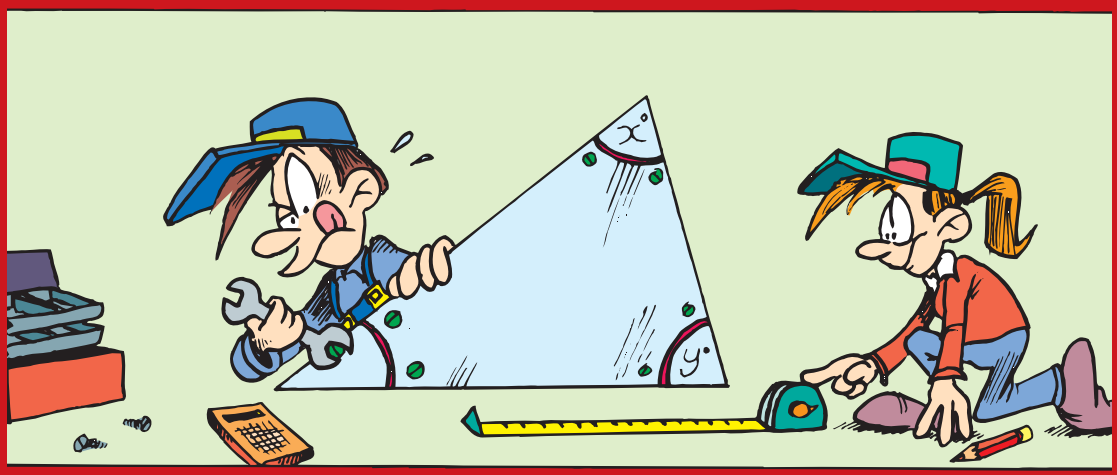
a What percentage of fatalities were drivers?
b What percentage of fatalities were motor cyclists?
c Because fewer motor cyclists were killed than drivers, can you say that it is safer to be a motor cyclist than a driver? Why or why not?



- In a trifecta you have to pick 1st, 2nd and 3rd places. In how many different ways can a trifecta be chosen in:
  - a 3-horse race?
  - a 4-horse race?
  - a 5-horse race?
  - a 10-horse race?

10

Geometry



Chapter Contents

Learning Outcomes

Students will be able to:

- Identify and name angles formed by the intersection of straight lines, including those formed by transversals on sets of parallel lines, and makes use of the relationships between them.
- Classify and determine the properties of triangles and quadrilaterals.
- Apply results related to the angle sum of interior and exterior angles of polygons.

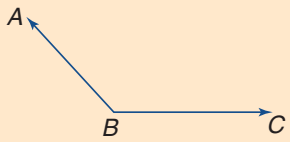
Areas of Interaction

Approaches to Learning, Homo Faber

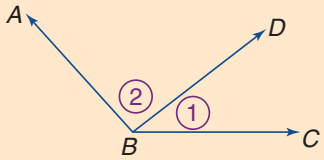
10:01 | Adjacent Angles

prep quiz 10:01

1 Name the vertex of this angle.
2 Name the arms of this angle.
3 Name this angle.



4 Name the angle marked (1).
5 Name the angle marked (2).



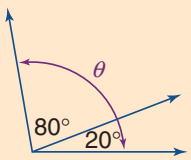
Give the size in degrees of:

6 a right angle

7 a straight angle

Find the value of θ in each of the following:

8



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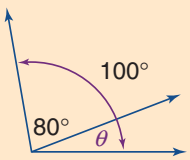

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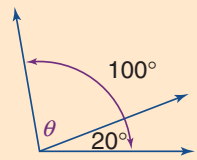
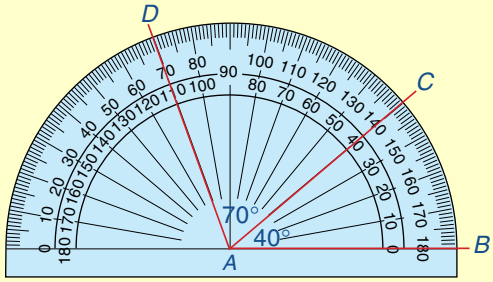


The diagram on the right shows an important relationship in geometry.

- The two smaller angles add to make the larger angle.
  ie $\angle BAC + \angle CAD = \angle BAD$
  $40° + 70° = 110°$
- The two smaller angles ($\angle$s *BAC* and *CAD*) are said to be **adjacent angles**.
- When two angles are adjacent they add together to make a larger angle.



Adjacent angles

1 They have a common vertex (or point), *A*.
2 They have a common arm, *AC*.
3 They lie on opposite sides of this common arm.

Two special cases occur when the adjacent angles add to give either a right angle or a straight angle. These are shown in the diagrams.

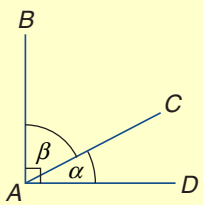


Figure 1

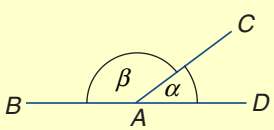


Figure 2

Greek letters such as θ, α, ϕ and β are used to represent the sizes of angles.

When Greek letters are used a degree symbol is not needed.

- When two angles add up to give 90° they are called **complementary angles**.
- When two angles add up to give 180° they are called **supplementary angles**.

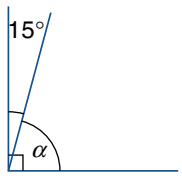
Hence:

- In Figure 1 we have **adjacent complementary angles** ($\alpha + \beta = 90°$).
- In Figure 2 we have **adjacent supplementary angles** ($\alpha + \beta = 180°$).

worked examples

Find the value of the pronumeral in each case. Give a reason for your solution.

1



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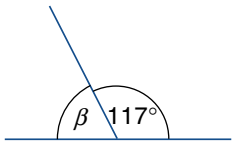

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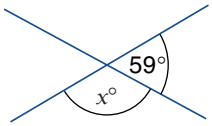


Solutions

1 $15° + \alpha = 90°$
$\therefore\ \alpha = 75°$
(adjacent complementary) angles)

2 $\beta + 117° = 180°$
$\therefore\ \beta = 63°$
(adjacent supplementary angles)

3 $x° + 59° = 180°$
$\therefore\ x = 121$
(adjacent supplementary angles)

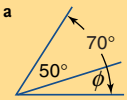
Exercise 10:01

Foundation Worksheet 10:01

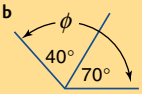
Adjacent angles

1 Find the size of ϕ in each of the following:

a

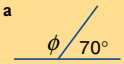


b



2 Find the size of ϕ in each of the following:

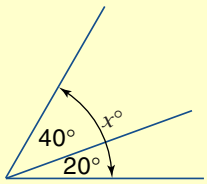
a



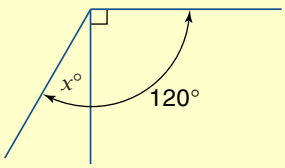
b

1 Find the value of the pronumeral in each of the following.

a



b



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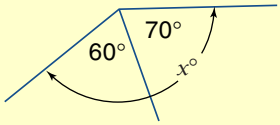
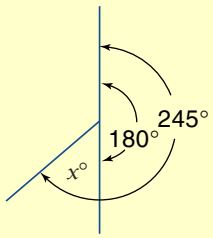
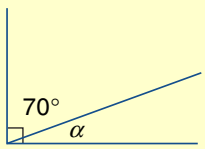


d

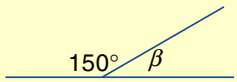


2 Find the value of the pronumeral in each. Give a reason for your answer.

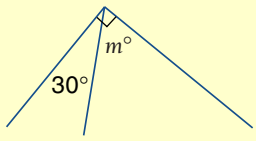
a



b



c



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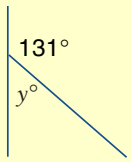
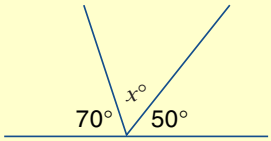


e



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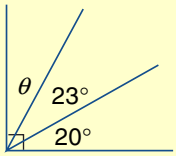
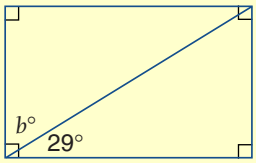
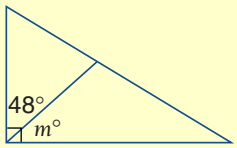


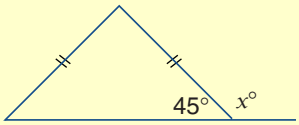
g



h



i

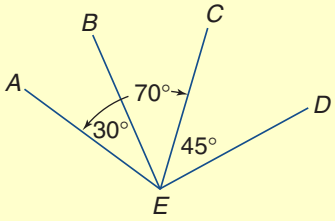


3 In the diagram find the size of:

a $\angle BEC$

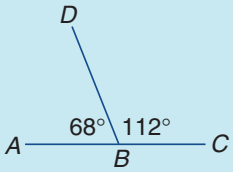
b $\angle BED$

c $\angle AED$



If two adjacent angles add up to 180°, then together they form a straight angle.

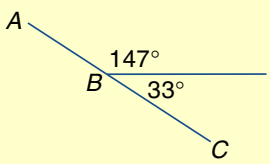
eg **68° + 112° = 180°**



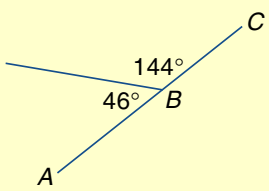
Since the supplementary angles are adjacent, *ABC* is a straight line.

4 In which of the following is $\angle ABC$ a straight angle?

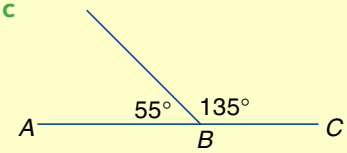
a



b



c



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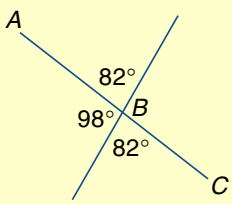
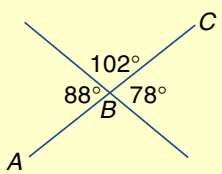


e



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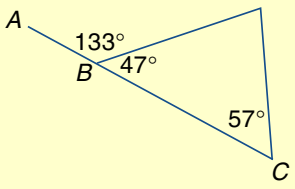
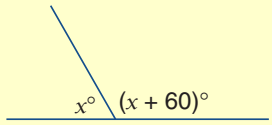
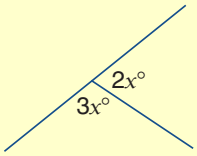


5 Find the value of the pronumeral and in each case give a reason.

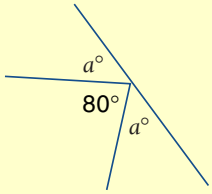
a



b



c



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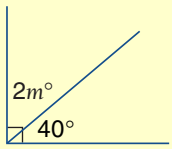
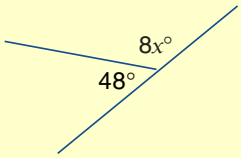


e



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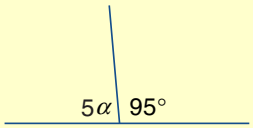
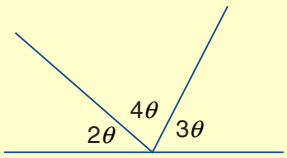
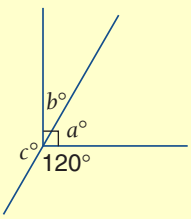


6 Find the value of the pronumerals, and give reasons in each case.

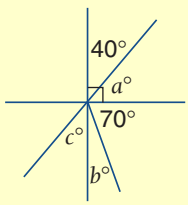
a



b



c

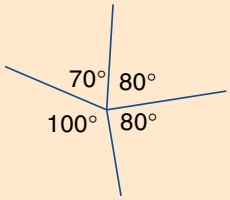


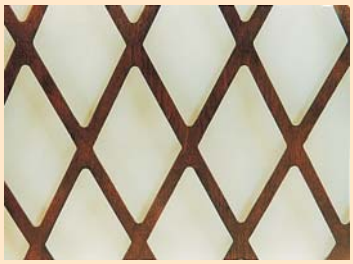
10:02 | Angles at a Point and Vertically Opposite Angles

prep quiz 10:02

1 What is the size of an angle of revolution?

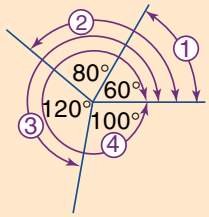
2 Why is this diagram impossible?





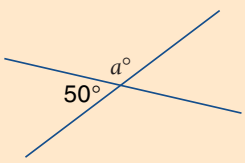
In the diagram on the right, what is the size of the angle marked:

3 ①? **4** ②? **5** ③? **6** ④?

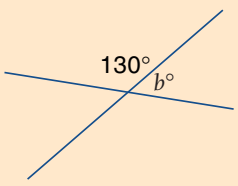


Find the value of the pronumeral in each of the following.

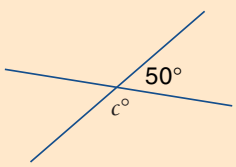
7



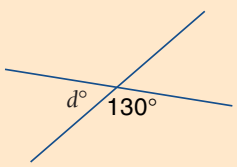
8



9

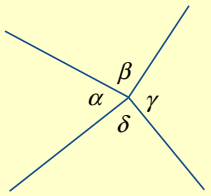


10



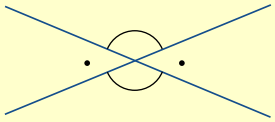
The Prep Quiz on the previous page shows how a knowledge of adjacent angles produces two further results.

- If a series of adjacent angles completes a revolution, the angles must sum to 360°. We say that the **angles at a point sum to 360°**.



Angle sum = 360°

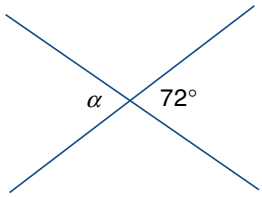
- When two straight lines cross, two pairs of equal angles are formed. These are called **vertically opposite angles**.


Vertically opposite angles are equal.

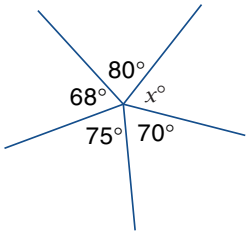
worked examples

Find the value of the pronumeral in each case.

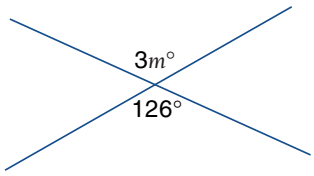
1



2



3



Solutions

1 $\alpha = 72°$
(vertically opposite ∠s)

2 $x + 70 + 75 + 68 + 80 = 360$
(angles at a point)
$\therefore\ x + 293 = 360$
$x = 67$

3 $3m = 126$
(vertically opposite ∠s)
$m = 42$

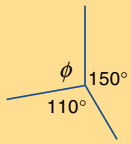
Exercise 10:02

Foundation Worksheet 10:02

Angles at a point

1 Find the size of ϕ in the following:

a

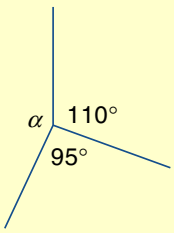


2 Find the size of ϕ in the following:

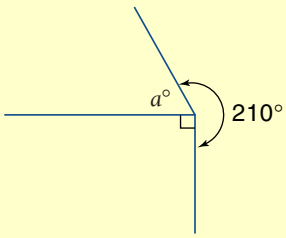
a

1 Find the value of the pronumeral and, in each case, give a reason.

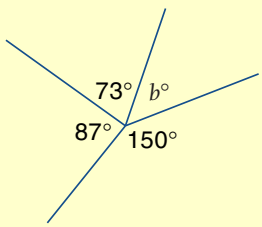
a



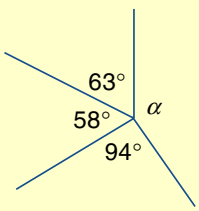
b



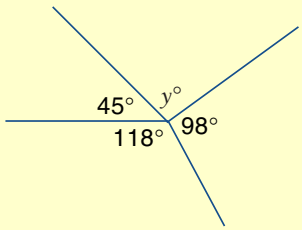
c



d

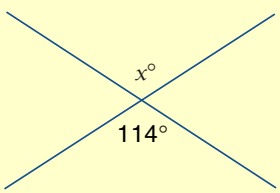


e



2 Find the value of the pronumeral and in each case give a reason.

a



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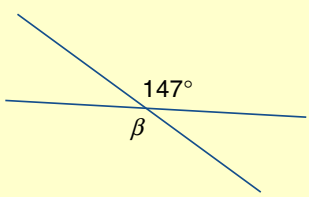

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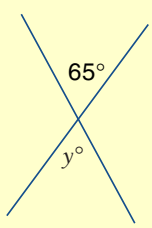

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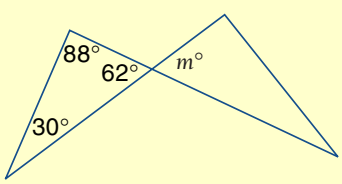
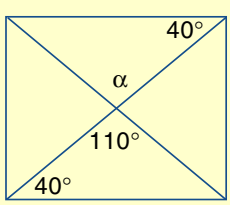


e



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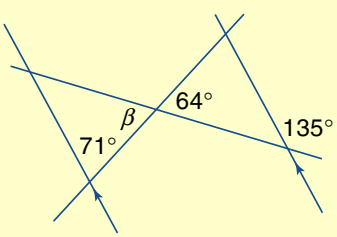
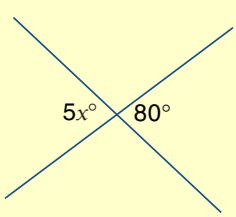
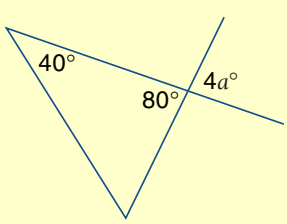


3 Find the value of each pronumeral. Give a reason for your answer in each case.

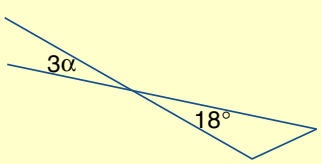
a



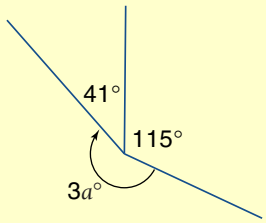
b



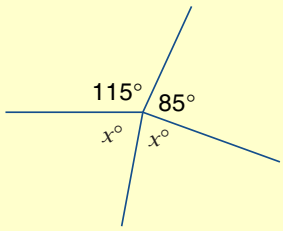
c



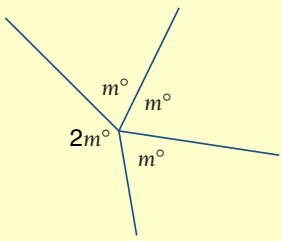
d



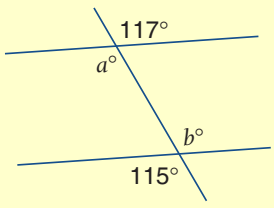
e



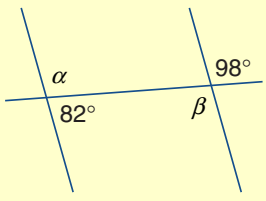
f



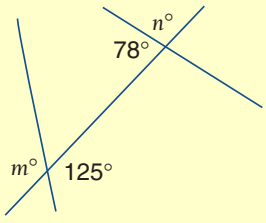
g



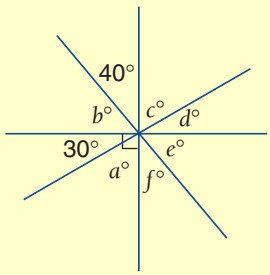
h



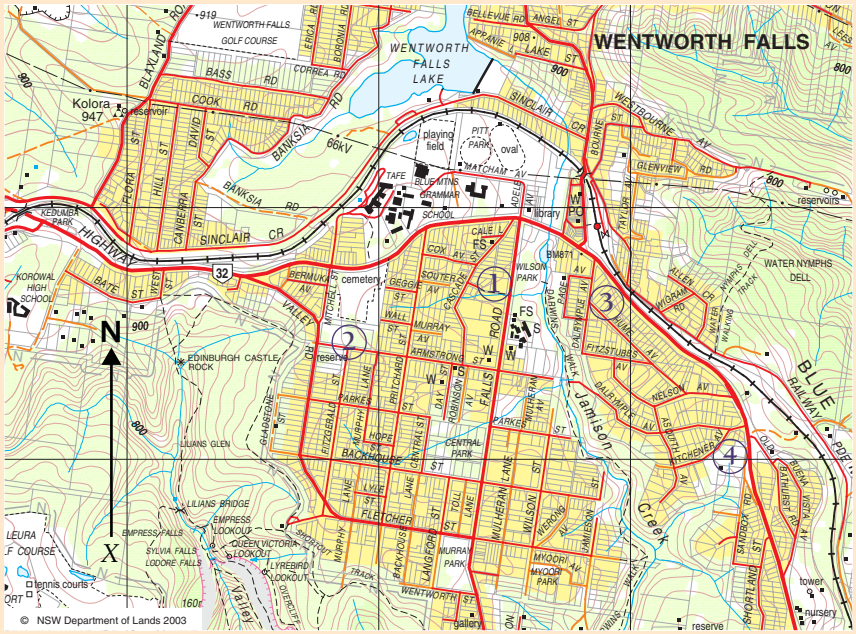
i



4 Find the value of each pronumeral. Explain which pronumeral you found the value of first. What reason could you give for finding its value?

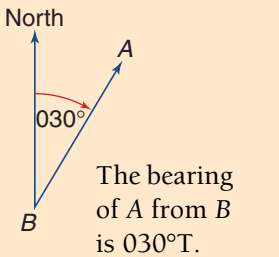


Practical Activity 10:02 | Using angles to measure direction

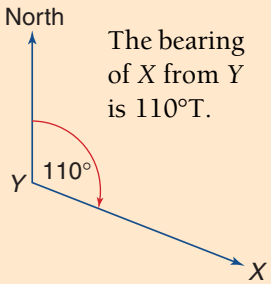


The direction of point A from point B is called the bearing of A from B.

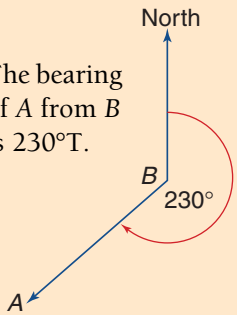
The **true bearing** of a line is the angle between the line and true north. A true bearing can take values from 000° to 360° and is always measured in a clockwise direction from true north. The letter T is written after the number to indicate that it is a true bearing.



The bearing of A from B is 030°T.



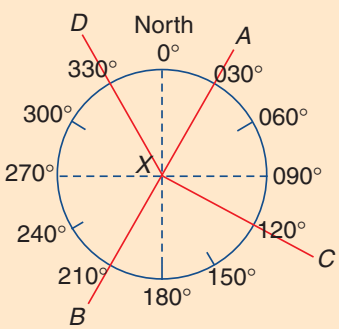
The bearing of X from Y is 110°T.



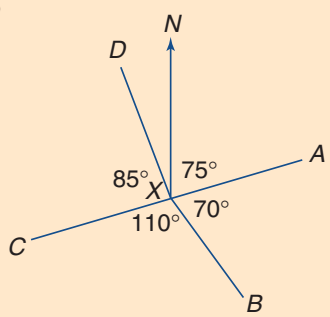
The bearing of A from B is 230°T.

1 Give the true bearings of the points A, B, C and D from X in each of the following.

a



b



2 On the map estimate the bearing from X of:

a Falls Rd (1) **b** Armstrong St (2) **c** Hume Ave (3) **d** Kitchener Ave (4)

10:03 | Alternate, Corresponding and Co-interior Angles

When a line cuts two (or more) lines, a number of angles are formed. These are given special names as described in the following section.

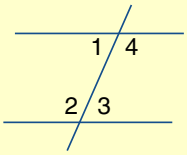


Figure 1

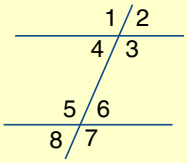


Figure 2

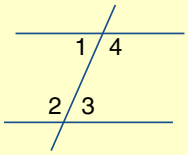


Figure 3

■ A transversal is a line cutting two or more other lines.

FIGURE 1

1 and 3 are alternate angles.
2 and 4 are alternate angles.
Alternate angles are on opposite sides of the transversal and between the other two lines.

FIGURE 2

The pairs of corresponding angles are:
1 and 5, 2 and 6,
3 and 7, 4 and 8.
Corresponding angles are in corresponding positions at each intersection.

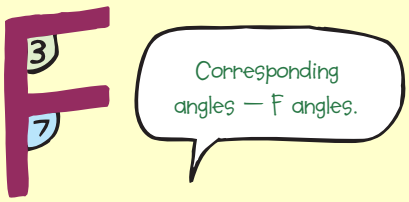


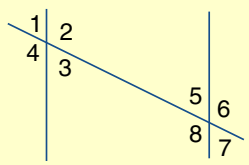
FIGURE 3

1 and 2 are co-interior (or allied) angles.
3 and 4 are co-interior angles.
Co-interior angles are on the same side of the transversal and between the other two lines.

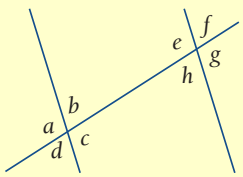
Exercise 10:03

1 Which pairs of angles are alternate?

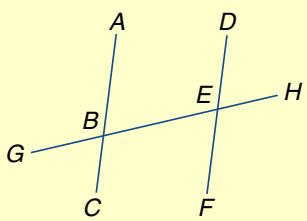
a



b

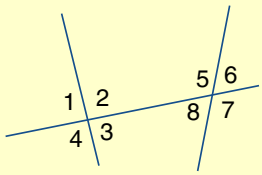


c

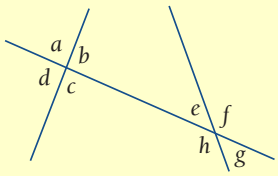


2 Which pairs of angles are corresponding?

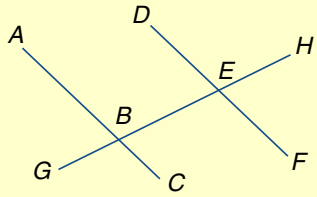
a



b

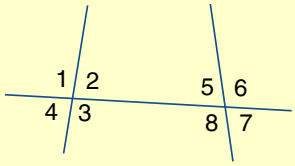


c

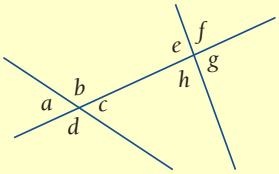


3 Which pairs of angles are co-interior?

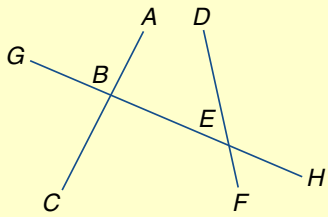
a



b

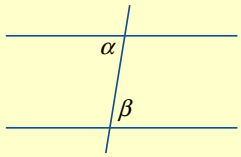


c

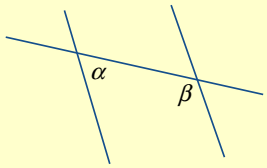


4 In each, say whether α and β are alternate, corresponding or co-interior.

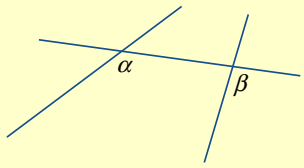
a



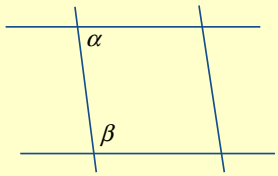
b



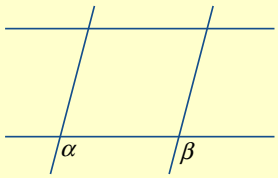
c



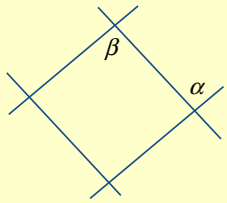
d



e

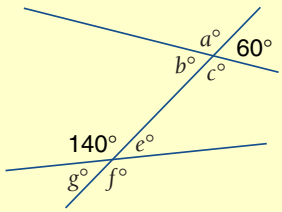


f

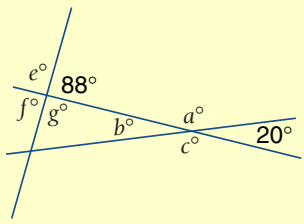


5 Find the values of the pronumerals.

a



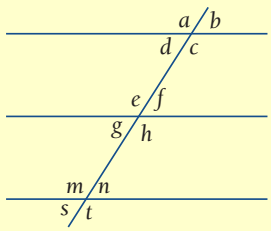
b



6 **a** Write down two angles which correspond to f?

b Which angle is alternate to h?

c Which two angles are co-interior to d?



10:04 | Angles Around Parallel Lines

When a transversal cuts two **parallel lines**, the alternate, corresponding and co-interior angles formed have special properties. These are investigated in the following exercise.

Exercise 10:04

1 In the series of diagrams below, *AB* and *CD* are two lines that are cut by the transversal *EF*. Imagine that *AB* and *EF* are fixed but *CD* can pivot about *F*.
We will look at the two corresponding angles marked ① and ② as *CD* pivots about *F*.

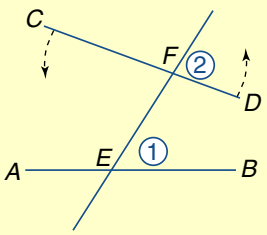


Figure 1

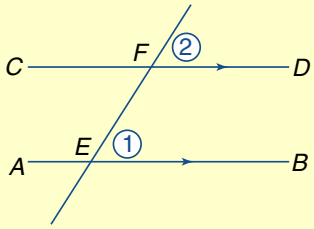


Figure 2

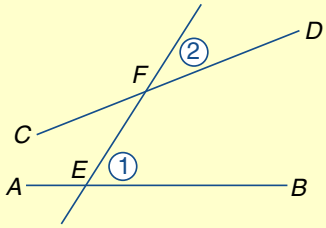


Figure 3

a In Figure 1, do angles ① and ② look equal in size?

b In Figure 1, do *AB* and *CD* look parallel? If not, on which side of *EF* would they meet?

c In Figure 3, do angles ① and ② look equal in size?

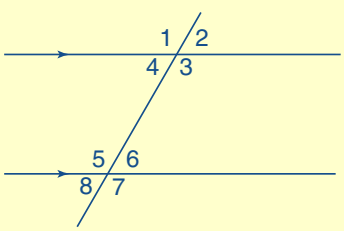
d In Figure 3, do *AB* and *CD* look parallel? If not, on which side of *EF* would they meet?

e In Figure 2, *AB* and *CD* are parallel. Do angles ① and ② look equal in size?

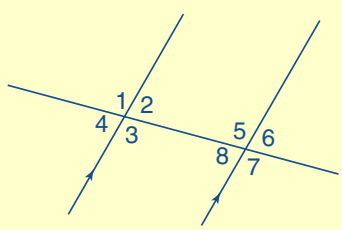
f What does this suggest about the corresponding angles ① and ②, when *AB* is parallel to *CD*?

2 In each of the following, measure the angles marked and complete the tables.

a



b



Angle number	1	2	3	4	5	6	7	8
Angle size								

Angle number	1	2	3	4	5	6	7	8
Angle size								

3 Use the measurements in question **2** to answer the following.

a Check the four pairs of corresponding angles in each diagram. What do you notice?

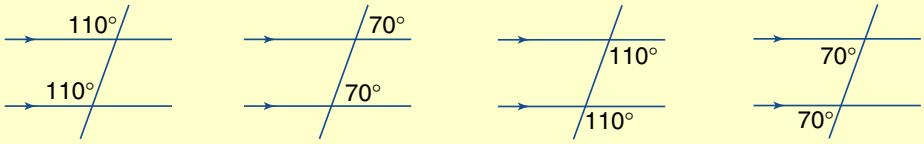
b Check the alternate angles in each diagram. What do you notice?

c Check the co-interior angles in each diagram. What do you notice?

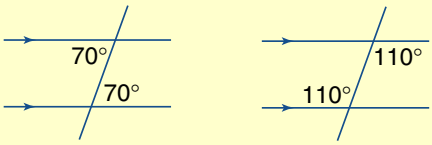
Summary

The last exercise should have convinced you of the following rules.

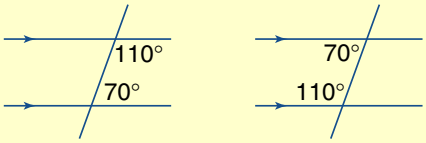
If a transversal cuts two parallel lines, then corresponding angles are equal.



If a transversal cuts two parallel lines, then alternate angles are equal.



If a transversal cuts two parallel lines, then co-interior angles are supplementary.



- The girders of the Sydney Harbour Bridge demonstrate alternate angles and parallel lines.

10:05 | Angles and Parallel Lines

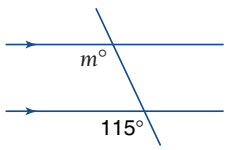
The results from the last exercise are used to:
- find the size of unknown angles
- prove that two lines are parallel

These are shown in the following worked examples.

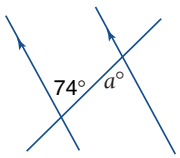
worked examples

Find the value of the pronumerals. Give reasons.

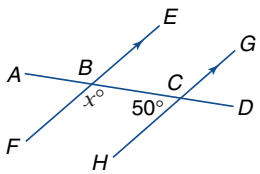
1



2



3



Solutions

1 $m = 115$
(corresponding angles and parallel lines)

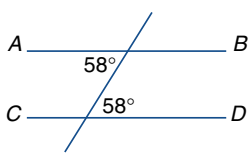
2 $a = 74$
(alternate angles and parallel lines)

3 $x + 50 = 180$
$\therefore \quad x = 130$
(co-interior angles and $FE \parallel HG$)

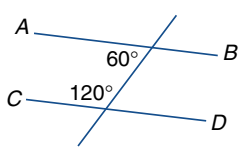
worked examples

4 In each case give a reason why $AB \parallel CD$.

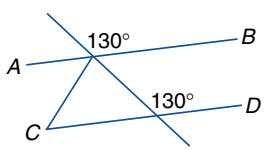
a



b



c



Solutions

1 a $AB \parallel CD$
(alternate angles equal)

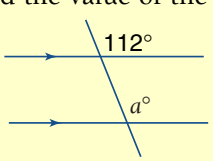
b $AB \parallel CD$
(co-interior angles supplementary)

c $AB \parallel CD$
(corresponding angles equal)

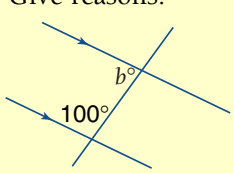
Exercise 10:05

1 Find the value of the pronumerals. Give reasons.

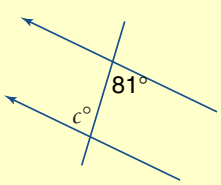
a



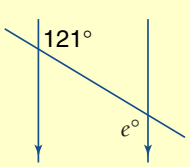
b



c



d

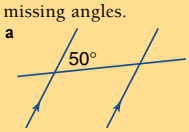


Foundation Worksheet 10:05

Angles and parallel lines

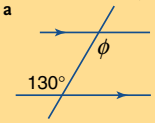
1 Write down the size of all the missing angles.

a

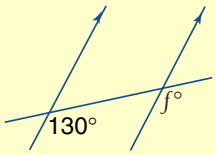


2 Find the size of ϕ.

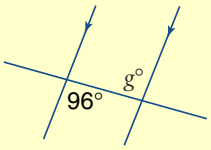
a



e



f



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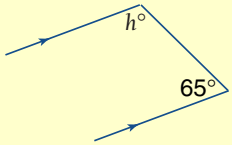
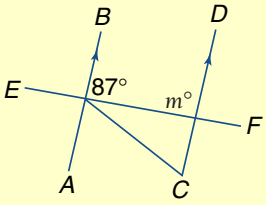


2 Find the value of the pronumerals. Give reasons.

a



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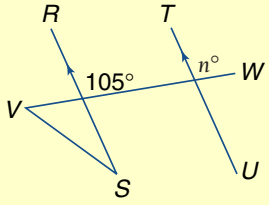

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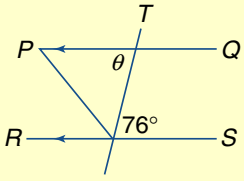
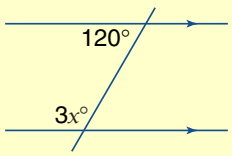


3 Find the value of the pronumerals. Give reasons.

a



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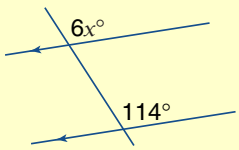

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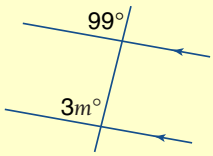

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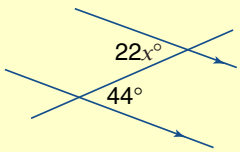

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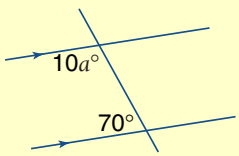

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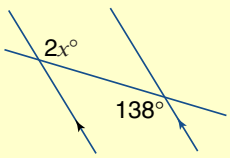

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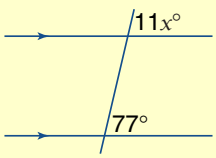

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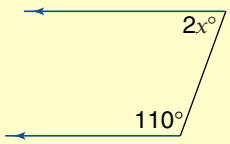

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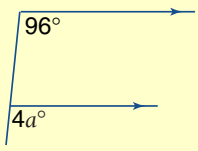

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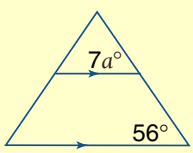

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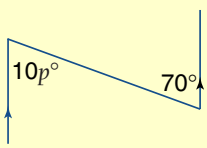

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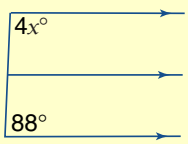
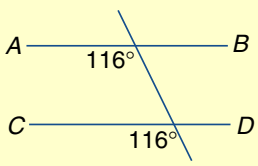


4 In each case, give a reason why $AB \parallel CD$.

a



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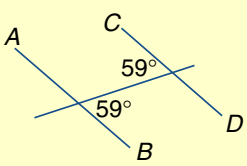

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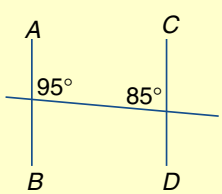

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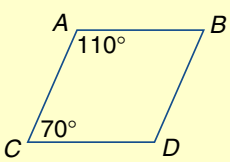

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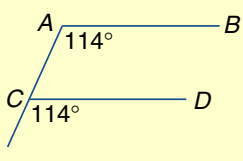

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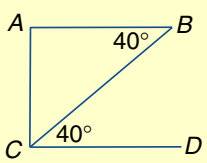
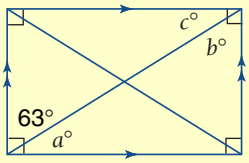


5 Find the value of each pronumeral.

a



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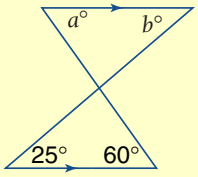

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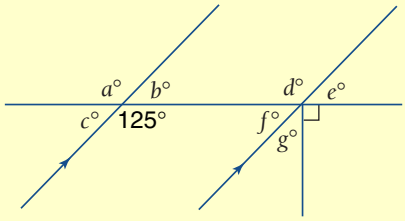


fun spot

10:05

Fun Spot 10:05 | Why are giraffes easy to feed?

Answer each question and put the letter for that question in the box above the correct answer.

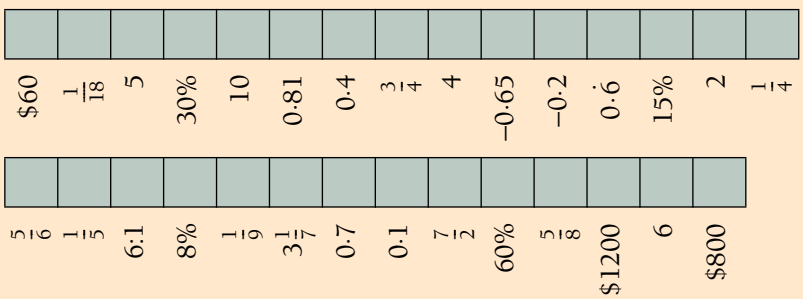
A $(0{\cdot}9)^2$	**A** $1 - 0{\cdot}3$
A $0{\cdot}8 \div 0{\cdot}2$	**A** 6% of 100
D $2\% + 6\%$	**E** $1 \div 0{\cdot}2$
E $\frac{1}{2}$ of $\frac{1}{2}$	**E** $\frac{1}{2} + \frac{1}{4}$
F $\frac{1}{2} + \frac{1}{3}$	**G** $(\frac{1}{3})^2$
G $1 - \frac{3}{8}$	**H** $\frac{1}{3} \times \frac{1}{6}$
L $0{\cdot}35 - 1$	**K** $\frac{1}{2}$ of $0{\cdot}8$
L $\frac{2}{3}$ of 3	**L** 10% of 1
I $15 - 15{\cdot}2$	**M** $11 \div 1{\cdot}1$

N Write $\frac{3}{5}$ as a percentage.	**O** Write $\frac{22}{7}$ as a mixed number.
O Simplify the ratio 30:5.	**O** Change $3\frac{1}{2}$ to an improper fraction.
O What fraction is 200 m of 1 km?	**T** Increase \$48 by 25%.
T Write $\frac{2}{3}$ as a repeating decimal.	**T** What percentage is 300 m of 2 km?

W 70% of my money is \$840. How much money do I have?

Y 24 men and 56 women were in the hall. What percentage were men?

Y \$2000 is divided between Naomi and Luke in the ratio 2:3. How much does Naomi receive?



10:06 | Triangles

prep quiz

10:06

Use ID Card 5 on page xvii to give the number for:

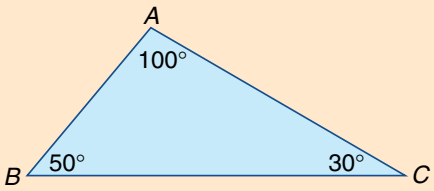
1 a right-angled triangle **2** an acute-angled triangle **3** an obtuse-angled triangle

From ID Card 4 on page xviii, give the name for the triangle in:

4 Figure 10 **5** Figure 11 **6** Figure 12

For the triangle shown, write:

7 the biggest angle
8 the biggest side
9 the smallest angle
10 the smallest side



In the Prep Quiz above you have been reminded that a triangle can be described according to the length of its sides or the size of its angles.

In this section we will look at the properties of some of these special triangles as well as some of the properties common to all triangles.

Questions 7 to 10 in the Prep Quiz should have reminded you of the first property.

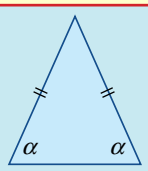
In a scalene triangle the largest side is opposite the largest angle and the smallest side is opposite the smallest angle.

In books 1 and 2 you would have found that the angles of a triangle sum to make a straight angle of 180°.

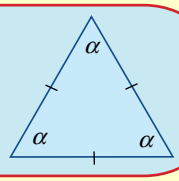
In any triangle, the angles add up to 180°.

The isosceles triangle and equilateral triangle have important properties.

An *isosceles triangle* has:
- **two equal sides**
- **two equal angles**
- **one axis of symmetry**

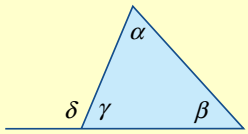


An *equilateral triangle* has:
- **three equal sides**
- **three equal angles**
- **three axes of symmetry**



A triangle has three interior angles. If a side of a triangle is produced, an exterior angle is formed. The exterior angle and interior angle are adjacent supplementary angles. This leads to the next important property of triangles.

$(\alpha + \beta) + \gamma = 180°$ (angle sum of a triangle)
$(\delta) + \gamma = 180°$ (adjacent supplementary angles)
$\therefore\ \delta = \alpha + \beta$ (both equal to $180 - \gamma$)

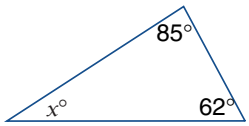


The exterior angle of a triangle is equal to the sum of the two interior opposite angles.

worked examples

1 Find the value of each of the pronumerals. Give reasons.

a



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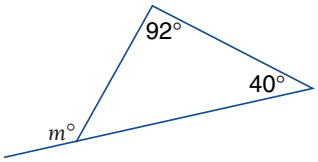

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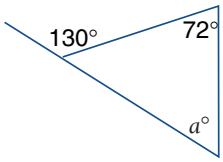


Solutions

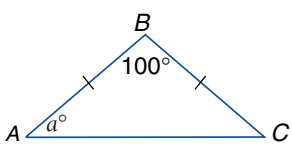
1 a $x + 85 + 62 = 180$
(angle sum of a Δ)
$\therefore\ x + 147 = 180$
$\therefore\ x = 33$

b $m = 92 + 40$
(exterior angle of a Δ)
$\therefore\ m = 132$

c $130 = 72 + a$
(exterior angle of a Δ)
$\therefore\ 58 = a$
$\therefore\ a = 58$

2 Find the value of each of the pronumerals. Give reasons.

a



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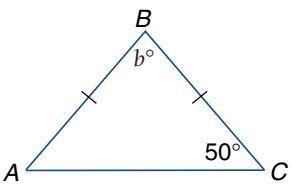

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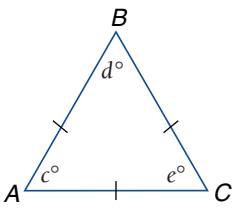


Solutions

2 a $\angle BCA = a°$
(base angles of isosceles Δ)
$\therefore\ a + a + 100 = 180$
$2a + 100 = 180$
$\therefore\ 2a = 80$
$\therefore\ a = 40$

b $\angle BAC = 50°$
(base angles of isosceles Δ)
$\therefore\ b + 50 + 50 = 180$
$b + 100 = 180$
$\therefore\ b = 80$

c $c = d = e = 60$
(angles of an equilateral triangle)

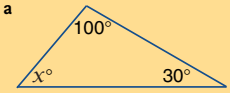
Exercise 10:06

Foundation Worksheet 10:06

Triangles

1 Find the value of the pronumeral.

a



2 Find the value of the pronumeral.

a

1 Sketch each of the following.

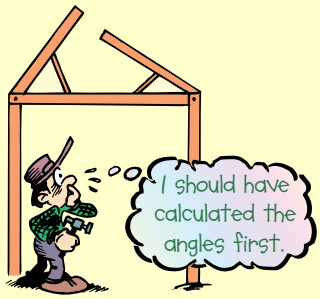
- **a** an acute-angled isosceles triangle
- **b** a right-angled isosceles triangle
- **c** an obtuse-angled isosceles triangle
- **d** an acute-angled scalene triangle
- **e** a right-angled scalene triangle
- **f** an obtuse-angled scalene triangle

2 Why can't an equilateral triangle be right-angled or obtuse-angled?

3 **a** An isosceles triangle has *one* side 10 cm and *one* angle 25°. Sketch a triangle that shows all this information.

b A triangle is right-angled and isosceles. It is known to have a side 5 cm long but you are not sure which side it is. Sketch the two possible triangles, showing the position of the 5 cm side.

c How many different triangles can be drawn that have one side 5 cm in length and two angles that measure 90° and 60°?



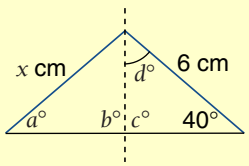
4 **a** In ΔABC, $AB = 6$ cm, $BC = 10$ cm and $AC = 9$ cm. Name the smallest angle.

b In ΔABC, $\angle A = 40°$, $\angle B = 60°$ and $\angle C = 80°$. Which is the largest side?

c How can you use a ruler to find which is the largest angle in a triangle?

5 In each of the following, the dotted lines are axes of symmetry. Find the value of each pronumeral.

a



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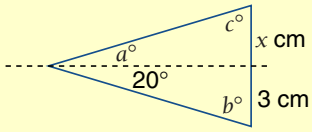

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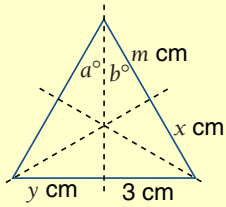
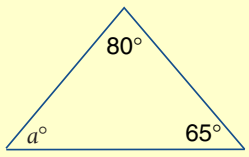


6 Find the value of the pronumerals. Give reasons.

a



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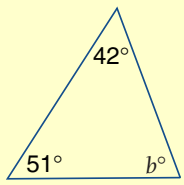

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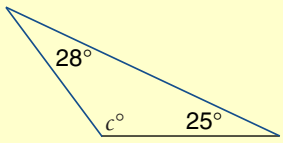

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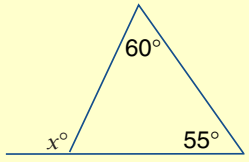

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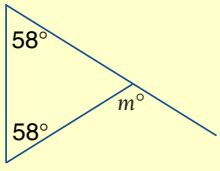

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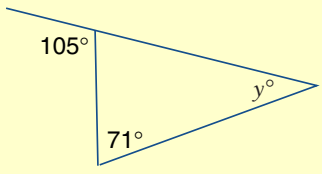

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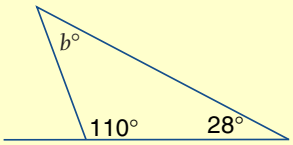

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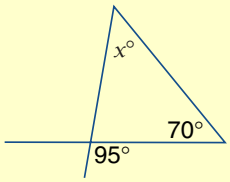

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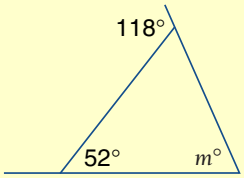

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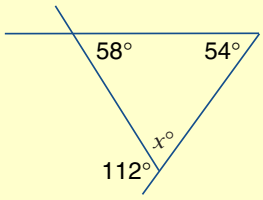

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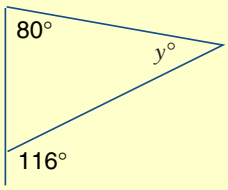

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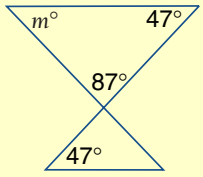
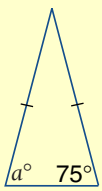


7 Find the value of the pronumerals. Give reasons.

a



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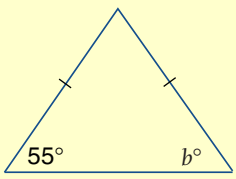

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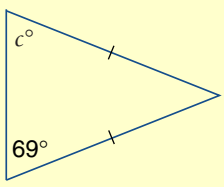

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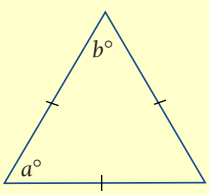

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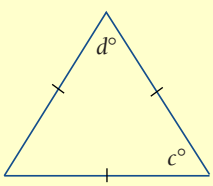

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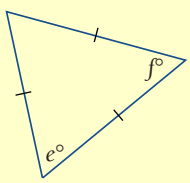
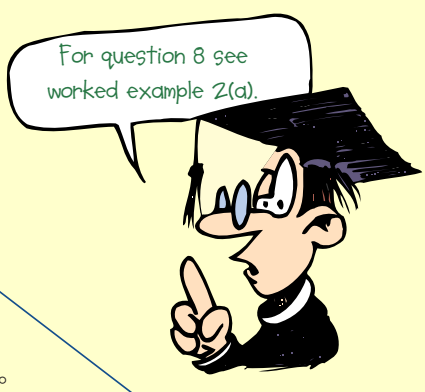
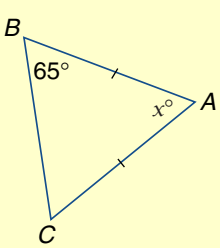




8 Find the value of the pronumerals, giving reasons for your answers.

a



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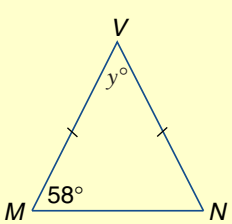

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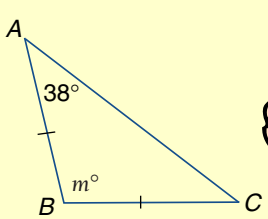

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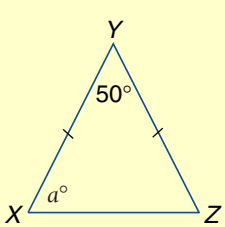

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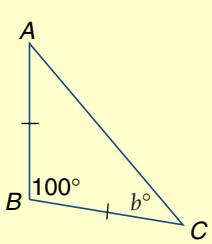

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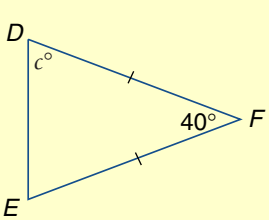
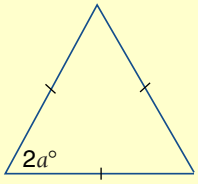


9 Find the value of the pronumerals. Give reasons.

a



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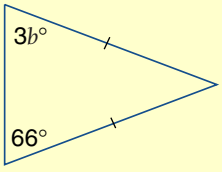

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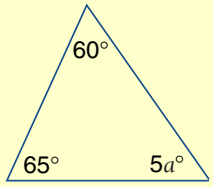

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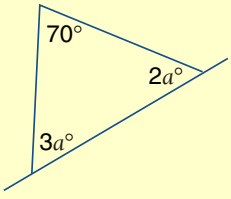

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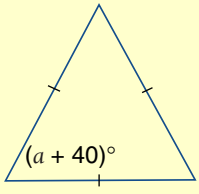

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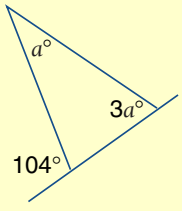

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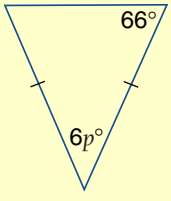

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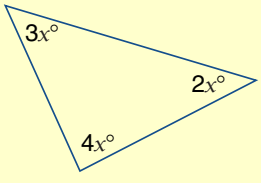

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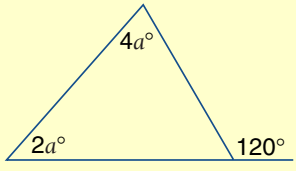
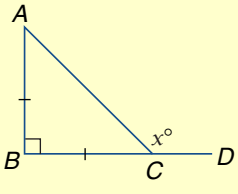


10 In each case, find the value of x. Give reasons.

a



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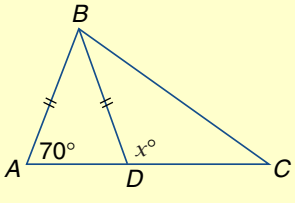

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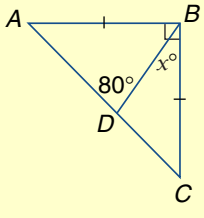
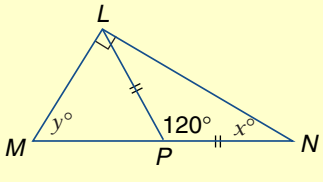


11 Find the value of x and y, giving reasons for your results.

a



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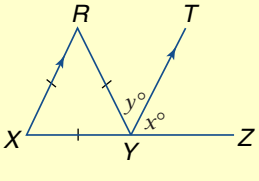

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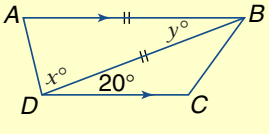
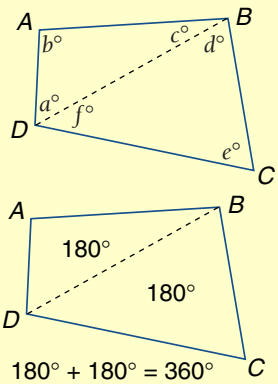


• Isosceles triangles are often found in architecture.

10:07 | Angle Sum of a Quadrilateral

A quadrilateral is a plane figure bounded by four straight sides. The angles in a quadrilateral add up to 360°. This comes from the result for the angle sum of a triangle as outlined in the following proof.

Proof:



$ABCD$ is a quadrilateral. The diagonal BD has been drawn in. It divides the quadrilateral into two triangles.

Let $a°, b°, c°, d°, e°, f°$ be the size of the angles in the figure.

Now $a° + b° + c° = 180°$ (angle sum of ΔABD)

and $d° + e° + f° = 180°$ (angle sum of ΔBCD)

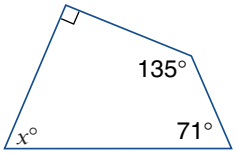
$$\begin{aligned}\angle A + \angle B + \angle C + \angle D &= b° + (c° + d°) + e° + (a° + f°)\\ &= (a° + b° + c°) + (d° + e° + f°)\\ &= 180° + 180°\\ &= 360° \quad \text{Q.E.D.}\end{aligned}$$

The angle sum of a quadrilateral is 360°.

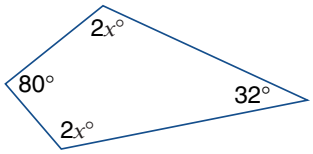
worked examples

Find the value of each of the pronumerals. Give reasons.

1



2



Solutions

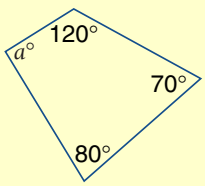
1 $x + 90 + 135 + 71 = 360$
(angle sum of a quadrilateral)
$\therefore x + 296 = 360$
$\therefore x = 64$

2 $2x + 80 + 2x + 32 = 360$
(angle sum of a quadrilateral)
$\therefore 4x + 112 = 360$
$\therefore 4x = 248$
$\therefore x = 62$

Exercise 10:07

1 Find the value of each pronumeral. Give reasons.

a



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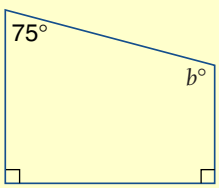

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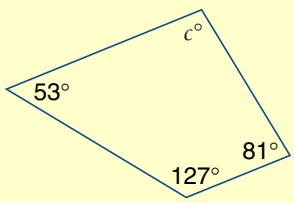

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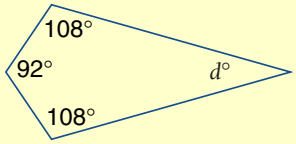

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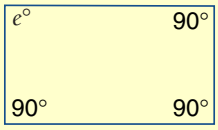

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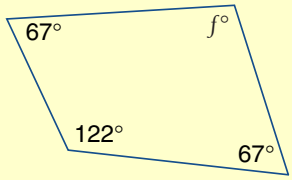

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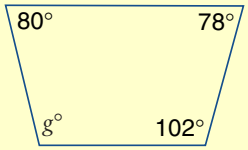

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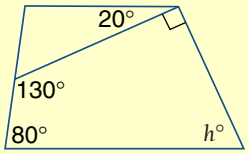

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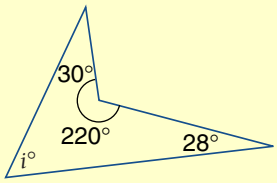
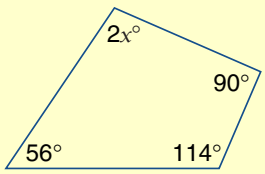


2 Find the value of each of the pronumerals. Give reasons.

a



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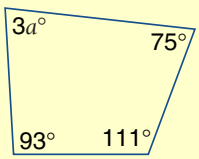

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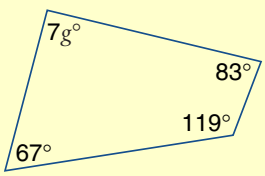

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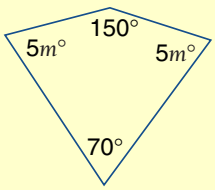

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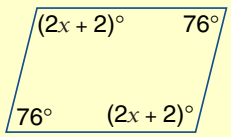

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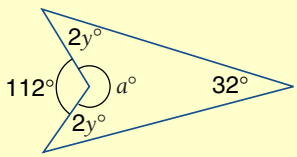
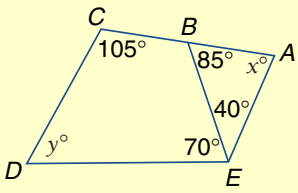


3 Find the value of each of the pronumerals. Give reasons.

a



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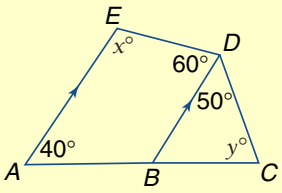
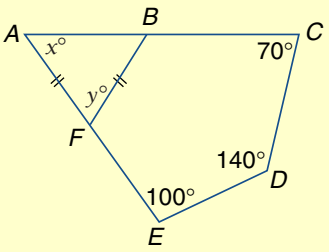


c



- Building frames need to be squared. This means builders have to make sure they are rectangles and not parallelograms.

10:08 | Types of Quadrilaterals

prep quiz

10:08

Use ID Card 4 on page xviii to give the figure number for:

1 a trapezium **2** a square **3** a rhombus

4 a parallelogram **5** a rectangle **6** a kite

7 How many diagonals does a quadrilateral have?

Which quadrilaterals have:

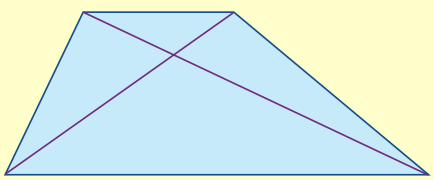
8 all angles equal? **9** all sides equal? **10** all angles and all sides equal?

Each of the six special quadrilaterals mentioned in the Prep Quiz has its own set of special properties. These properties refer to the angles, sides and diagonals of the quadrilateral.

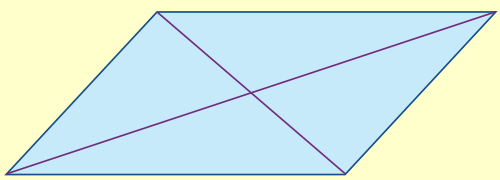
Furthermore, certain properties can be used to 'test' for each of the special quadrilaterals.

Exercise 10:08

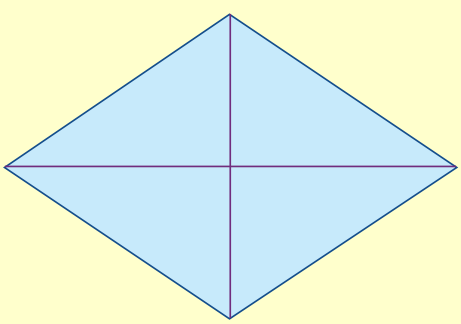
Use the diagrams below, where necessary, to answer the questions in this exercise.



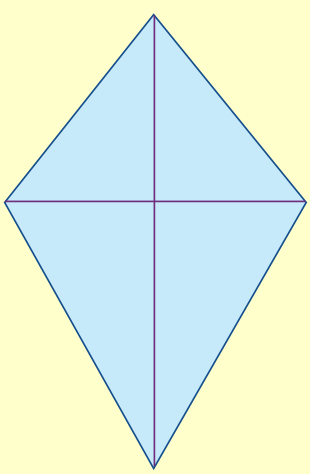
Trapezium



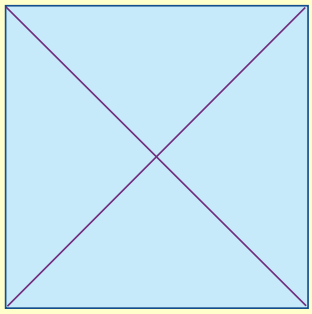
Parallelogram



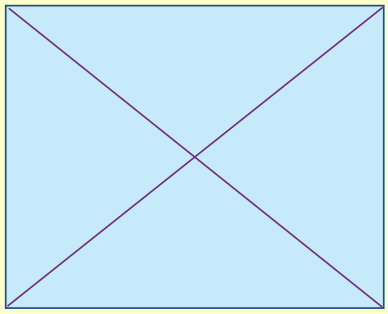
Rhombus



Kite



Square



Rectangle

1 Copy and complete this table by writing YES or NO in each space.

Properties	*Parallelogram*	*Rhombus*	*Rectangle*	*Square*
Opposite sides parallel				
Opposite sides equal				
Opposite angles equal				
Diagonals bisect one another				
All sides equal				
All angles right angles				
Diagonals perpendicular				
Diagonals bisect angles through which they pass				
Diagonals are equal				

2 Study the diagram of the kite and then answer the following questions.

a A kite has two pairs of adjacent sides equal. True or false?
b How do the diagonals of a kite intersect?
c Do the diagonals of a kite bisect each other?
d Does one diagonal of the kite bisect the angles through which it passes?
e What can you say about the angles of a kite?

3 Name two properties that the following have in common.

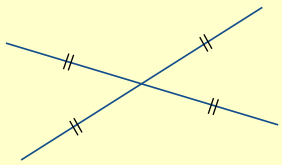
a square and rhombus
b square and rectangle
c rectangle and parallelogram
d parallelogram and rhombus

4 **a** Name two properties of a square that are not properties of a rectangle.
b Name two properties of a square that are not properties of a rhombus.
c Name two properties of a rectangle that are not properties of a parallelogram.

5 **a** In which of the special quadrilaterals are the diagonals axes of symmetry?
b How many axes of symmetry does the square have?
c How many axes of symmetry does the kite have?

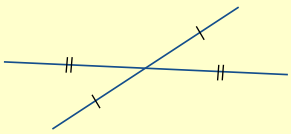
6 In each of the diagrams below, two intersecting diagonals have been drawn. If the end points of these diagonals were joined, what special quadrilaterals would be formed?

a



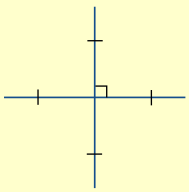
- Diagonals are equal.
- Diagonals bisect each other.

b



- Diagonals are not equal.
- Diagonals bisect each other.

c

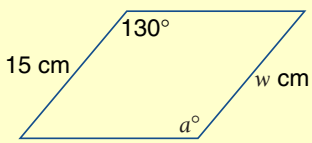


- Diagonals are equal.
- Diagonals bisect each other at right angles.

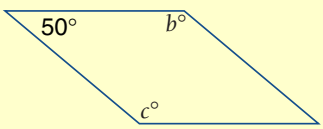
7 **a** What else can you say about a quadrilateral if you know that it has equal diagonals that bisect each other?
b A quadrilateral has all its angles equal. What type of quadrilateral can it be?
c A quadrilateral has all its sides equal and its diagonals are both axes of symmetry. What shape could it be?

8 Each of the following figures are parallelograms. Find the value of each pronumeral.

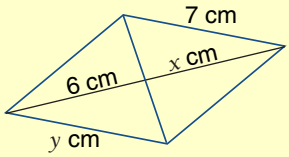
a



b



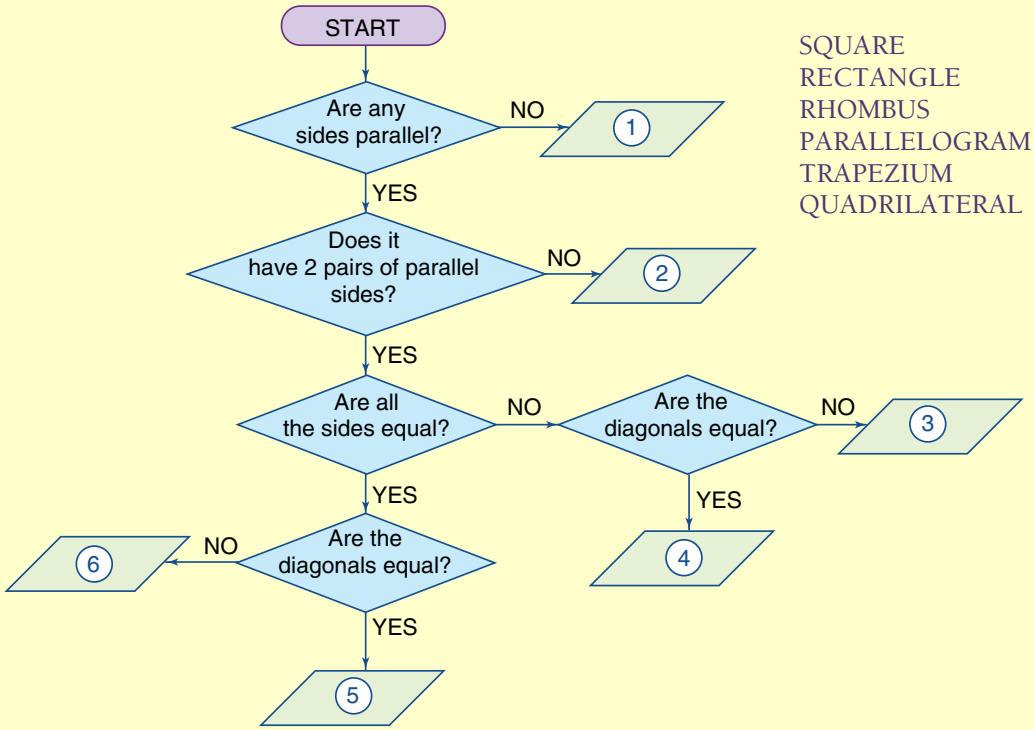
c



9 **a** Does a square have all the properties of a rectangle?
b Is a square a special rectangle?
c Is a rectangle a square? Give a reason for your answer.
d Does a square have all the properties of a rhombus?
e Is a square a special rhombus?
f Is a rhombus a square? Give a reason for your answer.

- Geometric shapes have been used extensively on the wall of this house.

10 Follow the flowchart below and choose the correct names from the list, to be inserted into the boxes (1) to (6).



SQUARE
RECTANGLE
RHOMBUS
PARALLELOGRAM
TRAPEZIUM
QUADRILATERAL

Certain properties can be used to 'test' if a quadrilateral is a parallelogram, rhombus or rectangle.

Tests for a parallelogram

1 Both pairs of opposite sides parallel *or* equal,
***or* 2 both pairs of opposite angles equal,**
***or* 3 one pair of opposite sides equal and parallel,**
***or* 4 diagonals bisect each other.**

Tests for a rhombus

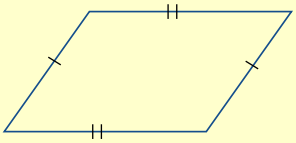
1 All sides equal,
***or* 2 diagonals bisect each other at right angles.**

Tests for a rectangle

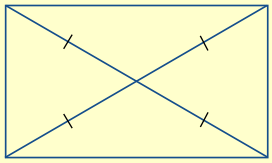
1 All angles equal,
***or* 2 diagonals are equal and bisect each other.**

11 Use the tests for a parallelogram to determine which of the following are parallelograms and, for each, write down the test used.

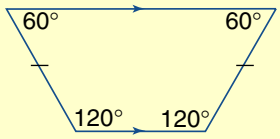
a



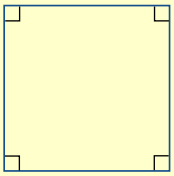
b



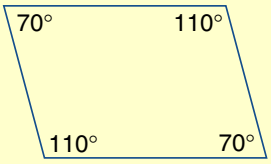
c



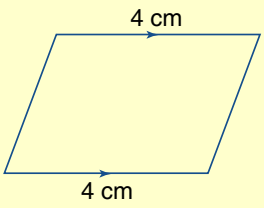
d



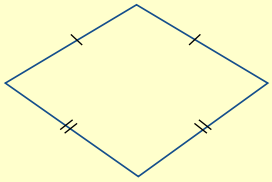
e



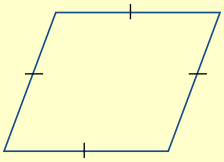
f



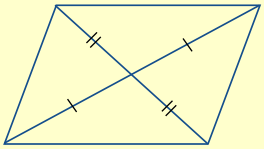
g



h

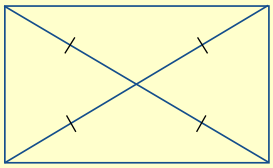


i

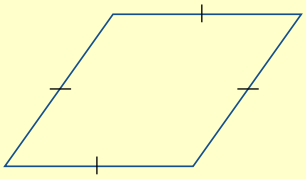


12 Use the tests for a rhombus to determine which of the following are rhombuses and for each write down the test used.

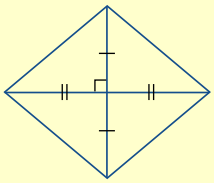
a



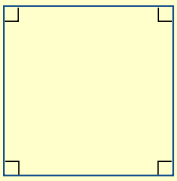
b



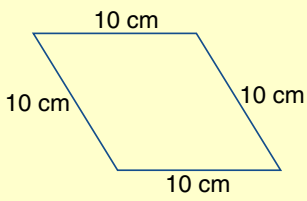
c



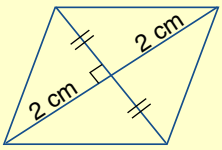
d



e



f



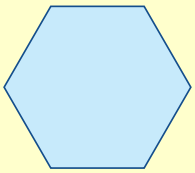
13 Which of the figures in question **12** are rectangles?

14 Carol and Sharon thought that a good test for a square would be 'equal diagonals that bisect each other at right angles'. Do you agree with their test?

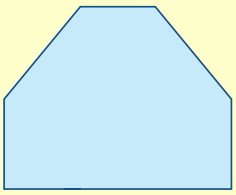
10:09 | Polygons

- The Thien Mu Pagoda in Hue, Vietnam is octagonal in shape.

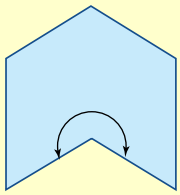
- **A polygon is a plane figure with straight sides.**
- **A polygon is said to be regular if all of its sides and angles are equal. (If they are not, it is said to be irregular.)**
- **Some polygons are named according to the number of sides they have.**



A regular hexagon



An irregular hexagon



A concave hexagon

- **A polygon can be concave or *convex*.**
- **In a *convex* polygon all the angles are acute or obtuse. If a polygon has any reflex angles it is said to be concave.**

Angle sum of a polygon

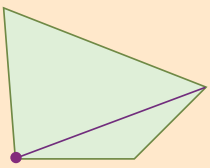
By noticing that all polygons can be built from triangles we can use the result for the angle sum of a triangle to work out the angle sum of a polygon.

investigation 10:09A

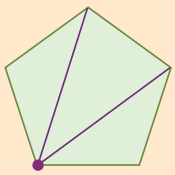
Investigation 10:09A | The angle sum of a polygon

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

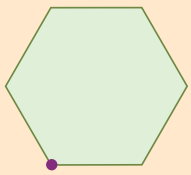
1 Divide each of these polygons into triangles by drawing diagonals from one vertex. The quadrilateral and pentagon have already been done.



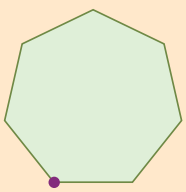
Quadrilateral



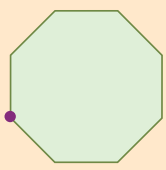
Pentagon



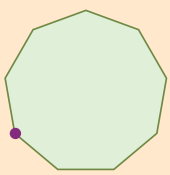
Hexagon



Heptagon



Octagon



Nonagon

2 Sketch a decagon (10 sides) and a dodecagon (12 sides) and divide them into triangles as above.

3 Use the results of your work to complete this table.

Number of sides in polygon	3	4	5	6	7	8	9	10	11
Number of triangles	1	2	3						

4 What do you think is the relationship between the 'number of sides' and the 'number of triangles'?

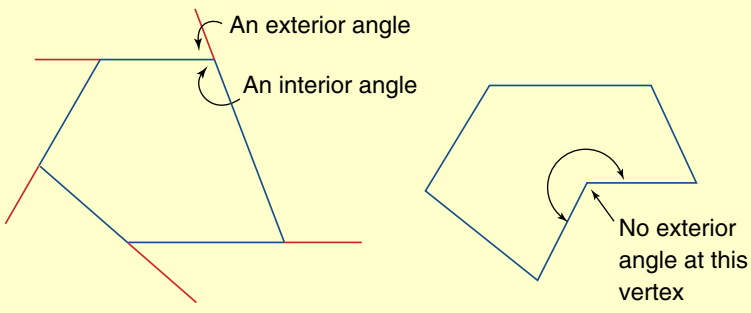
5 If a polygon had n sides how many triangles would be formed?

6 If the angle sum of one triangle is 180°, what would be the angle sum of an n-sided polygon?

7 If the polygon is regular (all angles and sides equal), what would be the size of each angle in an n-sided polygon?

The exterior angle sum of a convex polygon

- **An exterior angle of a polygon is an angle that forms a straight line with one of the interior angles.**
- **If the interior angle is a reflex angle then no exterior angle exists.**



Assessment Grid for Investigation 10:09A | The angle sum of a polygon

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	The student has not used a systematic approach and has had difficulty following instructions and calculating answers.	1	
			2	
	b	The student has used a systematic approach and has attempted to describe the patterns in the investigation.	3	
			4	
	c	A systematic approach has been used and the patterns identified have been described in words or symbols, with some conclusions drawn.	5	
			6	
	d	The correct patterns have been recognised and explained clearly using words, and generalised in symbols.	7	
			8	
Criterion C Communication	a	No working out is shown and presentation is poor with little use of diagrams and symbols.	1	
			2	
	b	Working out is shown and explanations are given. Presentation is good with some structure shown.	3	
			4	
	c	Results are well interpreted and communicated using diagrams, symbols and words. Work is well structured.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt has been made to explain the steps and to check the reliability of the results.	1	
			2	
	b	The majority of the steps are justified and results have been checked with some success.	3	
			4	
	c	A thorough, reasoned justification has been given for all steps used and the accuracy of all results has been checked thoroughly.	5	
			6	

As all the angles in a convex polygon are acute or obtuse then an interior angle/exterior angle pair will exist at each vertex.

The sum of the exterior angles of a convex polygon is explored in the following Investigation.

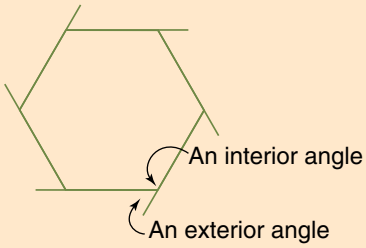
investigation 10:09B

Investigation 10:09B | The exterior angle sum of a convex polygon

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

1 At each vertex of the hexagon, a side has been produced to form an exterior angle. One interior angle/exterior angle pair is formed at each vertex. How many interior angle/exterior angle pairs does a hexagon have?

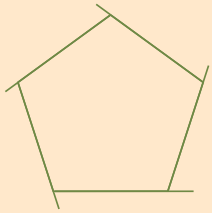
2 At each vertex the exterior angle and interior angle make a straight angle and so sum to 180°. What is the sum of all the interior and exterior angles in the hexagon?



3 Use the result of the last investigation to find the sum of all of the interior angles.

4 Use the answers to questions **2** and **3** to find the sum of all of the exterior angles.

5 Repeat steps 1 to 4 above for the pentagon shown.



6 What do the results above suggest about the sum of the exterior angles of any polygon?

7 Read through the following.
Let the sum of the interior angles be I.
Let the sum of the exterior angles be E.
For an n-sided polygon:

$$I = (n - 2) \times 180$$
$$I + E = 180n$$
$$E = 180n - (n - 2) \times 180$$
$$= 180n - 180(n - 2)$$
$$= 180n - 180n + 360$$
$$= 360$$

- The sum of the interior angles of a polygon of n sides is: $(n - 2) \times 180°$.
- The sum of the exterior angles of any convex polygon is 360°.

worked examples

1 Find the sum of the interior angles of an octagon and the size of an interior angle, if the octagon is regular.

2 A regular polygon has an exterior angle of 20°. How many sides does the polygon have?

Solutions

1 Sum of interior angles $= (n - 2) \times 180°$
For an octagon, n is equal to 8.
Sum of interior angles $= (8 - 2) \times 180°$
$= 1080°$
If the octagon is regular, all angles are equal.
$\therefore$ Size of an interior angle $= 1080° \div 8$
$= 135°$

2 Sum of exterior angles = 360°
For a polygon, the number of sides is equal to the number of exterior angles.

$$\text{Number of angles} = \frac{360°}{20°}$$
$$= 18$$

$\therefore$ Number of sides = 18

Assessment Grid for Investigation 10:09B | The exterior angle sum of a convex polygon

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	The student has not used a systematic approach to solve the problems and has had difficulty working through the investigation.	1	
			2	
	b	The student has adopted a systematic approach and has attempted to describe any patterns in the results.	3	
			4	
	c	A systematic approach has been used and the patterns identified have been described in words or symbols, with some conclusions drawn.	5	
			6	
	d	The correct patterns have been recognised and explained clearly using words and symbols, including a prediction in question 6 and why $E = 360°$ in question 7.	7	
			8	
Criterion C Communication	a	Little working out is shown and presentation is poor with little explanation.	1	
			2	
	b	Working out is shown and explanations are given with some interpretation of the results.	3	
			4	
	c	Results and interpretations are well communicated using diagrams and symbols. Work is well structured.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt has been made to explain the steps and to check the reliability of the results.	1	
			2	
	b	The majority of the processes are justified and results have been checked with some success.	3	
			4	
	c	A thorough, reasoned justification has been given for all steps used, specifically in question 7, with results applied through further examples.	5	
			6	

Exercise 10:09

Foundation Worksheet 10:09

Formulae

1 Count the number of sides, n, for each of the following polygons.

a  b 

2 Evaluate $(n - 2) \times 180$ if n is: **a** 5 **b** 8

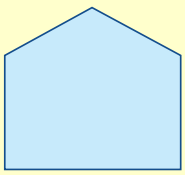
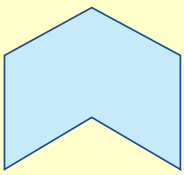
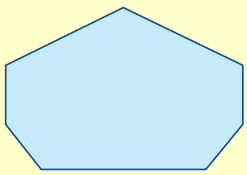
3 Find the angle sum of the polygons in question 1.

1 Evaluate $(n - 2) \times 180$ if $n =$

a 6 **b** 10 **c** 15

2 For each of the following polygons find:

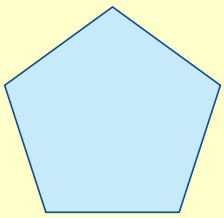
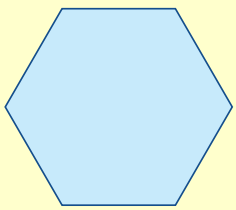
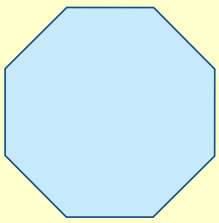
i the number of sides **ii** the interior angle sum

a  **b**  **c** 

3 Find the interior angle sum of:

a an octagon **b** a decagon **c** a heptagon

4 For each of the following regular polygons, find the size of the interior angle.

a  **b**  **c** 

d a decagon (10 sides) **e** a nonagon (9 sides) **f** a regular polygon with 17 sides

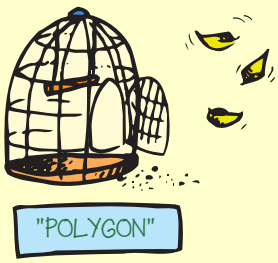
5 A polygon has 20 sides. Find:

a the sum of the interior angles

b the sum of the exterior angles

c the size of an interior angle if the polygon is regular

d the size of an exterior angle if the polygon is regular

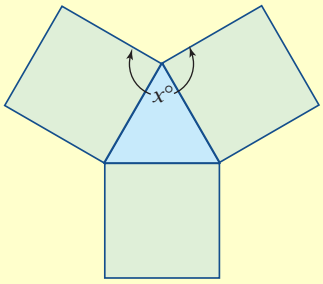


6 Find the number of sides in a regular polygon which has an exterior angle of:

a 60° **b** 40° **c** 10° **d** 15°

7 Tiles in the shapes of regular polygons have been used to make the following shapes. Find the value of the pronumerals.

a



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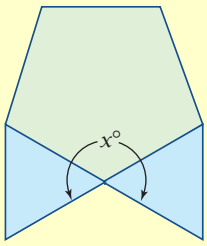
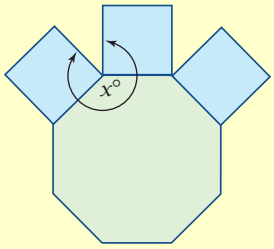
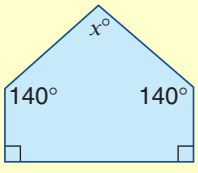


c



8 Find the value of the pronumeral in each of the following:

a



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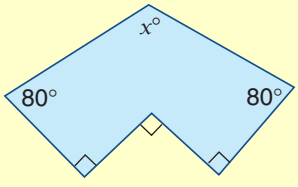

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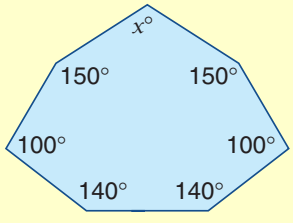
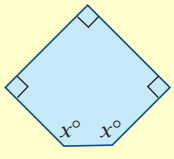
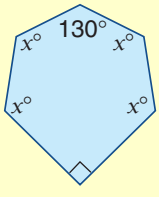


d



e



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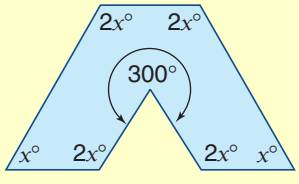
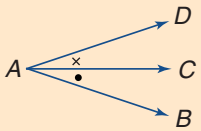


mathematical terms 10

Mathematical terms 10

adjacent angles
- Share a common arm and vertex.
- Lie on opposite sides of the common arm.

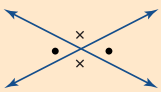


$\angle DAC$ and $\angle BAC$ are adjacent angles.

complementary angles
- angles that add up to give 90°.

supplementary angles
- angles that add up to give 180°.

vertically opposite angles
- Two pairs of equal angles formed when two straight lines cross.



bearing
- An angle used to measure the direction of a line from north.

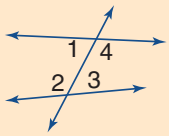
parallel lines
- Lines in the same plane that do not meet.

transversal
- A line that cuts two (or more) lines.

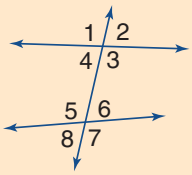
exterior angle
- An angle formed when the side of a convex polygon is produced.

polygon
- A plane figure with straight sides.
- *Regular* polygons have all sides and angles equal.
- *Convex* polygon has all its angles either acute or obtuse.
- Some polygons have special names. (See Investigation 10:09A.)

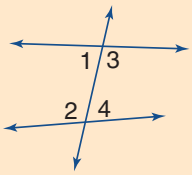
alternate angles
- A pair of angles on opposite sides of the transversal between the other two lines.
- In the diagram the alternate angles are 1 and 3, 2 and 4.



corresponding angles
- Angles that are in corresponding positions at each intersection.
- In the diagram the corresponding angles are: 1 and 5, 2 and 6, 3 and 7, 4 and 8.



co-interior angles
- A pair of angles on the same side of the transversal and between the other two lines.
- In the diagram the co-interior angles are 1 and 2, 3 and 4.



triangle
- A polygon with 3 sides.
- *Equilateral*, *isosceles* and *scalene triangles* have 3 sides, 2 sides and no sides equal in length respectively.
- *acute-angled triangle* has three acute angles.
- *right-angled triangle* has one right angle.
- *obtuse-angled triangle* has one obtuse angle.

quadrilateral
- A polygon with 4 sides.
- There are six special quadrilaterals. (See 10:08 or ID Card 4.)

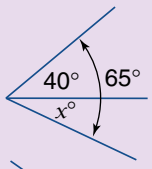
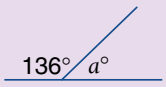
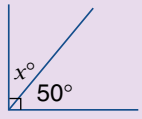
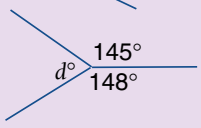
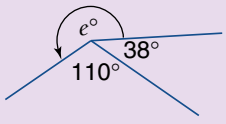
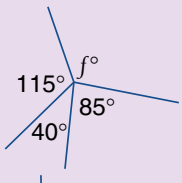
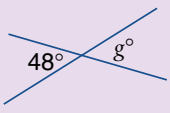
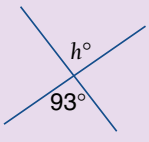
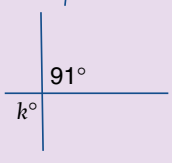
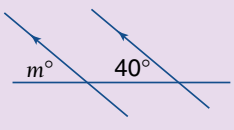
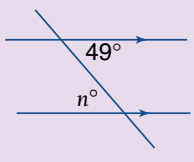
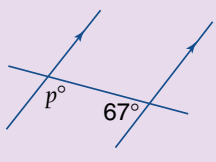
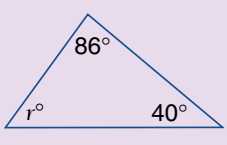
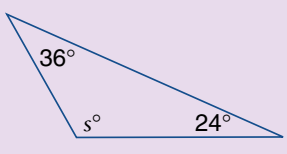
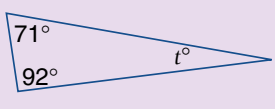
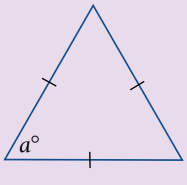
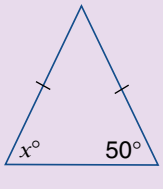
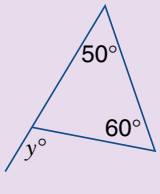
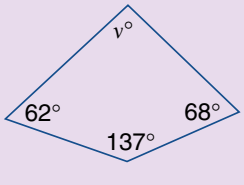
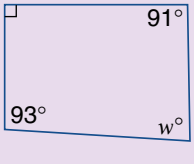
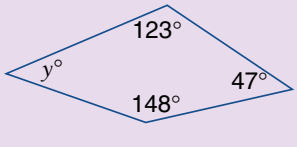
Technology Applications Drag and Drops

Mathematical terms 10

- An octagonal light well in a library building.

Diagnostic Test 10: | Geometry

- Each part of this test has similar items that test a certain type of question.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

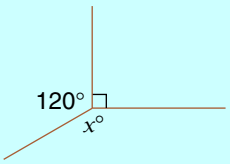
Find the value of each of the pronumerals in questions 1 to 7 below.	Section
1 a  **b**  **c** 	10:01
2 a  **b**  **c** 	10:02
3 a  **b**  **c** 	10:02
4 a  **b**  **c** 	10:05
5 a  **b**  **c** 	10:06
6 a  **b**  **c** 	10:06
7 a  **b**  **c** 	10:07
8 a What is the sum of the interior angles of a hexagon? **b** What is the size of an interior angle in a regular octagon? **c** What is the sum of the exterior angles of a pentagon?	10:09

assignment 10A

Chapter 10 | Revision Assignment

1 Find the value of the pronumeral and, in each case, give a reason.

a



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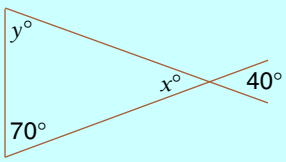

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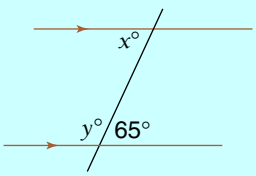

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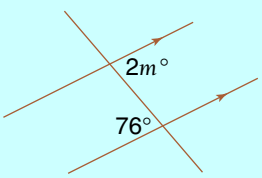

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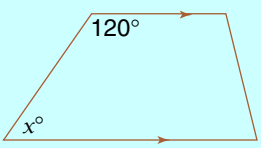

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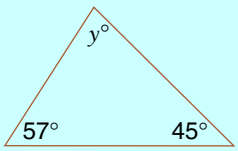
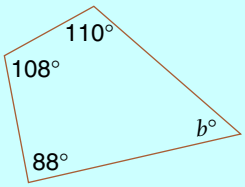
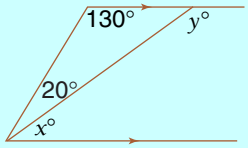


g



2 In each of the following, find the value of x and y. Give reasons for your answer.

a



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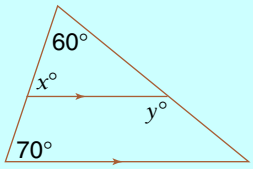
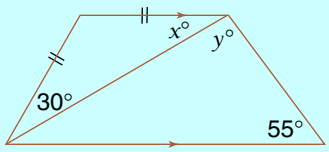
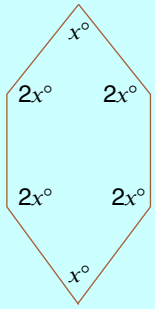


c

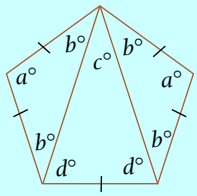


3 **a** A regular polygon has an interior angle of 175°. How many sides does it have? (Hint: Find the size of its exterior angles first.)

b A special hexagon is made with four of its angles equal and the remaining two angles are both half the size of the others. This is shown in the diagram. Find the size of the angles.



c Sharon makes a regular pentagon from three isosceles triangles as shown. Find the sizes of the angles in the triangles.



Technology Applications Drag and Drops

1 Angles 1
2 Angles 2
3 Angles 3
4 Triangles
5 Angle sum of a quadrilateral
6 Properties of quadrilaterals
7 Angle sum of polygons

Quadrilaterals
Flowcharts

Technology Applications Activities
Chapter Review Questions

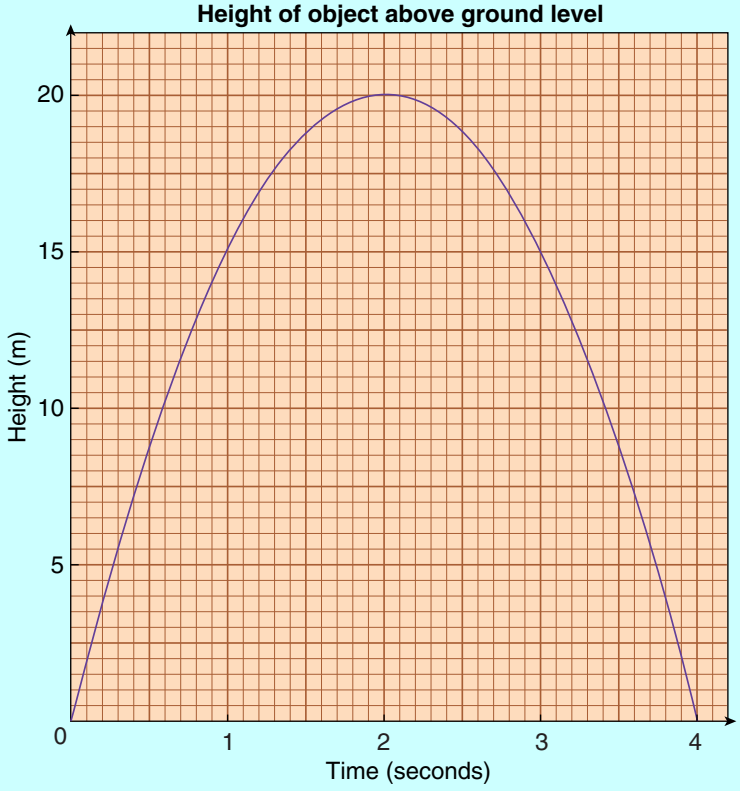
assignment 10B

1 Describe mathematically the shape of the lamp-shade in the photograph.

2 The ISCF group has 20 members. The school choir has 16 members. Only Sue and Graham are members of both groups. How many different people belong to these two groups altogether?

3 The petrol tank of my car holds 45 litres. I drove into a petrol station and filled the tank. It cost me $9.90. If the petrol cost 82·5 cents per litre, how many litres did I buy? How much was in the tank before it was filled?

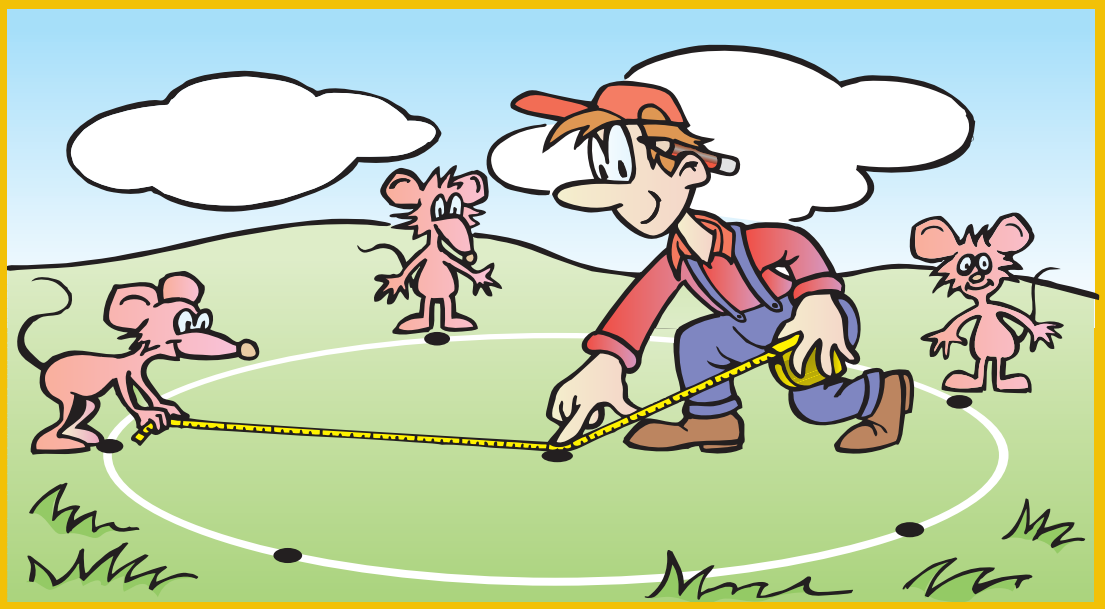
4 The graph shows the height of an object fired into the air.
- a How high is the object above ground level after 1 second?
- b At what times is the object 10 m above ground level?
- c What is the greatest height reached by the object?
- d How long does it take for the object to fall from its maximum height to ground level?
- e For how long is the object above a height of 17·5 m?



Teacher's Resource Practice Test

11

Locus



Chapter Contents

Learning Outcomes

Students will be able to:

- Construct bisectors of angles and intervals.
- Construct perpendiculars to an interval.
- Construct angles of 30°, 60°, 90°, 120°.
- Apply constructions to locus problems.

Areas of Interaction

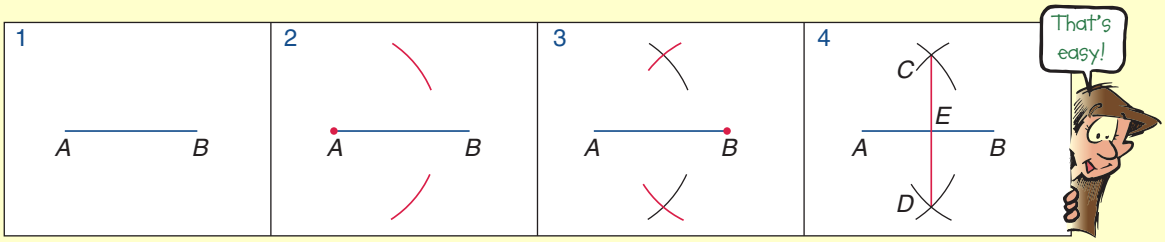
Approaches to Learning, Homo Faber, Environment

11:01 | Review of Bisecting Intervals and Angles

Bisect means to cut in half. In Book 1 you would have bisected an interval (or found the halfway point) using a ruler. To bisect (or halve) an angle you would have used a protractor.

The ancient Greeks knew how to bisect intervals and angles using compasses and a ruler. The steps that need to be followed to do this are given below in diagrammatic form with an explanation following.

Bisecting an Interval



- If we join the points A, C, B and D we get a rhombus.

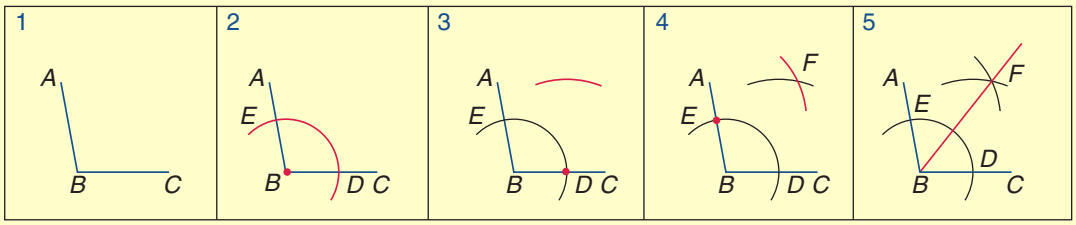
Step 1 The interval AB.

Step 2 With the point of the compasses at A, draw an arc above and below the interval.

Step 3 With the point of the compasses at B, draw two arcs to cut the arcs already drawn at C and D.

Step 4 Joint C to D. The line CD is called the **perpendicular bisector** of AB, while E is the **midpoint** of AB.

Bisecting an Angle



Step 1 The angle ABC.

Step 2 With the point of the compasses at the vertex B, draw an arc to cut both arms of the angle at D and E.

Step 3 From D draw an arc.

Step 4 From E draw an arc to cut the arc drawn in step 3 at F.

Step 5 Join B to F.

Practical Activity 11:01 | Bisectors

You will need: compasses, ruler, protractor, pencil, eraser

1 Draw three intervals similar to the ones shown below and construct the perpendicular bisector of each.

a

b

c

2 **a** Draw any triangle ABC and bisect the three sides.

b The bisectors should meet at a point called the **circumcentre**. What do you notice about the distances of the points A, B and C from the circumcentre?

c Use the circumcentre to draw a circle (called a **circumcircle**) that passes through the points A, B and C.

3 **a** Draw a circle that will pass through the points X, Y and Z. (You will need to trace the positions of X, Y and Z in your book and use the method used in Question **2**.)

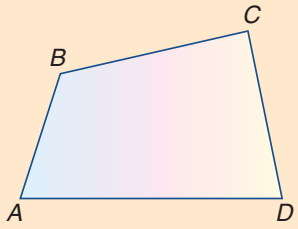
b Mark any three points on a page (they cannot be in a straight line) and construct a circle that passes through these points.

• X

• Y

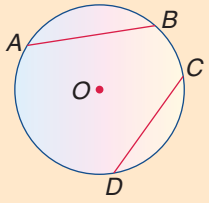
• Z

4



Draw any quadrilateral *ABCD* and by constructing the perpendicular bisectors, locate the midpoint of each side. What shape is obtained by joining the midpoints?

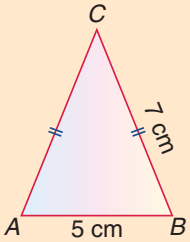
5 **a**

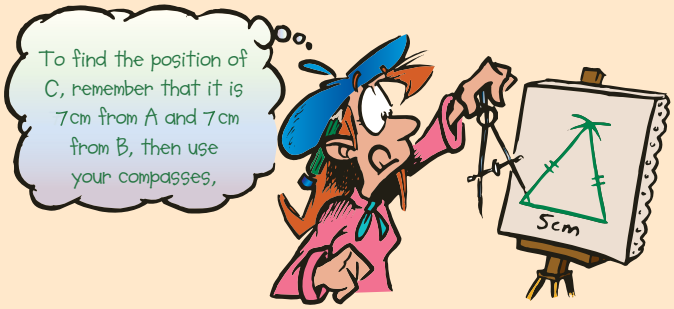


Draw any circle and mark the centre, *O*.
Draw any two chords *AB* and *CD*.
Bisect the chords *AB* and *CD*. What do you notice?

b Using what you have learnt in Part **a**, describe how you could find the centre of a circle.

6 **a** Construct the isosceles triangle *ABC* shown.
b Bisect the base *AB*.
c Does the perpendicular bisector pass through *C*?





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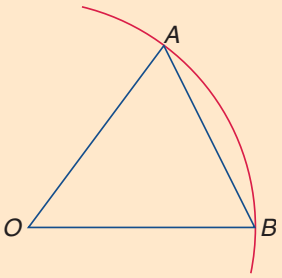
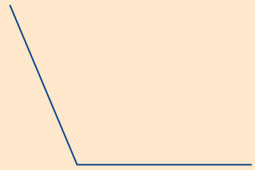


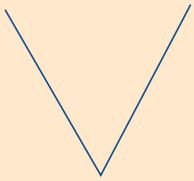
a Draw an arc of a circle. Choose two points on the arc and join them to the centre *O* as shown in the diagram.
b What type of triangle is ΔAOB? Why?
c Bisect *AB*.
d Does the perpendicular bisector pass through *O*?

8 Trace the angles below and bisect them.

a



b



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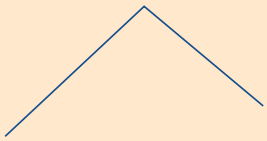

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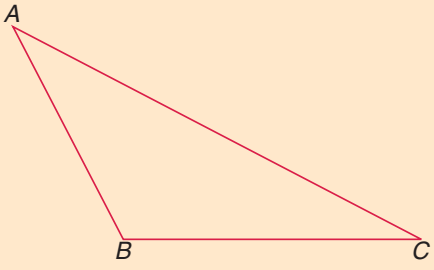
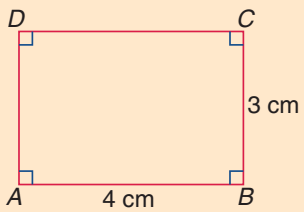
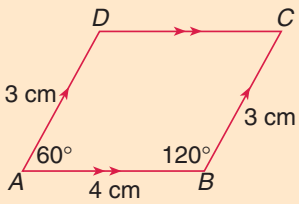


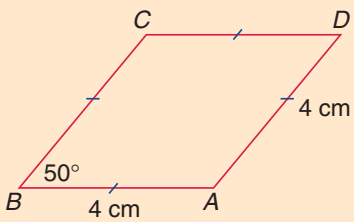
a Use your compass to copy ΔABC.
b Bisect $\angle ABC$.
c Bisect $\angle BCA$.
d Bisect $\angle BAC$.
e The bisectors meet at a point called the **incentre**. Use the incentre to draw the **incircle** (the circle that touches the three sides of the triangle on the inside).

10 **a** Use compasses and protractor to construct a square.
b Bisect one of the angles of the square.
c Does the bisector also bisect the opposite angle?
d Does the diagonal of the square bisect the angles through which it passes?

11 **a** Construct the rectangle, parallelogram and rhombus shown.

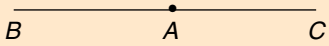






b Bisect $\angle ABC$ in each figure.
c In which figure does the bisector bisect the opposite angle?
d In which figure does the diagonal bisect the angles through which it passes?

12 Draw an interval BC and mark a point A on it.
a Bisect the straight angle BAC.
b Check the size of the bisected angle with a protractor. What size should it be?



11:02 | Review of Constructing angles of 60°, 120°, 30° and 90°

prep quiz

11:02

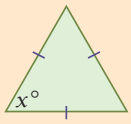
1 What does 'bisect' mean?

2 $\frac{1}{2}$ of 60°

3 $\frac{1}{2}$ of 180°

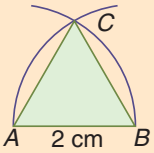
4 $\frac{1}{2}$ of 90°

5 Find the value of x.



6 What type of triangle is ΔABC?

7 What is the size of $\angle CAB$?



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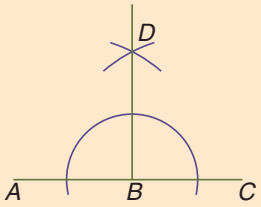
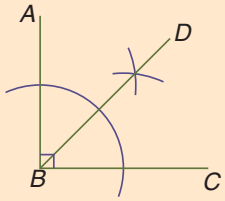


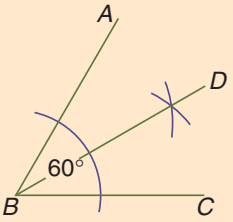
The diagram shows the bisection of the straight angle ABC. What is the size of $\angle ABD$?

9



The diagram shows the bisection of the right angle ABC. What is the size of $\angle ABD$?

10



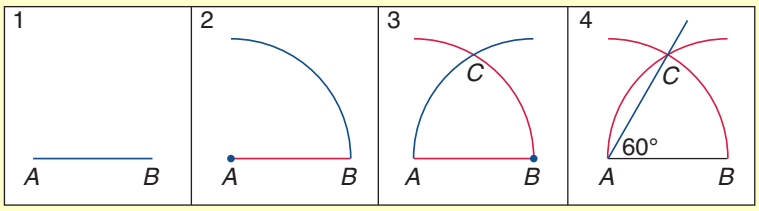
The diagram shows the bisection of a 60° angle. What is the size of $\angle ABD$?

In this section we will learn how to construct some angles with special sizes.

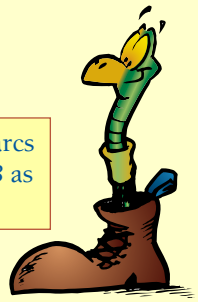
Question **6** in the Prep Quiz should have reminded you how to construct an equilateral triangle. When we remember that the angles in an equilateral triangle are all 60° in size, it means that all we have to do to construct a 60° angle is to construct an equilateral triangle.

This is shown in the series of diagrams below.

Constructing a 60° Angle



■ The arcs have AB as radius.



Step 1 Draw the arm AB.

Step 2 With the point of the compasses at A, draw an arc that passes through B.

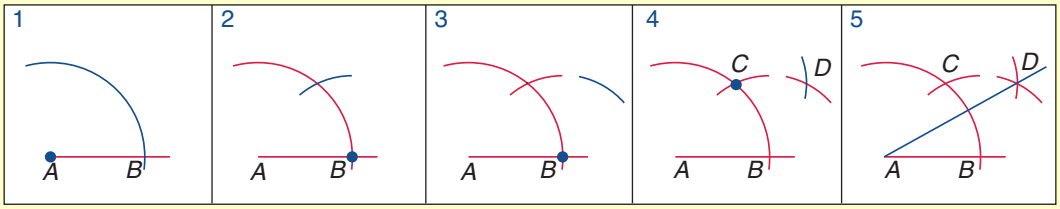
Step 3 With the point of the compasses at B, draw an arc that passes through A. Draw this arc so that it cuts the arc drawn in step 2 at C.

Step 4 Join A to C. The angle CAB is a 60° angle (since ΔCAB is equilateral).

Once you know how to construct a 60° angle you can also construct 30° and 120° angles.

Constructing a 30° Angle

To construct a 30° angle, first construct a 60° angle and then bisect this angle. In practice, this procedure can be shortened, as shown below.



Step 1 With the point of the compasses at *A* draw an arc. The point where the arc cuts the arm is *B*.

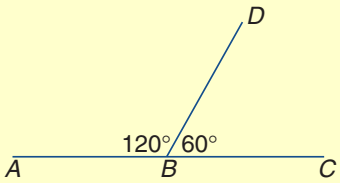
Step 2 With the point of the compasses at *B*, draw an arc to cut the arc already drawn at *C*.

Step 3 With the point of the compasses still at *B*, draw another arc as shown.

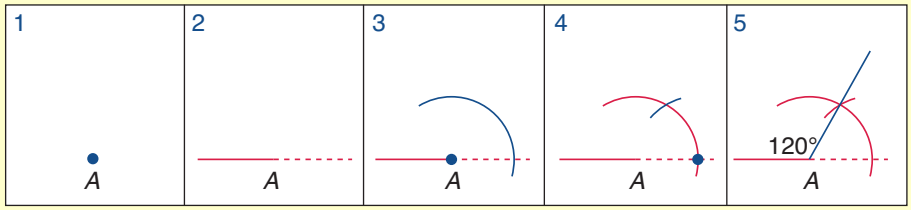
Step 4 With the point of the compasses at *C*, draw an arc to cut the arc drawn in step 3 at *D*.

Step 5 Join *D* to *A*. The angle *DAB* is a 30° angle.

Constructing a 120° Angle

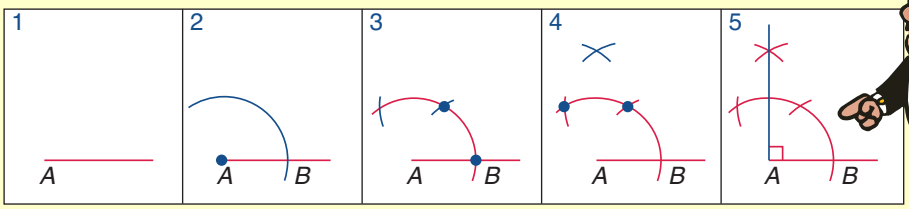


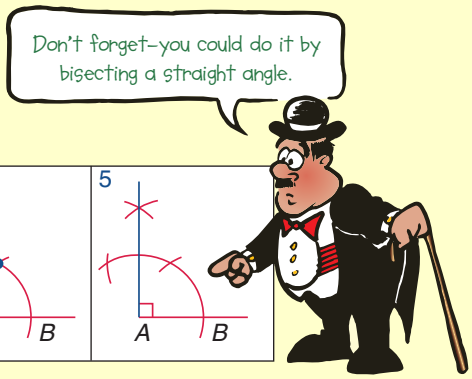
To construct a 120° angle, we remember that a 60° angle and a 120° angle, when they are adjacent, form a straight angle. Therefore, if we wish to construct a 120° angle, we can construct a 60° angle and then extend one of its arms. This is shown in the diagrams below.



Constructing a 90° Angle

To construct a 90° angle, we can either bisect a straight angle or follow the steps shown in the following diagrams.

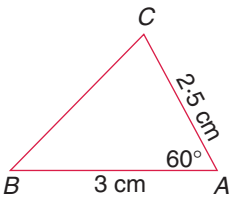




worked examples

Construct the following figures using compasses and ruler only.

1



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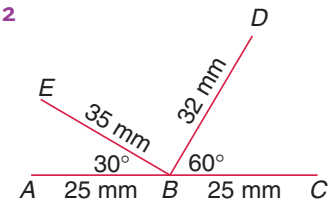
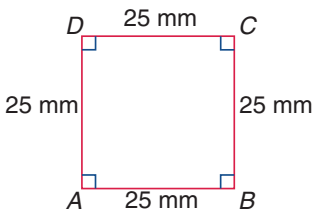
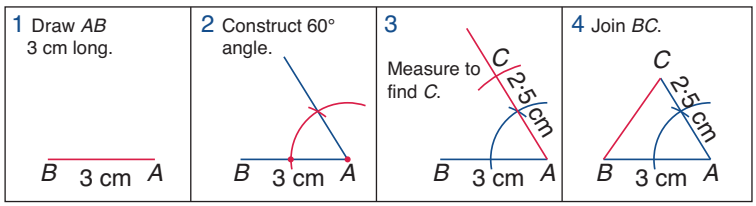


3



Solutions

1



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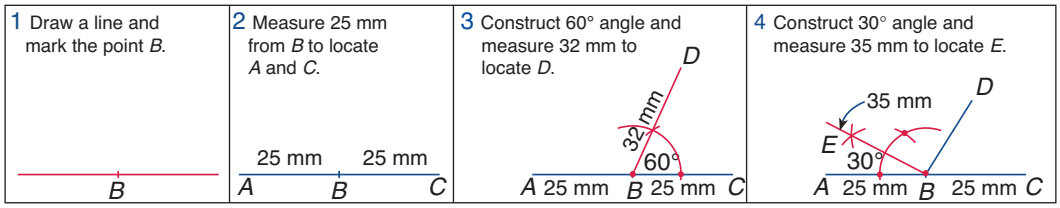
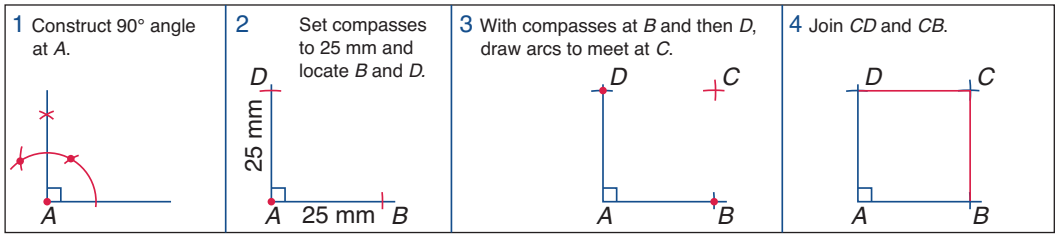


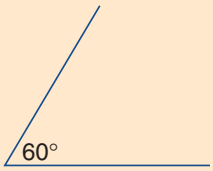
3



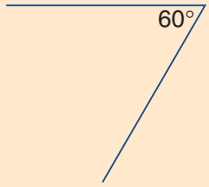
Practical Activity 11:02 | Be constructive

1 Construct each of the following 60° angles in your book.

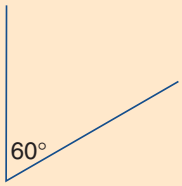
a



b

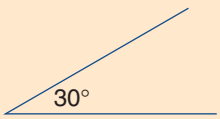


c

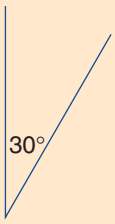


2 Construct each of the following 30° angles in your book.

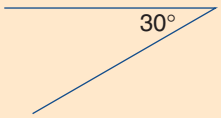
a



b

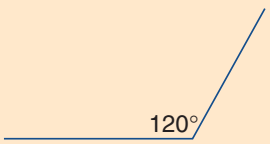


c

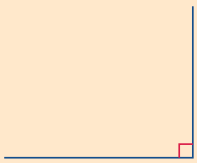


3 Construct each of the following angles in your book.

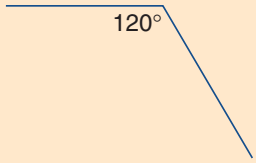
a



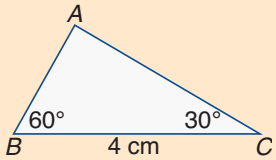
b



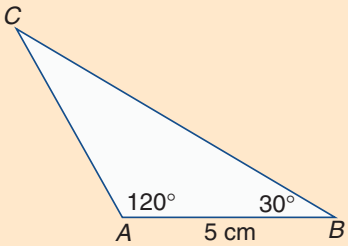
c



4 **a** Construct the triangle *ABC* shown.
b What should the size of ∠*BAC* be? Check with your protractor.
c Measure the lengths *AB* and *AC* to the nearest millimetre.

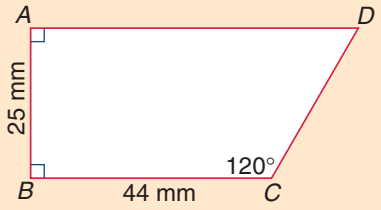


5



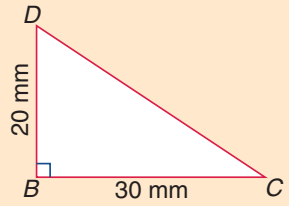
a Construct the triangle *ABC* shown.
b What should the size of ∠*ACB* be? Check with your protractor.
c Measure the lengths *AC* and *BC* to the nearest millimetre.

6 **a** Construct the trapezium shown.
b Measure $\angle ADC$. What should it be?
c Measure AD and DC to the nearest millimetre.

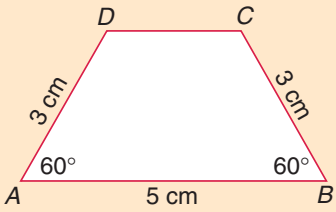


7 Construct the following figures:

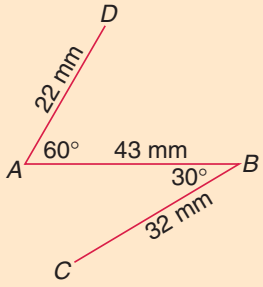
a



b



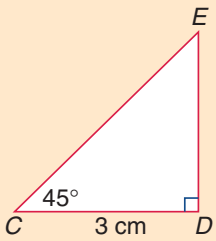
c



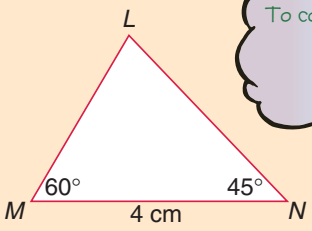
Check your accuracy by measuring the length of CD in each figure.

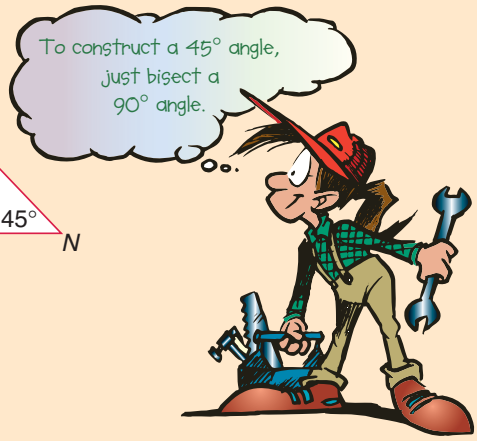
8 Construct the following figures.

a



b

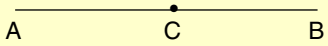




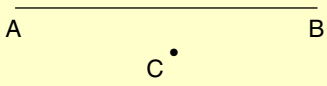
11:03 | Review of Constructing Perpendicular and Parallel Lines

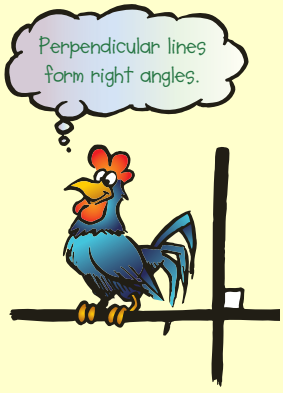
A line is perpendicular to another line when it intersects or meets it at right angles. You need to be able to construct a perpendicular to a line from:

1 a point on the line, eg perpendicular to AB through C.



2 a point not on the line, eg perpendicular to AB through C.

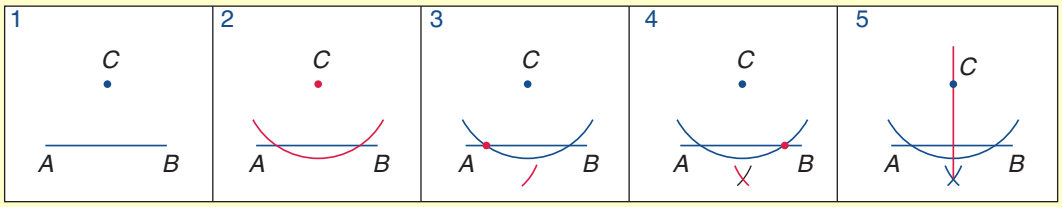




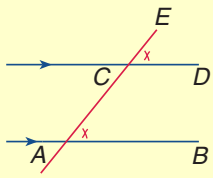
The first of these constructions can be done by either:
a constructing a right angle at C, or
b bisecting the straight angle ACB.

The second of these constructions is shown below.

Constructing the Perpendicular from *C* to *AB*



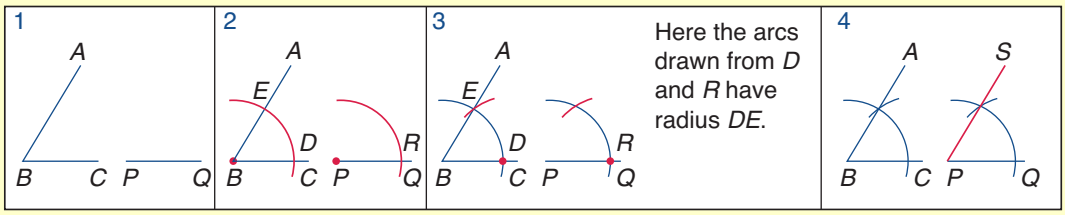
To construct a pair of parallel lines, we need to remember that when parallel lines are cut by a transversal, the corresponding angles formed are equal (see diagram on the right).



Therefore, to make *CD* parallel to *AB* we must make $\angle ECD$ equal to $\angle CAB$. That is, we must make a copy of $\angle CAB$.

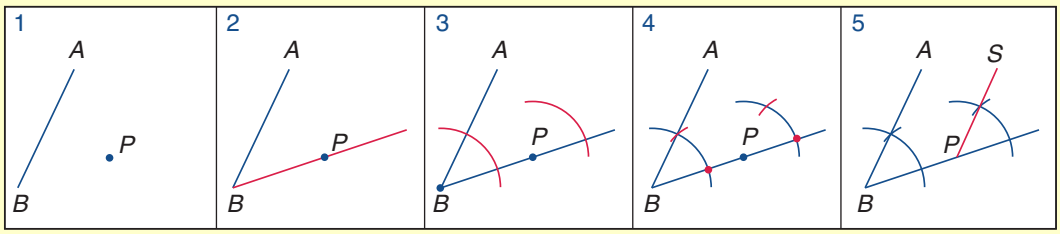
To copy an angle we follow the steps shown below.

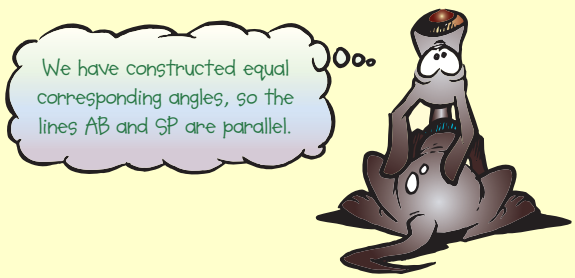
Copying an Angle



This construction can also be used to draw a line parallel to a given line through a given point.

Drawing a Line Parallel to *AB* through *P*

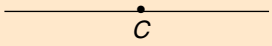




Practical Activity 11:03 | Perpendiculars

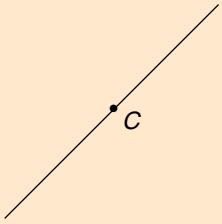
1 Draw figures similar to those below and then construct the perpendicular through C.

a

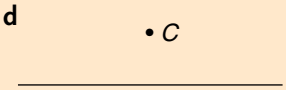


b

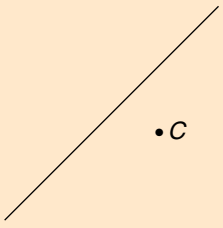
c



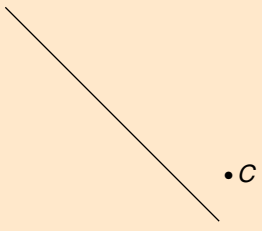
d



e

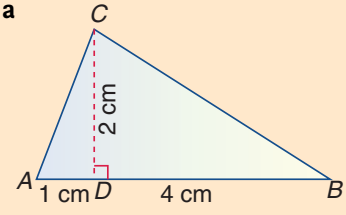


f

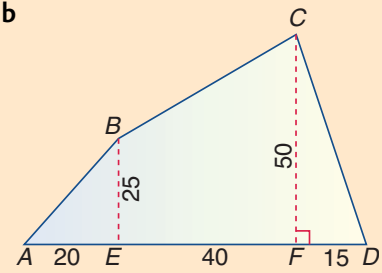


2 Construct the following figures. Measurements are in mm for **b** and **c**.

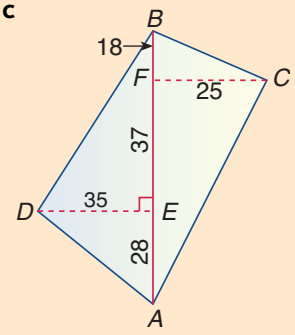
a



b

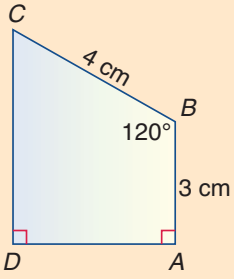


c

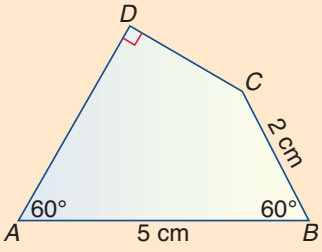


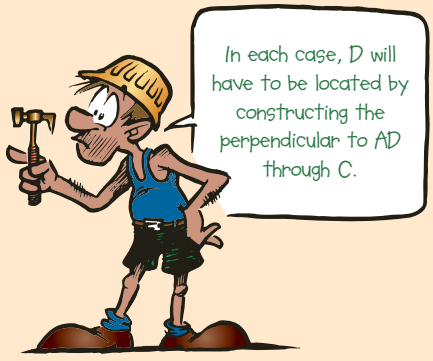
3 Construct the following figures.

a



b

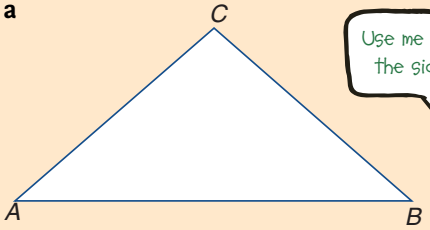


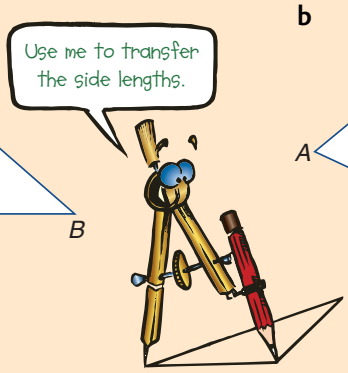


4 Draw three angles in your book. Copy them using the construction for copying an angle.

5 Use your compasses only to copy the figures given.

a





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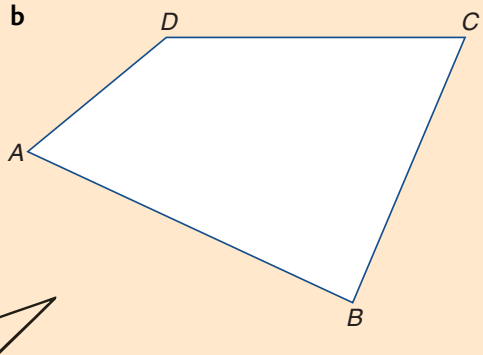
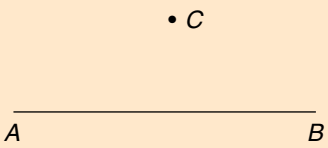
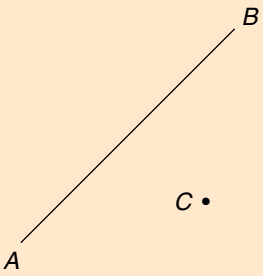


6 Draw three diagrams similar to those given below, then construct a line through C parallel to AB in each.

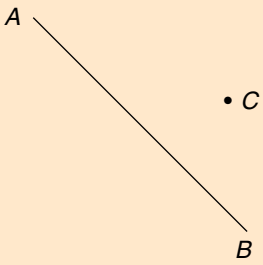
a



b



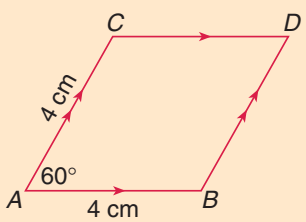
c



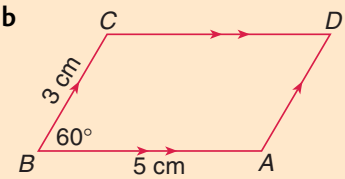
7 Draw a line in your book. Construct a line parallel to this line which is 3 cm away from it.

8 Construct the following figures.

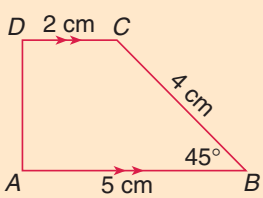
a



b



c



Investigation 11.03 | Special centres

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

11:03

Part A

1. Construct any scalene triangle with sides at least 4 cm long.
2. Bisect the angles of this triangle.
3. What do you notice about the angle bisectors?
4. Try this with two more triangles and see if it still works.
5. You should have found a special point. Search for the name of this point in your library or elsewhere.
6. This point is the centre of a special circle. What is special about it?
7. Draw this special circle on the three triangles you drew.

Part B

1. Construct another scalene triangle with sides at least 4 cm long.
2. Construct the perpendicular bisectors of the sides of this triangle. Make the bisectors very long.
3. What do you notice about the perpendicular bisectors?
4. Try this with two more triangles and see if it still works.
5. You should have found a special point. Search for the name of this point in your library or elsewhere.
6. This point is the centre of a special circle. What is special about it?
7. Draw this special circle on the three triangles you drew.

Can you explain these results? If you can't, the next Investigation in this chapter will help.

Assessment Grid for Investigation 11:03 | Special centres

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	The student had difficulty performing basic constructions.	1	
			2	
	b	The student has attempted to use an organised approach and to define the points.	3	
			4	
	c	An organised approach has been used and conclusions regarding the special points have been defined.	5	
			6	
	d	An understanding of the construction of circles is shown and there is a coherent explanation for why the special points occur in terms of locus, although the term locus may not have been used.	7	
			8	
Criterion C Communication	a	No working out or construction lines are shown.	1	
			2	
	b	Working out is shown and there is some interpretation of the results accompanied by diagrams.	3	
			4	
	c	Working out is shown and presentation is good. Interpretation of the results is given with correct notation and terminology	5	
			6	
Criterion D Reflection in Mathematics	a	An attempt has been made to give some explanation of the construction and to check the construction for accuracy	1	
			2	
	b	The processes used are explained and the constructions have been checked for accuracy.	3	
			4	
	c	Concise justifications of the results have been given, the constructions performed are evaluated and related to practical applications.	5	
			6	

11:04 | Locus

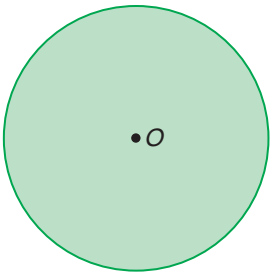
The **locus of a point or object** is the path that point or object makes as it occupies all possible places defined by a set of rules. A locus can be a shape or an area. If there is more than one locus, we say **loci**.

worked examples

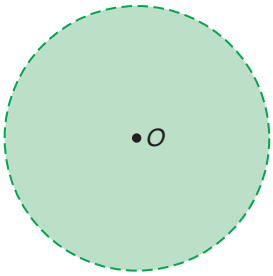
1 Construct and describe in words all points which are 3 cm from the point O.

2 Describe the following shaded areas as a locus. The circles have radius r cm.

a

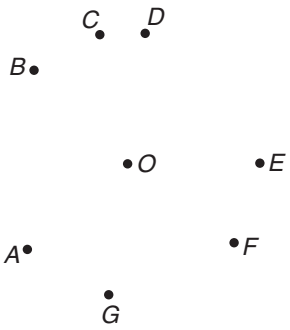


b



Solutions

1



First of all plot some points that follow the rule: points 3 cm from O.

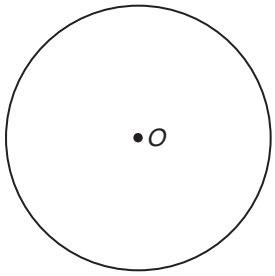
It looks like this locus is a circle.

Does this make sense?

So the *description in words* is:

The locus is a circle, centre O radius 3 m.

The construction is:

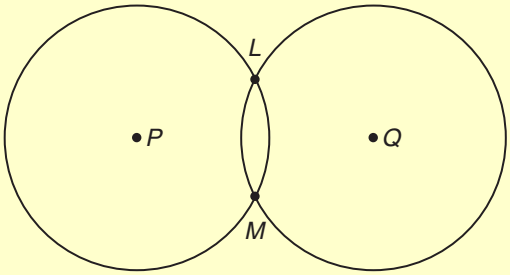


2 a The locus of points r cm or less from O.
The solid border indicates that all points on the circumference are included.
The shaded area indicates that all points inside the circle are included.

b The locus of points less that r cm from O.
The broken border indicates that all points on the circumference are not included.
The shaded area indicates that all points inside the circle are included.

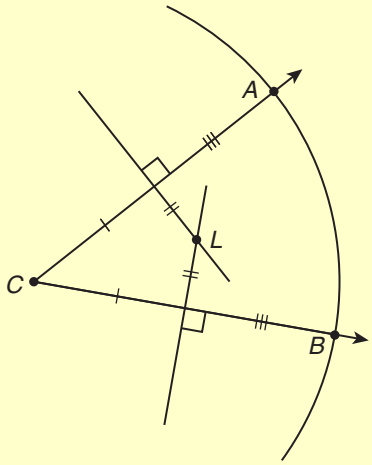
Exercise 11:04

1 The diagram shows two points *P* and Q.
Circle 1 shows the locus of all points a certain distance from *P*.
Circle 2 shows all the points the same distance from Q.



a What can be said about the points *L* and *M*?
b Do the same construction with different sized circles with centres *P* and Q.
c What do you notice about the locus of points that are equidistant from *P* and Q?
d Draw the line *PQ*. What is the relationship between the line *PQ* and the locus discovered in (c)?
e Complete this statement: 'The locus of points equidistant from two points *P* and Q is …'

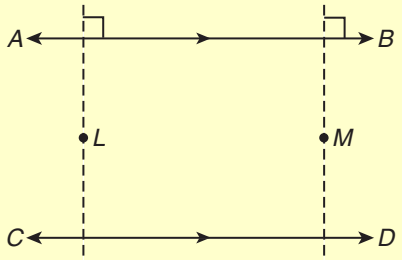
2 The diagram shows two half lines, *CA* and *CB* which are both joined at *C*. These half lines are called *rays*.
A and *B* are located equidistant from *C* using compasses as shown.
The perpendicular bisectors of *CA* and *CB* are then constructed as shown.
The point of intersection of these two bisectorsis labelled *L*.



a Since *L* lies on the perpendicular bisector of *CA*, it is equidistant from which two points?
b Since *L* lies on the perpendicular bisector of *CB*, it is equidistant from which two points?
c Is *L* equidistant from the rays *CA* and *CB*?
d Copy this construction into your book on a big diagram.
e Locate two other points like *A* and *B* and label them *D* and *E*.
f Construct the perpendicular bisectors of *CD* and *CE* and label their point of intersection *M*.
g Is *M* is equidistant the rays *CA* and *CB*?
h Repeat this construction once more, locating *F* and *G* in the same way as *A* and *B* and constructing perpendicular bisectors of *CF* and *CG* and label the point of intersection *N*.
i Is *N* is equidistant from the rays *CA* and *CB*?
j If you were to construct more points like *L*, *M* and *N*, and joined them up, what would result?
k Complete this statement: 'The locus of points equidistant from the rays *CA* and *CB* is …'

3 **a** Construct a pair of parallel lines *AB* and *CD* as shown, at least 5 cm apart.
b Construct two perpendiculars to both *AB* and *CD*.
c Find the midpoint of these perpendiculars and label them *L* and *M*.

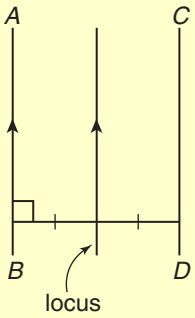
d Are L and M are both the same distance from AB and CD?
e Join L and M together to obtain a third parallel line.
f This third line represents what locus?



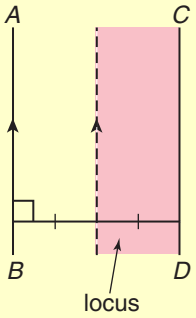
4 **a** Draw two points A and B which are at least 5 cm apart.
b Show the locus of points 3 cm from A.
c Show the locus of points 3 cm from B.
d Join A and B to form the interval AB.
e Construct the locus of all points 3 cm from the interval AB.

5 Describe the following loci.

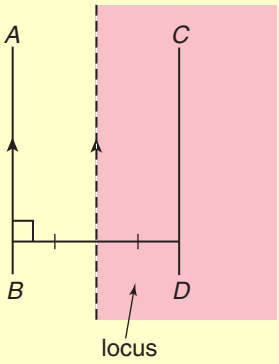
a



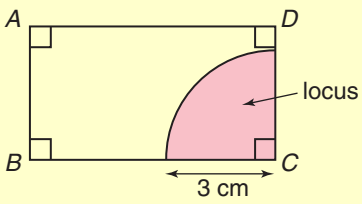
b



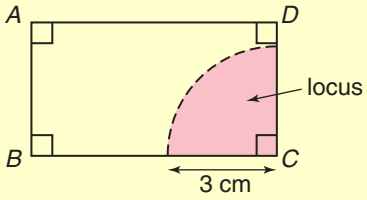
c



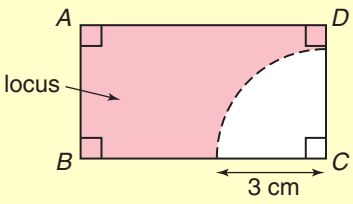
d



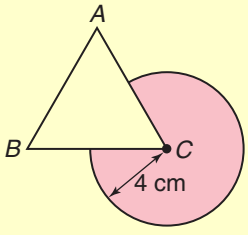
e



f



g



11:05 | Solving Problems Using Locus

For this exercise all the diagrams must be to an easy scale for the students to use.

worked example

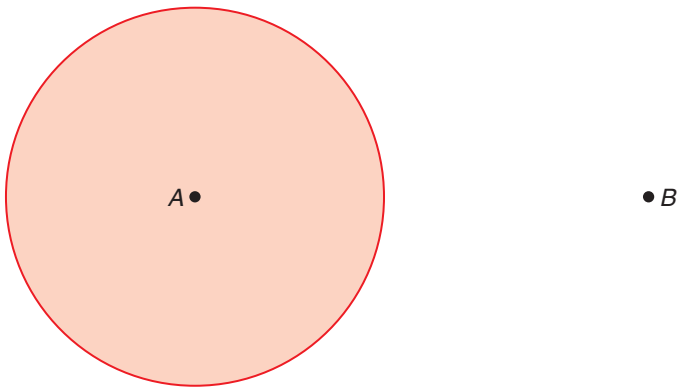
Towns A and B are 60 km apart. The high school at town A takes in all students that live within a radius of 25 km from the town. The high school at town B is much bigger and takes students who live within a radius of 45 km from the town.

Use a scale drawing to show the area in which students may go to either school.

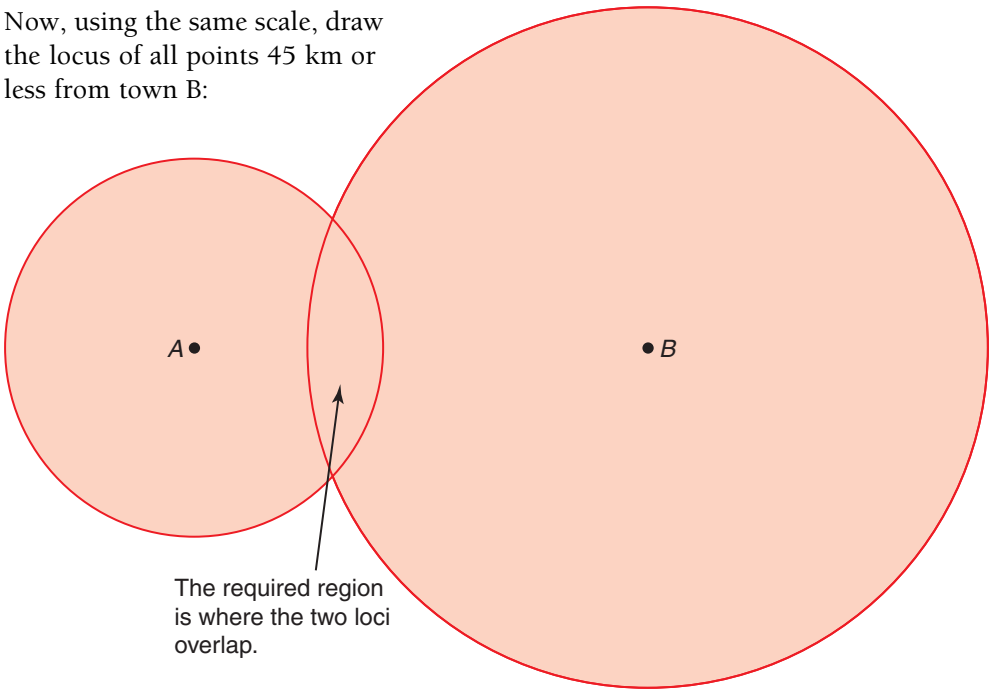
Solution

Using a scale of 1 cm = 10 km, sketch the position of the two towns:

Using the same scale draw the locus of all points 25 km or less from town A:



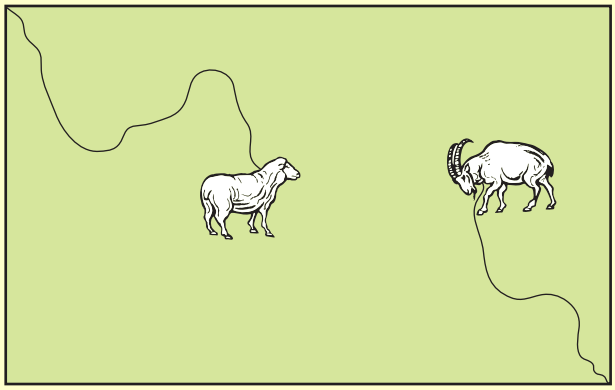
Now, using the same scale, draw the locus of all points 45 km or less from town B:



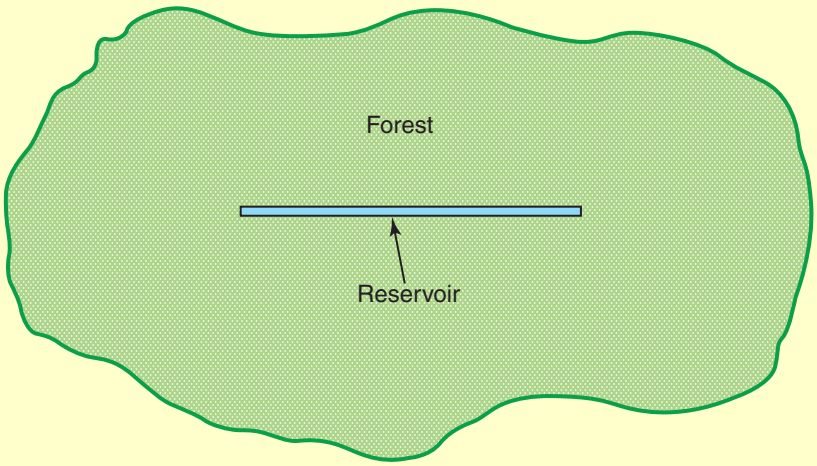
The required region is where the two loci overlap.

Exercise 11:05

1 A goat is tied by a 5 m rope to a post in one corner of an 8 m long and 5 m wide area as shown. A sheep is tied to the diagonally opposite corner with a 6 m rope. Make a scale drawing and show the area that can be grazed by both animals.

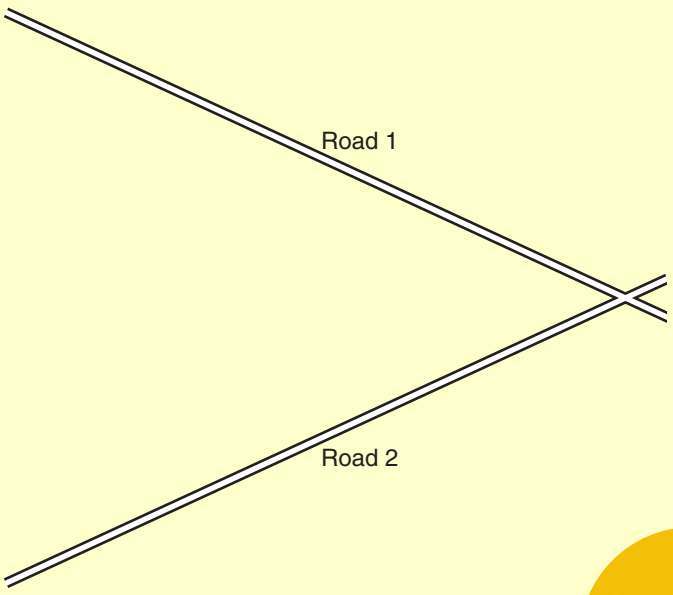


2 In the hot season, a country town uses a helicopter to put out forest fires in the local area. The helicopter can carry water a maximum of 9 km from a reservoir. The town has a 15 km long reservoir from which to get water. The reservoir is surrounded by forest as shown. Trace this diagram into your book and use locus and scale to show the area of forest protected by the helicopter.

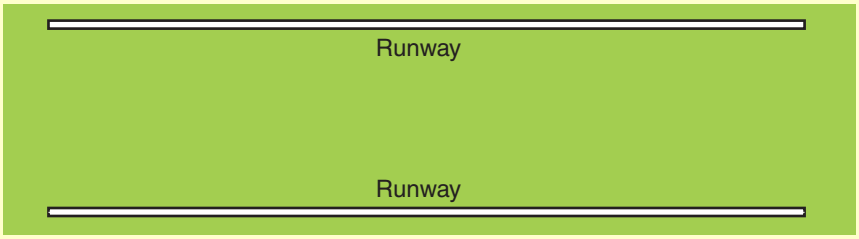


Scale: 3 mm : 1 km

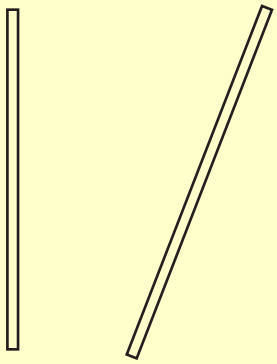
3 A bushwalker has become lost and uses his mobile phone to inform the rescue service of his location. He tells them that he knows he is closer to road 1 than he is to road 2 and that he has walked between 5 and 8 km from the intersection of the two roads. Copy the diagram and use a scale of 1 cm = 1 km to show where the rescue services should look for him.



4 At an airport, a drain must be laid equidistant from two parallel runways that are shown in the diagram below. Copy the diagram and show where the drain will be laid.

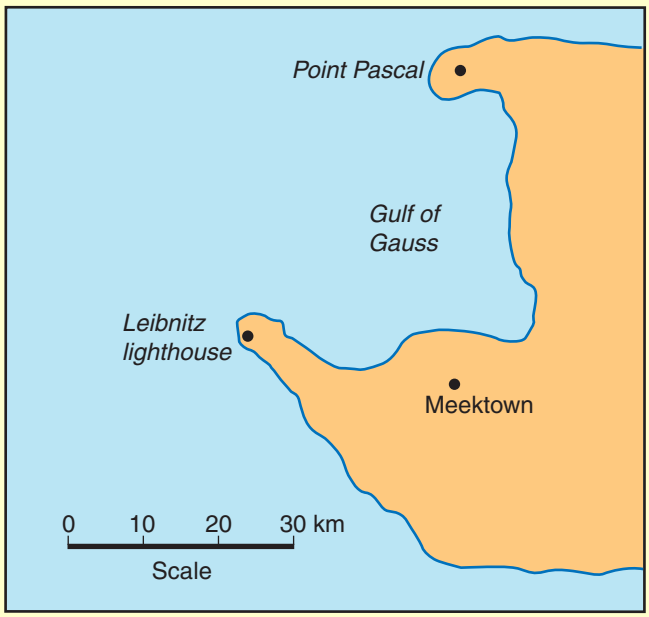


5 Another airport has runways that are not parallel as shown in the diagram below. A drain must be built that is equidistant from both. Copy the diagram into your exercise book and construct the path of the drain.



6 A boat has broken down in the Gulf of Gauss. It is known that the boat was travelling on a line equidistant from both Point Pascal and Leibnitz Lighthouse. It is also known that the boat was 20 km from Meektown.

Copy the map into your exercise book and locate the boat.



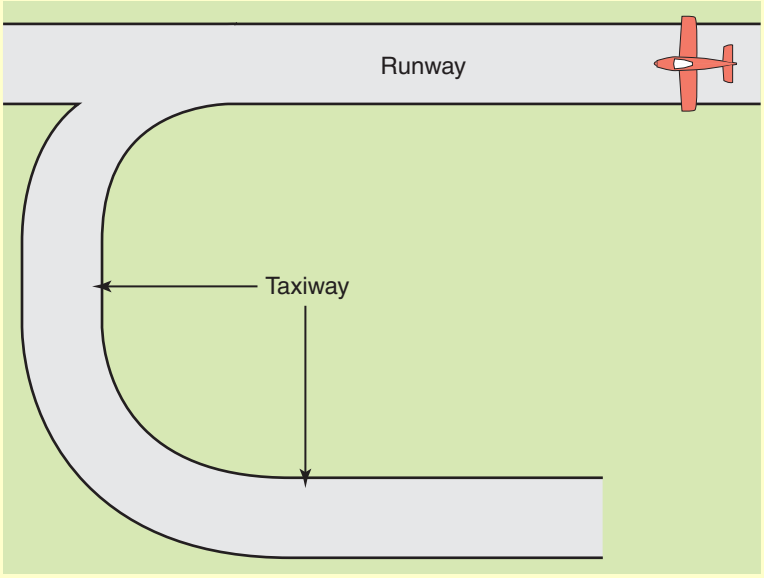
7 Grant's garden is a triangle ABC in which AB is 15 m, and BC and AC are both 10 m. Grant and Barry are playing a game in which Grant has hidden a €5 note which, given some clues, Barry has to find.

Here are the clues:

- The note is closer to BC than it is to AC.
- The note is closer to angle B than it is to the other two angles.

By making a scale drawing of Grant's garden, construct and shade the locus of places Barry should look.

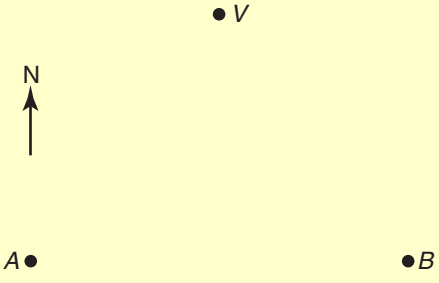
8 After it lands, a plane taxis so that it is always in the middle of the runway and taxiway. Copy the diagram of the runway and taxiway into your book and construct the locus of the plane as it lands and taxis.



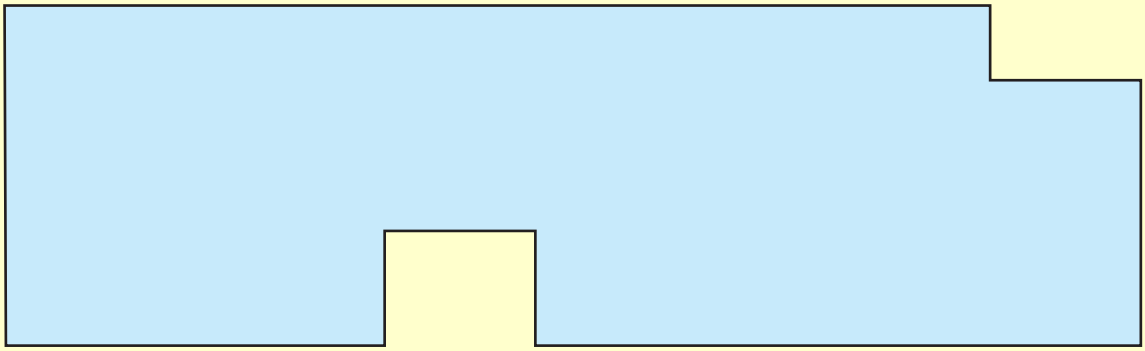
9 Two towns A and B are as shown in the scale diagram below. A volcano at V is also shown.

A plane, heading due North has to fly on a path equidistant from the two towns, but is not allowed to be less than 50 km from the volcano.

If the scale of the drawing is 1 cm = 25 km, copy the diagram and construct the locus of the minimum distance the plane can fly on its northerly route.



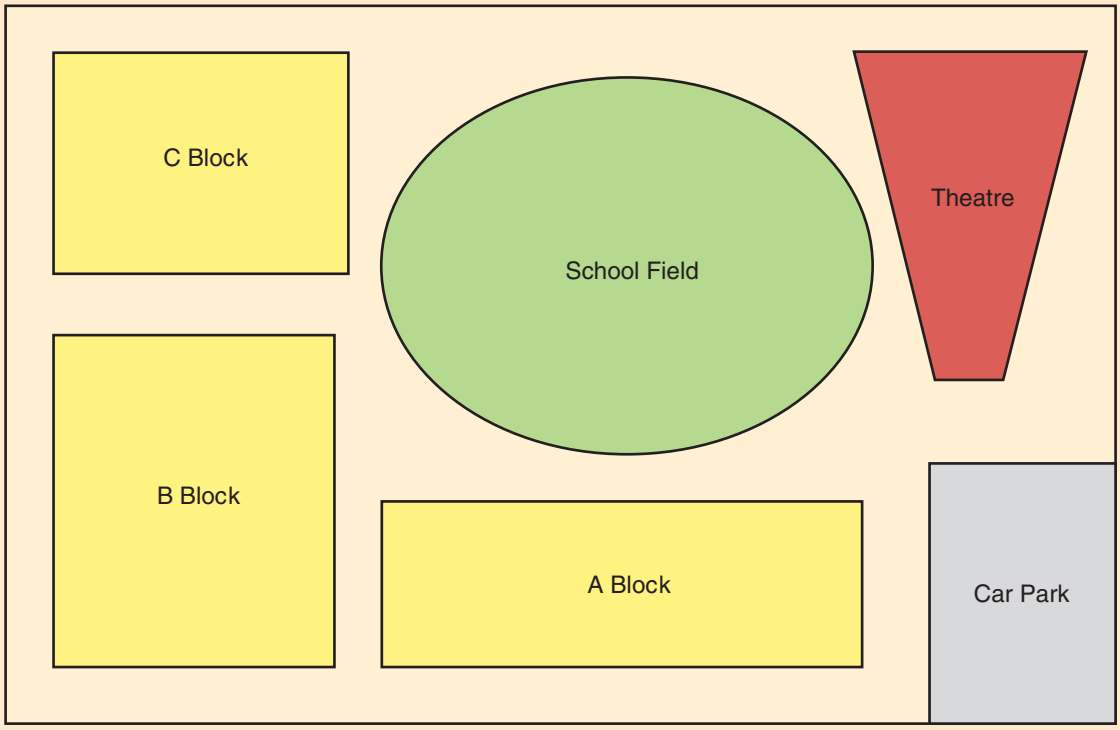
10 Martina's local pool is in the shape shown in the diagram below. She swims so that she is always 1 metre from the edge of the pool. If the scale of the diagram is 1 cm = 1 m, construct the locus of where she swims.



Investigation 11.05 | Wireless Network

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Below is a scale 1:2000 diagram of Julian's school. A wireless computer network is to be installed at the school so that all the buildings are serviced by the network. The school field and the car park do not need to be covered. Each access point for the network has a radius of 50 m. By using the scale diagram below, try to locate a minimum number of access points which cover all the buildings.



Assessment Grid for Investigation 11:05 | Wireless Network

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	No systematic or organised approach is made to solving the problem. There has been limited success.	1	
			2	
	b	Some general ideas of locus are used in solving the problem. Some effort has been made to minimise the number of points.	3	
			4	
	c	A number of different solutions have been tried to try to minimise the number of access points needed and some conclusions have been drawn.	5	
			6	
	d	The ideas of locus have been explained throughout the exercise and the positions of access points are justified fully.	7	
			8	
Criterion C Communication	a	A basic description of the solution is given using some Mathematical terminology and diagrams.	1	
			2	
	b	A clear explanation is given, alongside diagrams, to show the development of the solution.	3	
			4	
	c	A wide range of terminology is used in an effective explanation. Appropriate technology and logical steps are used.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt is made to explain how the solution was found and to assess whether the solution is practical.	1	
			2	
	b	An attempt is made to explain the solution, how it was found and its evaluation with some success.	3	
			4	
	c	A detailed explanation of the solution steps and the reliability of the findings is given, and other possible approaches have been effectively considered, with an evaluation of the one adopted.	5	
			6	

Mathematical terms 11

bisect
- To divide into two equal pieces.

construction
- To follow an instruction in geometry, usually without measuring, using a straight edge and a pair of compasses.

equidistant
- The same distance. If a point is equidistant from *A* and *B*, it is the same distance from both *A* and *B*.

interval
- A line segment or part of a line.

locus
- The path that point or object makes as it occupies all possible places defined by a set of rules.

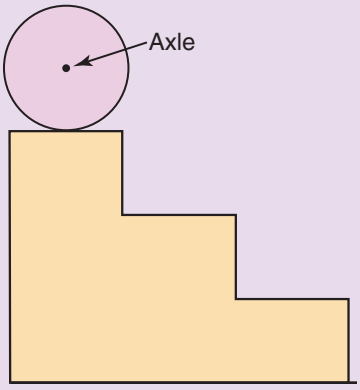
parallel
- Two lines that are parallel never meet.

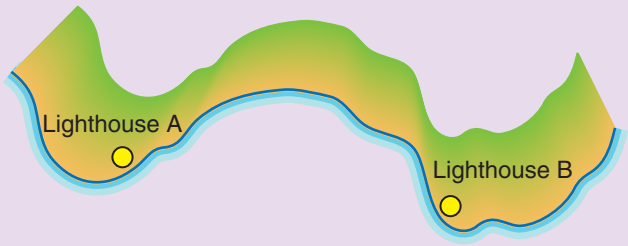
perpendicular
- If two lines are perpendicular to one another, they meet at right angles.

ray
- **A half line.**
- Two rays beginning at the same point make an angle.

Diagnostic Test 11

- Each section of the test has similar items that test a certain type of question.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

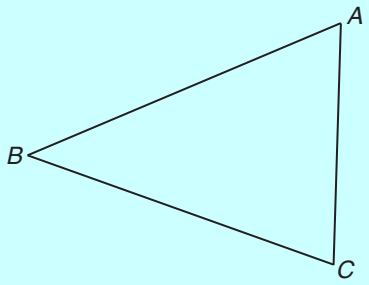
	Section
1 Draw a triangle *ABC* and: **a** bisect the side *AB*. **b** bisect the angle *ABC*. **c** shade the locus of all points inside the triangle that are closer to the side *AB* than they are to the side *BC*.	11:01
2 Construct a rectangle *PQRS* that is 6 cm long and 6 cm wide. Shade the locus of all points inside the rectangle that are 9 cm from both *P* and *R*.	11:01 & 11:04
3 **a** Draw an interval *XY* that is 7 cm long. **b** Construct two perpendiculars to *XY*, one at *X* labelled *AX*, and one at *Y*, labelled *BY*. **c** Bisect both the angles *AXY* and *BYX*. **d** Extend the bisectors until they meet at *Z*. **e** What type of triangle is *XYZ*?	11:01 & 11:04
4 A set of steps is shown with a wheel at the top. **a** Copy this diagram into your exercise book. **b** Draw the locus of the axle as the wheel rolls down the stairs. 	11:02 & 11:03

	Section
5 A boat has broken down at sea and is equidistant from lighthouse A and lighthouse B. Locate the position of the boat.  	11:05
6 A rectangular field is 10 m long and 15 m wide and has a fence around it. A goat is tied by a 12 m rope to one corner and is **outside** the fence. Make a scale drawing of the field and shade the area where the goat can graze.	11:05
7 A rectangle measures 8 cm wide and 10 cm long. Construct the locus of all points 3 cm from the edge of the rectangle.	11:04
8 Two towns A and B are 50 km apart. The fire brigade at town A responds to all fires within a 45 km radius. The brigade at town B responds to those within a 20 km radius. Use a scale diagram to show the area that is serviced by both brigades.	11:05
9 **a** Show the locus of points in the rectangle that are closer to *B* than *A*. **b** Show the locus of points in the rectangle that are closer to *B* than *A* and closer to *ZY* than *XY*.  	11:04

Chapter 11 | Revision Assignment

assignment 11A

Questions 1, 2 and 3 refer to the triangle *ABC*.



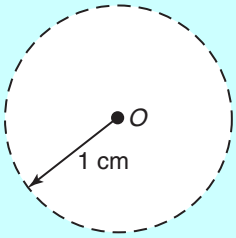
1 Bisect the angle *ABC*.

2 Bisect the side *AB*.

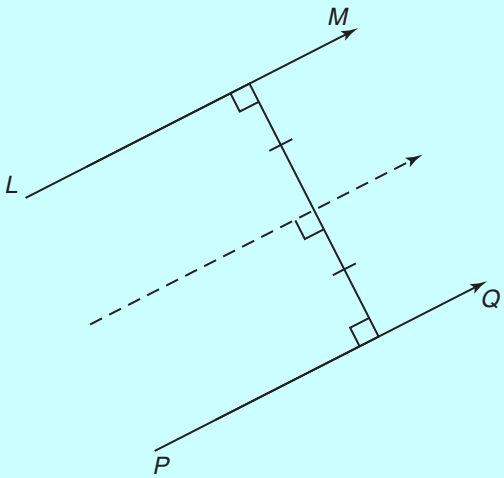
3 Construct a line through *B* that is perpendicular to *AC*.

4 Describe the following loci in words

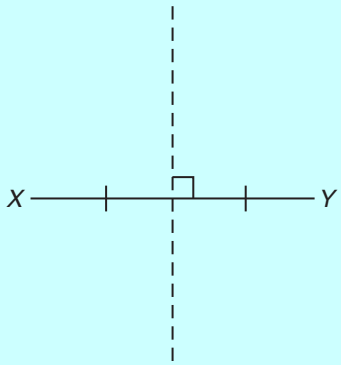
a



b



c



Draw three rectangles all labelled *ABCD* in which $AB = CD = 5$ cm and $BC = DA = 3$ cm.

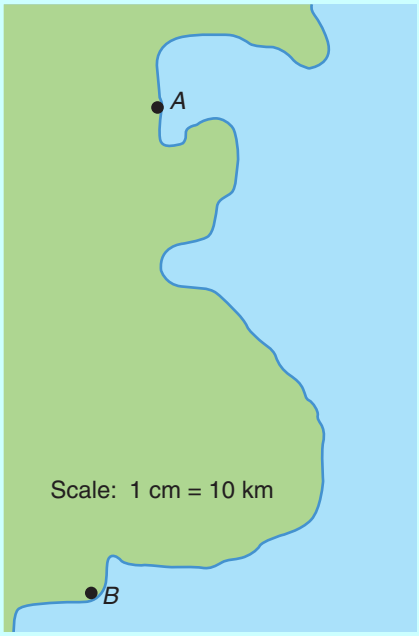
Use these rectangles to answer questions 5, 6 and 7.

5 Draw the locus of all points inside the rectangle that are closer to *AB* than *CD*.

6 Draw the locus of all points inside the rectangle that are more than 3 cm from *A*.

7 Draw the locus of all points inside the rectangle that are closer to *A* than *C*.

8 A piece of coastline is shown. The last know position of a ship that has sunk off shore was 40 km from port A and 60 km from port B. Indicate the area in which the rescuers should search for survivors.



assignment
11B

Chapter 11 | Working Mathematically

1 Paul is on holiday. He is travelling from New Zealand to several countries and has been advised to travel with US dollars, so he changes all his New Zealand dollars to US dollars.

His first stop is Thailand where he changes US$300 to Thai Baht. When he leaves Thailand he has Thai Baht 6300 left. After Thailand he travels to Europe where he changes all the money he has to Euros (€). If Paul had NZ$ 2900 at the start, how many Euros will he have when he gets to Europe?

Exchange rates are:
US$1 = NZ$1.45
US$1 = € 0.77
US$1 = Thai Baht 35
€1 = Thai Baht 45

2 In a money box there are only 10-cent pieces, 50-cent pieces and dollar coins. The ratio of 10-cent pieces to 50-cent pieces is 5:7. The ratio of 50-cent pieces to 1 dollar coins is 3:5. If there is 3 dollars in 10-cent pieces, how much money is in the box?

3 When she is 120 km from her destination, Elspeth has completed 87.5% of her journey. How far has she travelled so far?

4 A garden measures 5 m long and 8 m wide. It has a 1.5 m path along its two short sides and one long side. Find the area of the path.

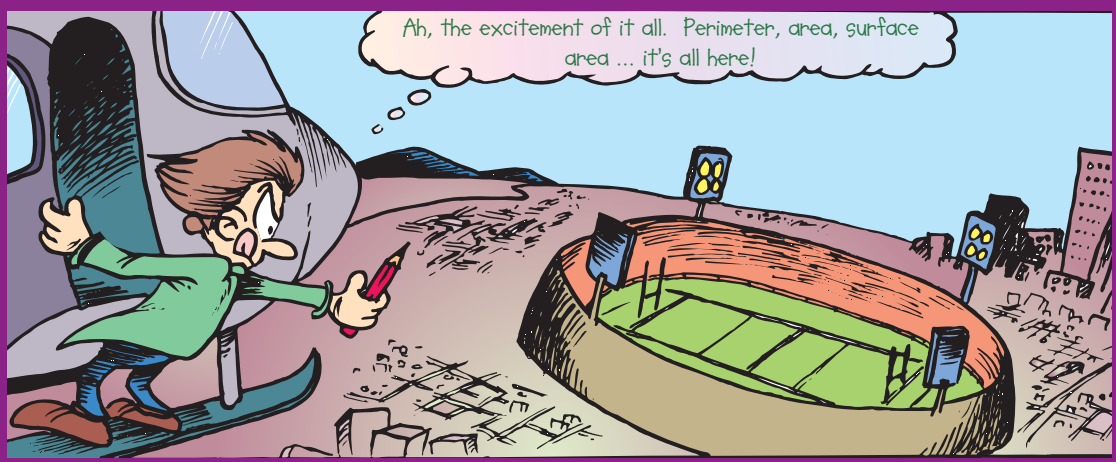
5 Find **two** pairs of consecutive numbers that have a product of 650.

6 In Jim's class there are 24 students. Three of the sports available at Jim's school are swimming, basketball and football. Eleven students do swimming, 9 play basketball and 15 play football. Three students swim and play basketball, 7 swim and play football, 6 play both football and basketball while 2 students participate in all three activities.

How many students are not involved in any of these sports?

12

Perimeter, Area and Surface Area



Chapter Contents

Learning Outcomes

Students will be able to:

- Use formulae and Pythagoras' theorem in calculating perimeter and area of circles and figures composed of rectangles and triangles.
- Calculate the surface area of rectangular and triangular prisms of right prisms and cylinders.
- Use formulae to calculate the area of quadrilaterals and find area and perimeter of simple composite figures.

Areas of Interaction

Approaches to Learning, Homo Faber, Environment

12:01 | Review of Perimeter

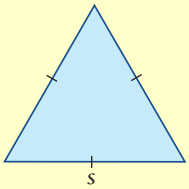
- The ability to calculate perimeters is necessary when calculating the stagger in staggered starts.

The perimeter of a plane figure is the length of its boundary.

To calculate the perimeter:
- find the lengths of all the sides
- add the lengths together

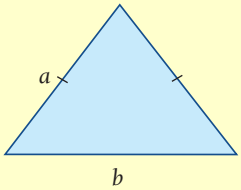
The geometric properties of some figures allow the perimeter to be calculated using a simple formula.

Equilateral triangle



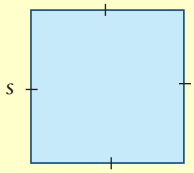
$P = 3s$

Isosceles triangle



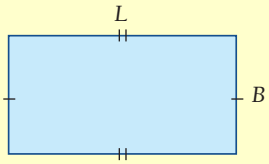
$P = 2a + b$

Square



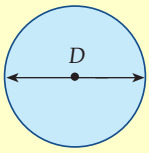
$P = 4s$

Rectangle



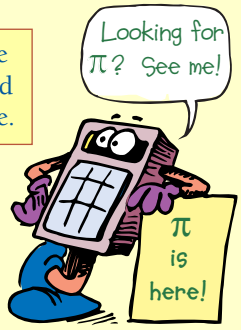
$P = 2L + 2B$
$= 2(L + B)$

Circle



$C = \pi D$

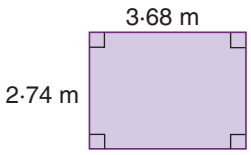
For a circle the perimeter is called the circumference.



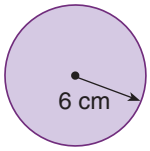
worked examples

Find the perimeters of the following figures.

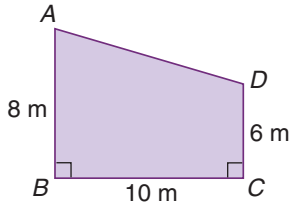
1



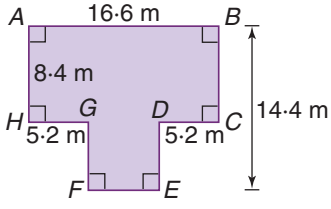
2



3



4

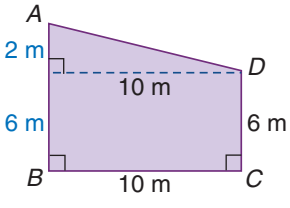


Solutions

1 Perimeter $= 2 \times (3{\cdot}68 + 2{\cdot}74)$
$= 2 \times (6{\cdot}42)$
$= 12{\cdot}84$ m

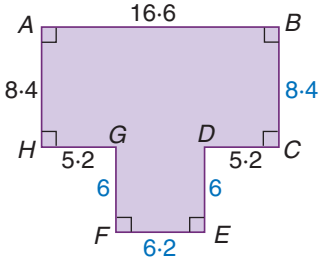
2 Perimeter = circumference
$= \pi D$
$= \pi \times 12$
$= 37{\cdot}7$ cm (correct to 1 decimal place)

3



Now, $AD^2 = 2^2 + 10^2$
$= 104$
$\therefore AD = \sqrt{104}$
$\therefore$ Perimeter $= 6 + 10 + 8 + \sqrt{104}$
$= 34{\cdot}20$ m (correct to 2 decimal places)

4



$5{\cdot}2 + FE + 5{\cdot}2 = 16{\cdot}6$
$FE = 6{\cdot}2$

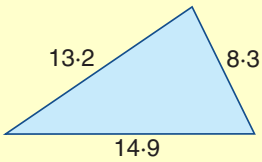
$8{\cdot}4 + GF = 14{\cdot}4$
$\therefore GF = 6$
$BC = 8{\cdot}4$
$DE = 6$

Perimeter = sum of horizontal sides
+ sum of vertical sides
Sum of horizontal sides $= 16{\cdot}6 + 5{\cdot}2 + 5{\cdot}2 + 6{\cdot}2$
$= 33{\cdot}2$
Sum of vertical sides $= 8{\cdot}4 + 6 + 8{\cdot}4 + 6$
$= 28{\cdot}8$
$\therefore$ Perimeter $= 33{\cdot}2 + 28{\cdot}8$
$= 62$ m

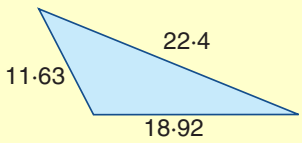
Exercise 12:01

1 Find the perimeter of each of the following figures. All measurements are in metres unless indicated otherwise.

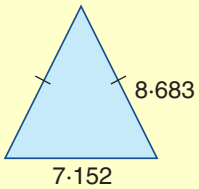
a



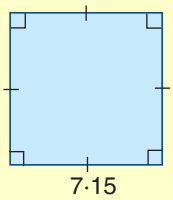
b



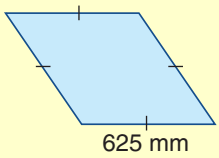
c



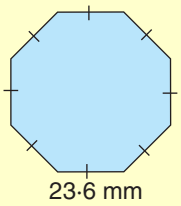
d



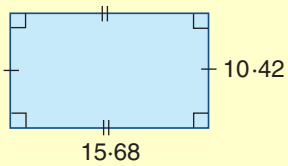
e



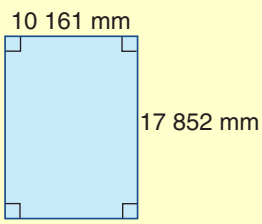
f



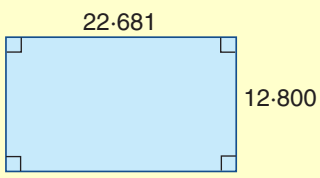
g



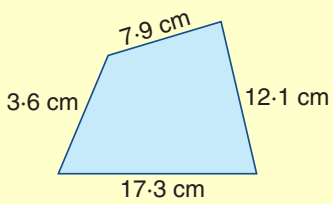
h



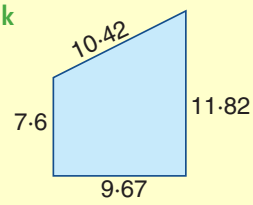
i



j



k



2 **a** An equilateral triangle has a side of length 4·68 m. What is its perimeter?

b Find the perimeter of a square that has a side of 563 mm.

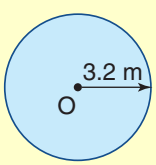
c A rectangle is measured and found to be 45·6 cm long and 23·4 cm wide. Find its perimeter.

d A rectangle is 7162 mm long and 4168 mm wide. Find its perimeter.

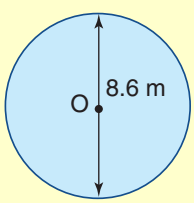
e A rectangle is measured and its dimensions are found to be: length 5·162 m, width 2·610 m. Find its perimeter.

3 Calculate the circumference of the following circles giving all answers correct to 3 significant figures.

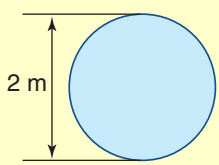
a



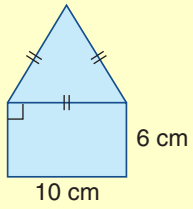
b



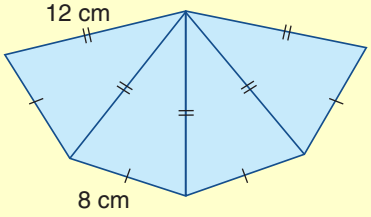
c



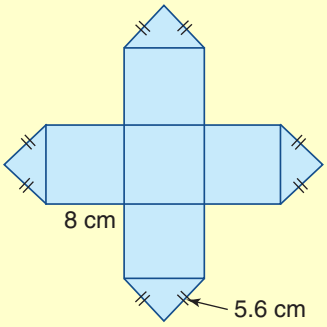
4 **a** A pentagon is made by placing an equilateral triangle on top of a rectangle. What is the perimeter of the pentagon?



b A hexagon is made by joining four isosceles triangles as shown in the diagram. What is the perimeter of the hexagon?

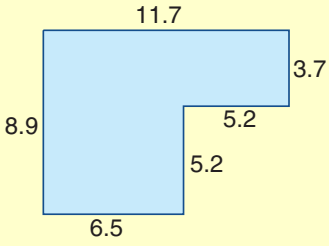


c Five squares and four isosceles triangles are used to make the figure shown. What is the perimeter of the figure?

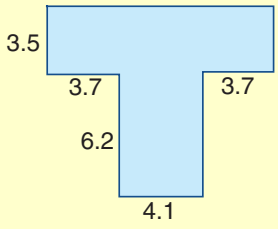


5 Calculate the perimeter of the following figures. All angles are right angles and all measurements are in metres unless indicated otherwise.

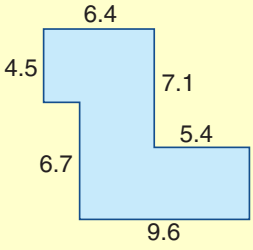
a



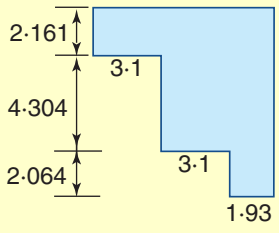
b



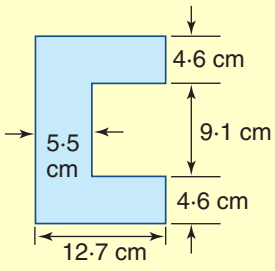
c



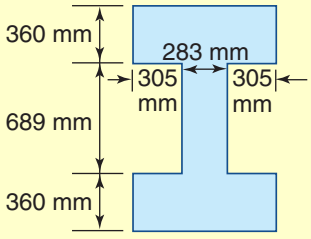
d



e



f



Remember: **Work out the lengths of all missing sides first.**

6 **a** A farmer has a field 450 m long and 255 wide. Find the perimeter of this field. How much would it cost to fence this field at 50c per metre?

b A jogger runs around a rectangular park that is 420 m long and 95 m wide. How far will he jog if he runs around the park 10 times?

c A building block is rectangular in shape. It is 26·4 m wide and 50·4 m deep. The block is to be fenced using welded fence panels. If the panels only come in lengths of 2·4 m, how many will be needed to fence the block?

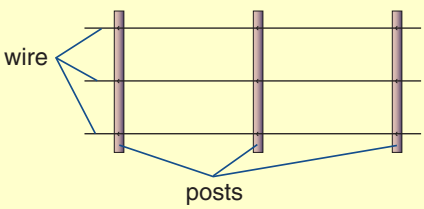
7 **a** The wheel of a steamroller has a diameter of 2·5 m. How far will the steamroller move if this wheel makes one revolution? Give your answer correct to 1 decimal place.

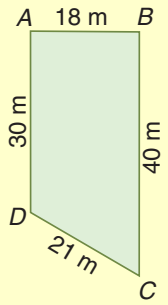
b The diameter of a car wheel is 35 cm. How many complete revolutions will this wheel need to make to travel 1 km? (*Hint:* Keep all units of length the same.)

c A circular bicycle track has a diameter of 125 m. Find how far a cyclist covers in completing one lap of the track. Give your answer to the nearest metre.

d If a circular running track has a diameter of 140 m, find how many complete laps must be run to cover at least 10 000 m.

8 A block of land (as shown in the diagram) is to be fenced with a fence of the type shown below.





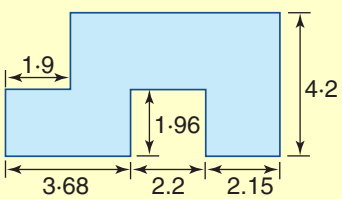
a Calculate the amount of wire needed to complete the fence.

b If posts are to be placed at 3 m intervals (as a maximum), calculate the number of posts needed for the fence.

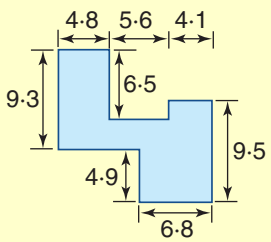
c Calculate the cost of the fence if the posts are $8.50 each and the wire is 95 cents a metre.

9 Find the perimeter of the following figures. All angles are right angles and all measurements are in metres.

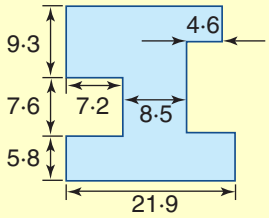
a



b



c



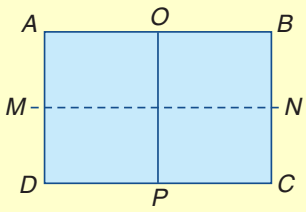
- The photo shows a floor plan like the ones above. The timber formwork is being erected ready for the pouring of the concrete.

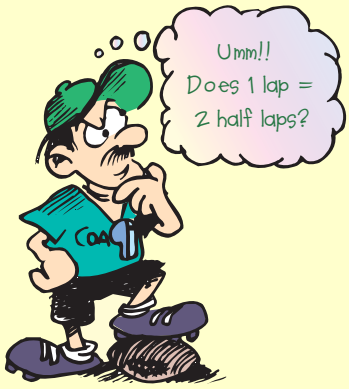
d A rugby field is rectangular in shape. It has a length of 90 m and a width of 60 m.

A coach arrived and told his team to run around the field 10 times to warm up. The team complained that this was too much, so the coach relented and told them to run around half of the field 15 times.

Some decided to run the half of the field *AOPDA*, while others decided to run the half *ABNMA*. Find:

- **i** the distance covered by those who followed the half *AOPDA*
- **ii** the distance covered by those who followed the half *ABNMA*
- **iii** the distance they would have run if they had followed the coach's original instruction to run 10 laps of the field *ABCDA*

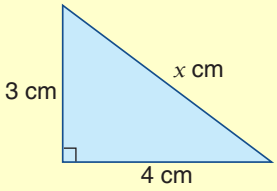




Using Pythagoras' theorem

10 Use Pythagoras' theorem to find the value of x and then calculate the perimeter.

a



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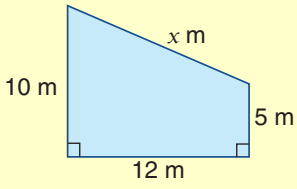
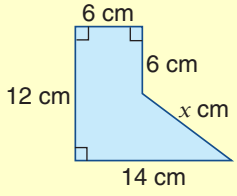
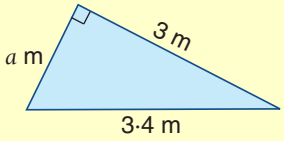


c

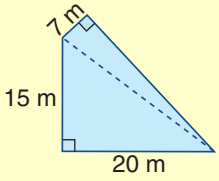


11 Calculate the perimeter of each of the following figures. (You will need to use Pythagoras' theorem to find a missing side.)

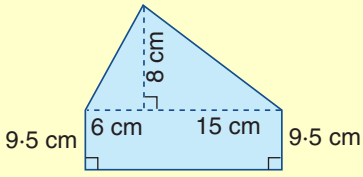
a



b

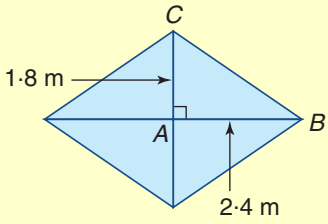


c



12 **a** The rhombus shown is made from four identical right-angled triangles. What is its perimeter?

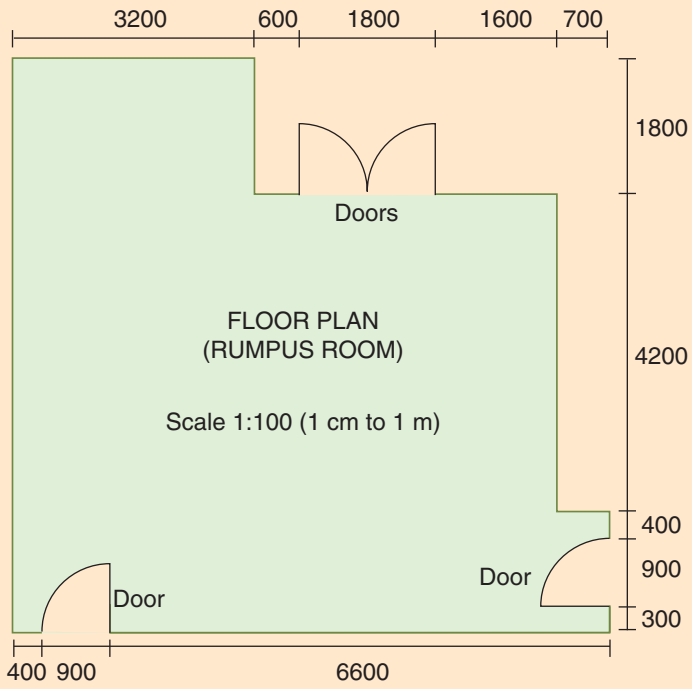
b A rhombus has diagonals that are 14·4 cm and 19·2 cm long. What is its perimeter?



investigation 12:01

Investigation 12:01 | Skirting board and perimeter

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.



- The diagram is a scale drawing of a rumpus room in a house. All measurements are in millimetres. Shehan Silva, a local builder, has been asked to fit skirting board to the room. (Skirting board is used to cover the gap between the wall and the floor of a building.)
- The skirting board is to be placed around the perimeter of the room except for the doorways. It can be ordered in lengths from 300 mm to 6·6 m at increments of 300 mm (ie 300 mm, 600 mm, 900 mm, 1·2 m and so on, up to 6·6 m).

Exercises

1 Shehan has been asked to do the job without any joins (except at corners). Is this possible? Give reasons for your answer.

2 What is the total length of skirting board required?

3 Shehan has nine 3·3 m lengths of skirting left from earlier jobs. Show how he could use these to do the job. What is the smallest number of joins he could have?

4 If Shehan has no skirting board, what would he need to order to complete the job with the smallest number of joins and the smallest amount of waste?

Assessment Grid for Investigation 12:01 | Skirting board and perimeter

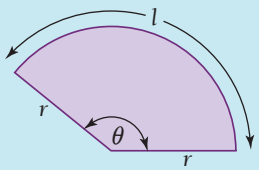
The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	An organised approach has not been used. The student has experienced difficulty performing routine calculations.	1	
			2	
	b	An organised approach has been attempted, ie diagrams and basic calculations have been used to answer part 2.	3	
			4	
	c	An organised approach has been used and conclusions for parts 1 and 3 have been described.	5	
			6	
	d	The correct results have been obtained through effective problem solving strategies, with sufficient justification throughout.	7	
			8	
Criterion C Communication	a	No working out is shown and presentation is poor with little or no use of diagrams or symbols.	1	
			2	
	b	Working out is shown with some explanation. Presentation is good with some structure to the work.	3	
			4	
	c	Work is well structured showing a clear progression and is well communicated using diagrams, words and symbols.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt has been made to explain the method used and to check the results.	1	
			2	
	b	The method used is justified and the results have been checked for reasonableness with some success.	3	
			4	
	c	Detailed explanations of the methods used have been given, and the reasonableness of the results obtained are evaluated and verified through further practical applications.	5	
			6	

12:02 | Perimeter of Sectors and Composite Figures

- A sector is part of a circle. It is bounded by two radii and an arc.
- To find the arc length of a sector, l, first find what fraction the sector is of the circle by dividing the sector angle θ by 360°. Then find this fraction of the circumference.

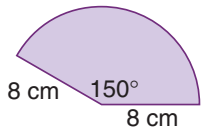
$$l = \frac{\theta}{360°} \times 2\pi r$$



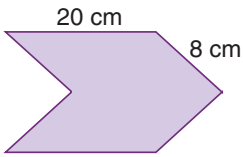
- Composite figures are formed by putting simple figures together or by removing parts of a figure. The calculation of the perimeter of composite figures is shown in the examples below.

worked examples

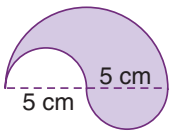
1 Find the perimeter of the sector in the diagram.



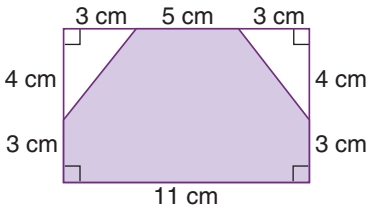
2 Find the perimeter of the 'arrow' made from joining two identical parallelograms.



3 Find the perimeter of the figure formed from three semicircles.



4 A rectangle has two of its corners removed to form a hexagon. Find its perimeter.



Solutions

1 Arc length $= \frac{150°}{360°} \times 2\pi \times 8$

$= 20{\cdot}94$ cm (correct to 2 decimal places)

$\therefore$ Perimeter $= 20{\cdot}94 + 2 \times 8$

$= 36{\cdot}94$ cm (correct to 2 decimal places)

2 Perimeter $= (2 \times 20) + (4 \times 8)$

$= 72$ cm

3 Arc length of large semicircle $= \frac{\pi D}{2}$

$= \frac{\pi \times 10}{2}$

$= 5\pi$

Arc length of small semicircle $= \frac{\pi \times 5}{2}$

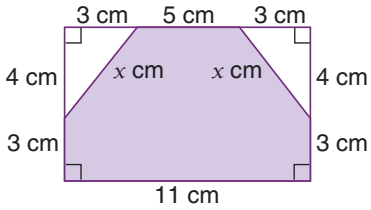
$= 2{\cdot}5\pi$

$\therefore$ Perimeter $= 5\pi + 2 \times 2{\cdot}5\pi$

$= 10\pi$

$= 31{\cdot}4$ cm (correct to 1 decimal place)

4



$x^2 = 3^2 + 4^2$ (Pythagoras' theorem)

$= 25$

$x = \sqrt{25}$

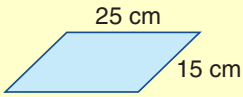
$= 5$

$\therefore$ Perimeter $= (11) + (2 \times 3) + (2 \times 5) + (5)$

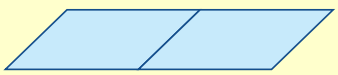
$= 32$ cm

Exercise 12:02

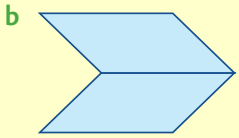
1 Identical parallelograms like the one shown have been joined to make the shapes below. Calculate the perimeter of each shape.



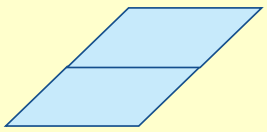
a



b

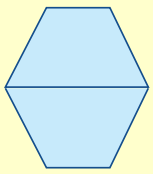


c

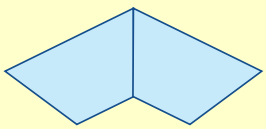


2 Tiles like the one shown were used to make the shapes below. Find the perimeter of each shape.

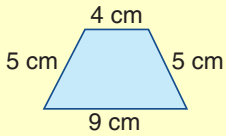
a



b

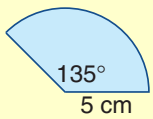


c

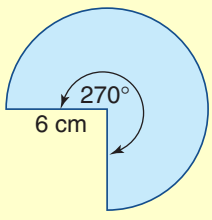


3 Use the formula $l = \frac{\theta}{360^\circ} \times 2\pi r$ to find the arc length of each sector. Answer correct to 1 decimal place.

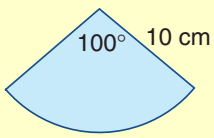
a



b



c



Foundation Worksheet 12:02

Perimeter

1 Find the perimeter of:

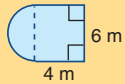
a

b

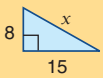
2 Find the perimeter of:

a

b



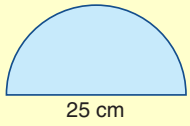
3



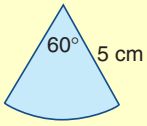
a Use Pythagoras' theorem to find x.

b Find the perimeter of the figure.

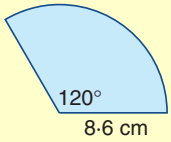
d



e



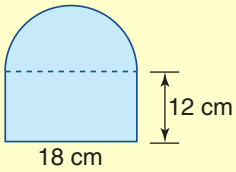
f



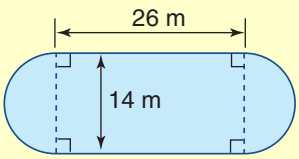
4 Find the perimeter of the sectors in question **3**.

5 Calculate the perimeter of the following composite figures.

a



b



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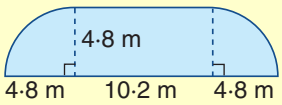
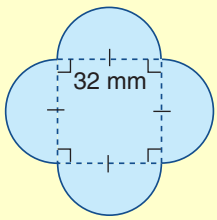
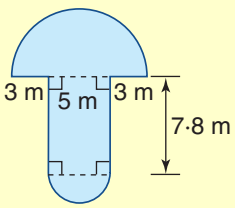


d



e



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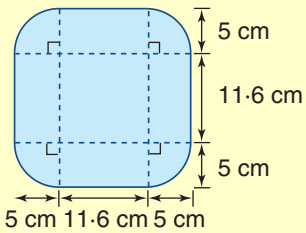
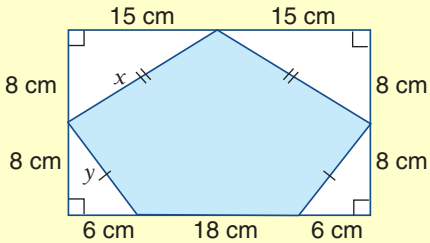
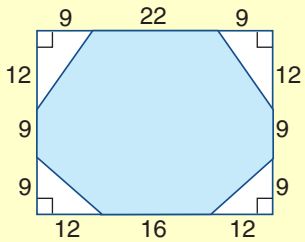


6 **a** Use Pythagoras' theorem to find the value of the pronumerals.

b Calculate the perimeter of the pentagon.

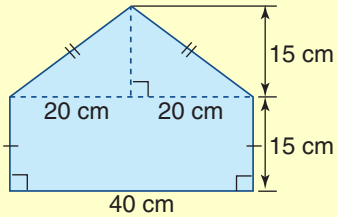


7 A rectangle has four of its corners cut off to form an irregular octagon. What is the perimeter of the octagon? (Measurements are in centimetres.)

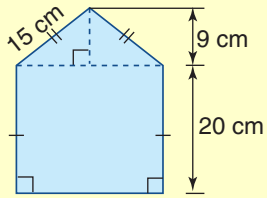


8 Find the perimeter of the pentagons below.

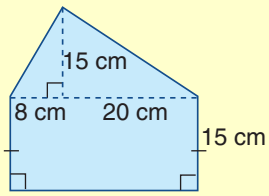
a



b

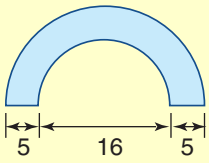


c

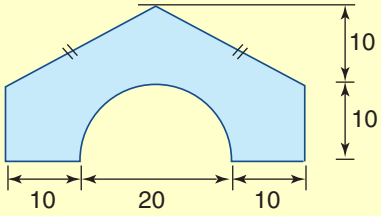


9 Find the perimeter of the figures below. Measurements are in centimetres. (Answer correct to 1 decimal place.)

a



b



fun spot

12:02

Fun Spot 12:02 | Why is it so noisy at tennis?

Answer each equation and put the letter for that question in the box above the correct answer.

A $(0{\cdot}1)^2$	**A** $6 - 1{\cdot}2$	**A** $0{\cdot}4 \div 0{\cdot}1$	**A** 120% of 10
A $\frac{1}{2} + \frac{1}{4}$	**A** $\frac{2}{3} \times 9$	**B** $7 \div \frac{1}{2}$	**C** $3 + 7 \times 2$
C $16 - 5 + 4$	**C** $24 \div (6 \div 2)$	**E** $(10 - 3)^2$	**E** $\frac{1}{2} - 0{\cdot}5$
E $6a + 2a$	**E** $7a - 2a$	**E** $8a - a$	**E** $3 \times 4a$
H $a \times 3^2$	**I** $12a \div 2$		

K Evaluate $4x - 5$ if $x = 3$.

L Evaluate a^2 if $a = -3$.

P Simplify $5(2a - 4) + 20$.

R Simplify $6a + 3b - 5a + 3b$.

R Simplify $a(a + 8) - 3(a - 2)$.

R Simplify $a(a + 8) - a(a - 8)$.

S Simplify $\frac{8}{a} \times \frac{a}{8}$.

S Simplify $3 - 2(a - 8)$.

S Simplify $a(a - 1) - a^2$.

T Simplify $\frac{3a}{5} + \frac{2a}{5}$.

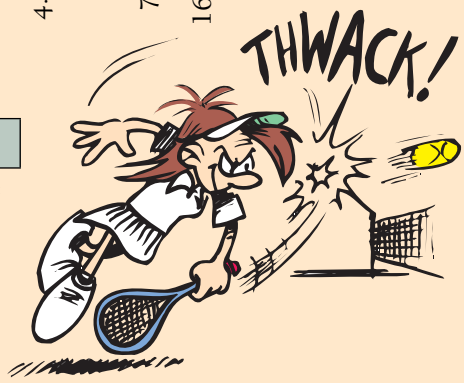
Of 15 people, 9 wore sand shoes, 8 wore shorts and 3 wore neither sandshoes nor shorts.

U How many people wore both sandshoes and shorts?

Y How many people wore shorts but not sandshoes?

14	$12a$	17	12	5	1	$5a$	$8a$	0·01	15	$9a$	$10a$	9	4·8	3	$7a$	$16a$

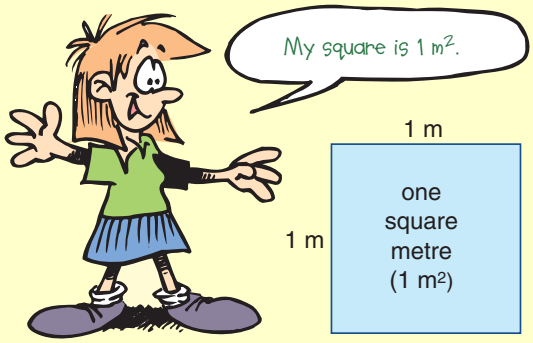
$a + 6b$	$\frac{3}{4}$	$6a$	$19 - 2a$	0	$-a$	6	$a^2 + 5a + 6$	4	8	7	49	a



12:03 | Review of Area

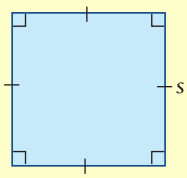
Measurement of area

- The area of a plane figure is the amount of space it occupies.
- Area is measured by calculating how many squares it would take to cover the figure. Small squares are used to measure small areas and large squares are used to measure large areas. It should not be surprising then that the units for measuring area are called square units.
- 1 cm^2 is the area within a square with 1 cm sides.
  1 m^2 is the area within a square with 1 m sides.
  1 ha is the area within a square with 100 m sides.
  1 km^2 is the area within a square with 1 km sides.
- Area is calculated using a formula.



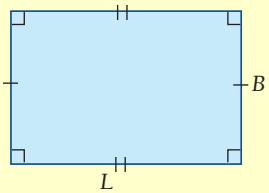
Area formulae

Square



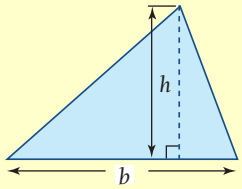
$A = s^2$

Rectangle



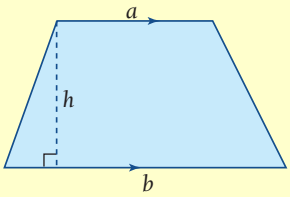
$A = LB$

Triangle



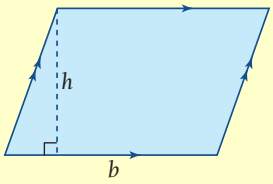
$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

Trapezium



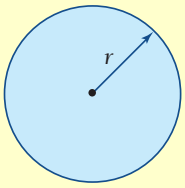
$A = \frac{1}{2}h(a + b)$

Parallelogram



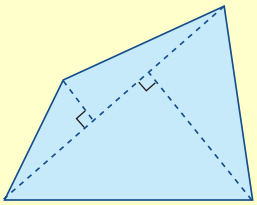
$A = bh$

Circle



$A = \pi r^2$

Quadrilateral



There is no formula. The area is found by joining opposite corners to form two triangles.

The area of each triangle is calculated and the two areas added to give the area of the quadrilateral.

prep quiz

12:03

1 $\frac{1}{2} \times 10{\cdot}6 \times 4{\cdot}8$

2 $\dfrac{3{\cdot}4 \times 1{\cdot}2}{2}$

3 $\frac{1}{2} \times 4{\cdot}6 \times (10{\cdot}7 + 3{\cdot}5)$

4 Simplify $L \times B$

Evaluate:

5 LB if $L = 4$ and $B = 5$

6 $\frac{1}{2}bh$ if $b = 5$ and $h = 8$

7 $\dfrac{bh}{2}$ if $b = 5$ and $h = 8$

8 ab^2 if $a = 3$ and $b = 4$

Complete the following:

9 1 m = . . . cm

10 1 cm = . . . mm

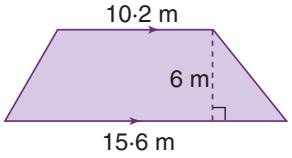
worked examples

1 Find the area of a square with sides of 3·1 m.

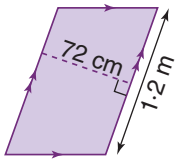
2 Find the area of a circle that has a radius of 5 m (take $\pi = 3{\cdot}14$).

3 Find the area of the following figures.

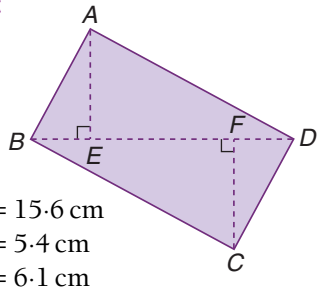
a



b



c



$BD = 15{\cdot}6$ cm
$AE = 5{\cdot}4$ cm
$FC = 6{\cdot}1$ cm

Solutions

1 $A = s^2$
$= (3{\cdot}1)^2$
$= 9{\cdot}61 \text{ m}^2$

2 $A = \pi r^2$
$= 3{\cdot}14 \times 5^2$
$= 3{\cdot}14 \times 25$
$= 78{\cdot}5 \text{ m}^2$

3 a $A = \frac{1}{2}h(a + b)$
$= \frac{1}{2} \times 6(10{\cdot}2 + 15{\cdot}6)$
$= 3(25{\cdot}8)$
$= 77{\cdot}4 \text{ m}^2$

Make sure all dimensions are measured in the same units.

b $A = bh$
$b = 1{\cdot}2$ m
$h = 72$ cm
$= 0{\cdot}72$ m
$\therefore A = 1{\cdot}2 \times 0{\cdot}72$
$= 0{\cdot}864 \text{ m}^2$
or
$b = 120$ cm
$h = 72$ cm
$A = 120 \times 72$
$= 8640 \text{ cm}^2$

c Area of $ABCD$
= area of ΔABD +
area of ΔBCD
Area of ΔABD
$= \dfrac{^{7{\cdot}8}\cancel{15{\cdot}6} \times 5{\cdot}4}{\cancel{2}_1}$
$= 7{\cdot}8 \times 5{\cdot}4$
$= 42{\cdot}12 \text{ cm}^2$
Area of ΔBCD
$= \dfrac{^{7{\cdot}8}\cancel{15{\cdot}6} \times 6{\cdot}1}{\cancel{2}_1}$
$= 7{\cdot}8 \times 6{\cdot}1$
$= 47{\cdot}58 \text{ cm}^2$
$\therefore$ Area of $ABCD$
$= 42{\cdot}12 + 47{\cdot}58$
$= 89{\cdot}7 \text{ cm}^2$

Exercise 12:03

1 Find the area of squares which have the following sides.

a 7 cm **b** 12·7 m **c** 0·75 m

2 Find the area of rectangles which have the following lengths and breadths.

a $L = 15$ m, $B = 4$ m **b** $L = 17$ cm, $B = 5{\cdot}2$ cm **c** $L = 25{\cdot}6$ m, $B = 7{\cdot}9$ m

d $L = 1{\cdot}5$ m, $B = 50$ cm **e** $L = 54$ cm, $B = 2{\cdot}2$ m **f** $L = 7{\cdot}1$ cm, $B = 25$ mm

3 Find the area of circles which have the following radii or diameters. Give your answer correct to 2 decimal places.

a $r = 6$ cm **b** $r = 3{\cdot}5$ cm **c** $r = 2{\cdot}6$ m

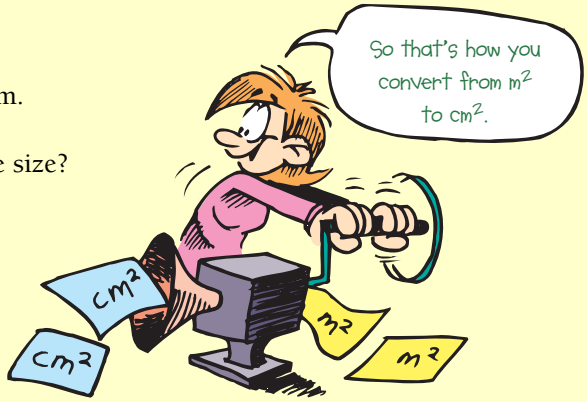
d $d = 7$ m **e** $d = 9{\cdot}1$ m **f** $d = 0{\cdot}9$ m

4 **a** Find the area of a square with side 1 m. (Answer in m^2.)

b Find the area of a square with side 100 cm. (Answer in cm^2.)

c Are the squares in parts **a** and **b** the same size?

d From your answers to parts **a**, **b** and **c** complete the following: $1\ m^2 = \ldots\ cm^2$.

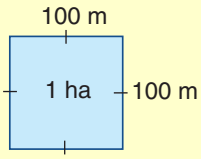


5 **a** A square of side 100 m has an area of 1 hectare (ha). How many square metres does it take to make 1 ha?

b A rectangle has a length of 700 m and a width of 300 m. What is the area of this rectangle in hectares?

c A rectangle is 750 m long and 420 m wide. Find its area in hectares.

d The length and width of a rectangle are 220 m and 75 m respectively. Calculate the area of this rectangle in square metres and hectares.



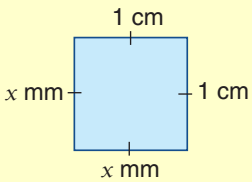
6 **a** What is the value of x?

b Calculate the area of the square in cm^2.

c Calculate the area of the square in mm^2.

d Complete the conversion below.

$1\ cm^2 = \ldots\ mm^2$

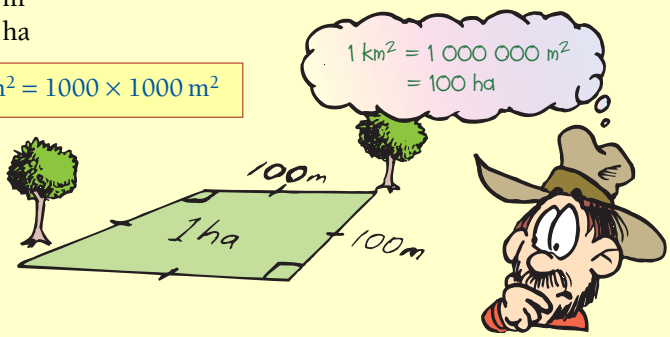


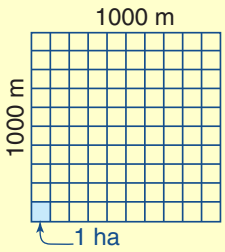
7 Complete the following.

a $1\ km^2 = \ldots\ m^2$

b $1\ km^2 = \ldots$ ha

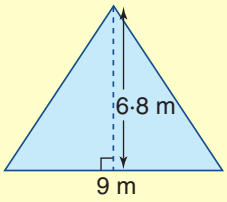
■ $1\ km^2 = 1000 \times 1000\ m^2$



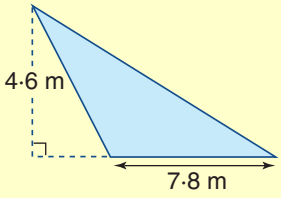


8 Calculate the areas of the following figures.

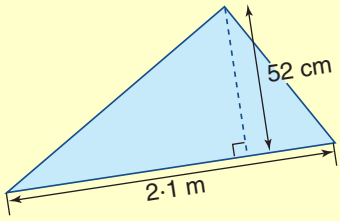
a



b

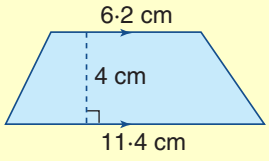


c

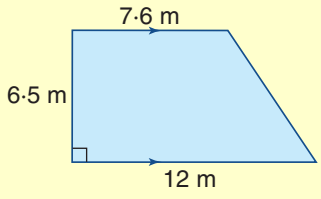


9 Find the area of each of the following trapeziums.

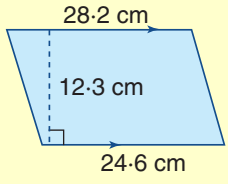
a



b

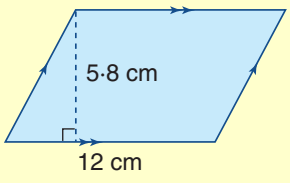


c

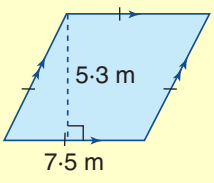


10 Calculate the area of each of the following figures.

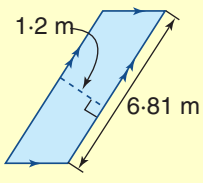
a



b

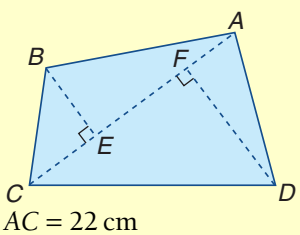


c



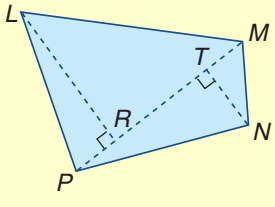
11 Work out the area of each of the following quadrilaterals.

a



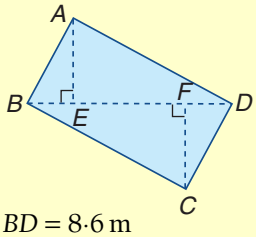
$AC = 22$ cm
$BE = 6$ cm
$FD = 16$ cm

b



$MP = 16{\cdot}8$ m
$LR = 7{\cdot}4$ m
$TN = 2{\cdot}3$ m

c



$BD = 8{\cdot}6$ m
$AE = 3{\cdot}7$ m
$CF = 6{\cdot}85$ m

challenge

12:03

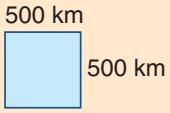
Challenge 12:03 | Area of scale drawings

1 The map below has a scale of 1 cm = 500 km. On the map, what area is represented by 1 cm^2?

2 Estimate the area of each of the South American countries shown on this map.

3 Estimate the distances across Brazil and Argentina from east to west and from north to south.

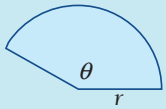
4 Find a map of Australia and use its scale to find the area.



12:04 | Area of Sectors and Composite Figures

- To find the area of a sector, first find what fraction the sector is of the circle by dividing the sector angle θ by 360°. Then find this fraction of the area of the circle.

$$\text{Area} = \frac{\theta}{360} \times \pi r^2$$



- The area of composite figures can be calculated by either of the two methods.

Method I (by addition of parts)

We imagine that smaller regular figures have been joined to form the figure, as in Figures 1 and 2.

1. Copy the figure.
2. Divide the figure up into simpler parts. Each part is a shape whose area can be calculated directly, eg square or rectangle.
3. Calculate the area of the parts separately.
4. Add the area of the parts to give the area of the figure.

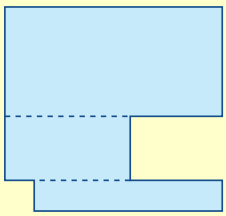


Figure 1

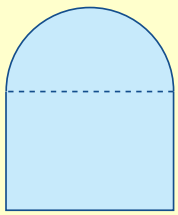
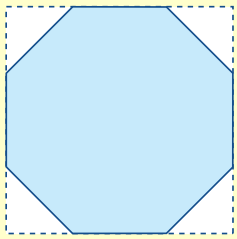


Figure 2

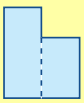
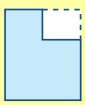
Method II (by subtraction)

We imagine the figure is formed by cutting away simple shapes from a larger complete figure, as shown.



1. Copy the figure and mark in the original larger figure from which it has been cut.
2. Calculate the area of the larger original figure.
3. Calculate the area of the parts that have been removed.
4. Area of figure = (area of original figure) − (area of parts that have been removed).

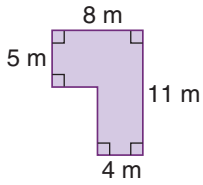
■ Think carefully before deciding which method to use.

1. ADDITION  or  2. SUBTRACTION

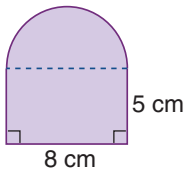
worked examples

Calculate the area of each figure. In 3 calculate the *shaded* area.

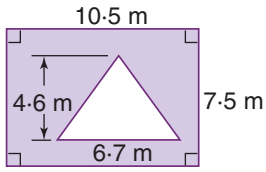
1



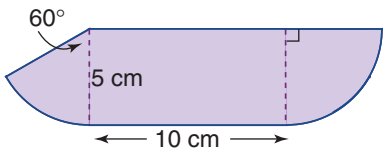
2



3

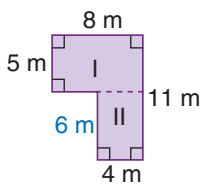


4



Solutions

1

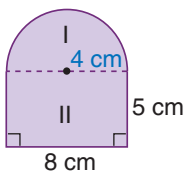


Area of rectangle I $= 8 \times 5$
$= 40\ \text{m}^2$

Area of rectangle II $= 6 \times 4$
$= 24\ \text{m}^2$

$\therefore$ Area of figure $= 40 + 24$
$= 64\ \text{m}^2$

2

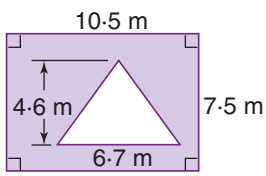


Area of semicircle I
$= \frac{1}{2}\pi r^2$
$= \frac{1}{2} \times \pi \times 16$
$= 25{\cdot}12\ \text{cm}^2$

Area of rectangle II
$= 8 \times 5$
$= 40\ \text{cm}^2$

$\therefore$ Area of figure
$= 40 + 25{\cdot}12$
$= 65{\cdot}12\ \text{cm}^2$
(correct to 2 decimal places)

3

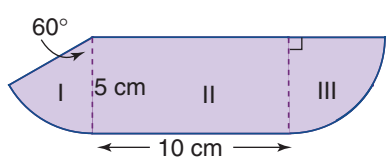


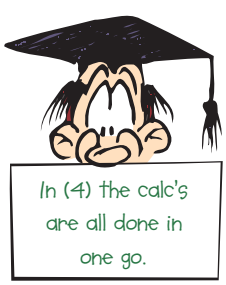
Area of rectangle $= 10{\cdot}5 \times 7{\cdot}5$
$= 78{\cdot}75\ \text{m}^2$

Area of triangle $= \dfrac{4{\cdot}6 \times 6{\cdot}7}{2}$
$= 15{\cdot}41\ \text{m}^2$

$\therefore$ Shaded area $= 78{\cdot}75 - 15{\cdot}41$
$= 63{\cdot}34\ \text{m}^2$

4





Area of figure = area of sector I + area of rectangle II + area of quadrant III

$= \left(\dfrac{60}{360} \times \pi \times 5^2\right) + (5 \times 10) + \left(\dfrac{1}{4} \times \pi \times 5^2\right)$

$= 82{\cdot}7\ \text{m}^2$ (correct to 1 decimal place)

Exercise 12:04

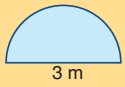
Foundation Worksheet 12:04

Area

1 Find the area of:

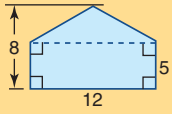
a

b

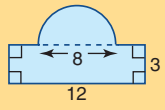


2 Find the area of:

a

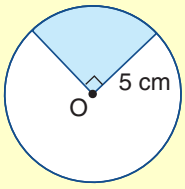


b

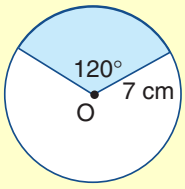


1 Use the formula $A = \frac{\theta}{360} \times \pi r^2$ to find the area of the sectors.

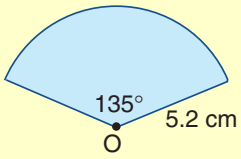
a



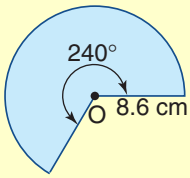
b



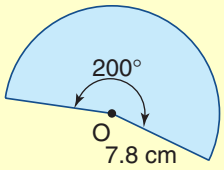
c



d

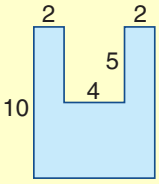


e

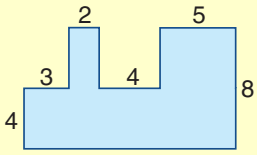


2 Calculate the area of each of the following figures. All measurements are in centimetres and all angles are right angles.

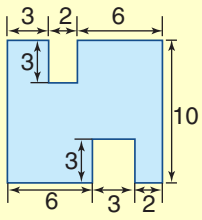
a



b

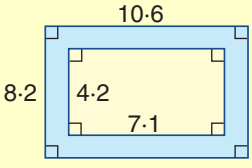


c

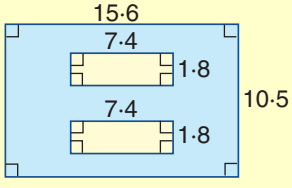


3 Calculate the shaded area of the following figures. All measurements are in metres.

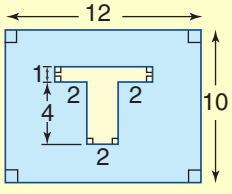
a



b

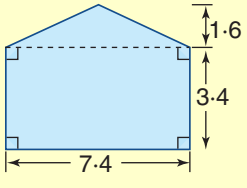


c

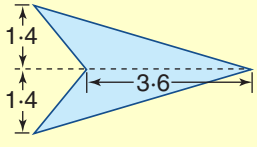


4 Calculate the area of each figure. All measurements are in metres.

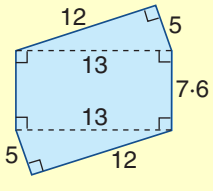
a



b

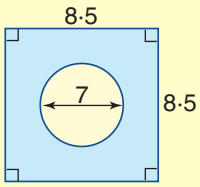


c

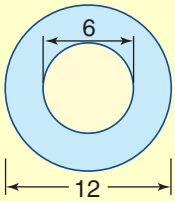


5 Calculate the area of each of the following figures. All measurements are in centimetres. Give answers correct to 2 decimal places.

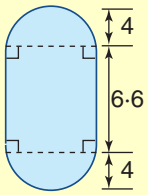
a



b

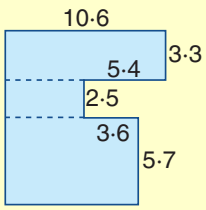


c

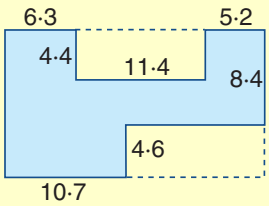


6 Calculate the area of the following composite figures. All angles are right angles and all measurements are in metres.

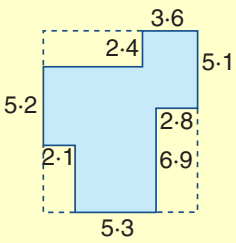
a



b

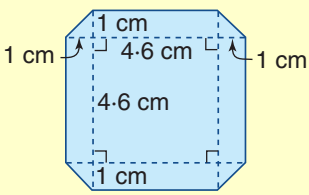


c

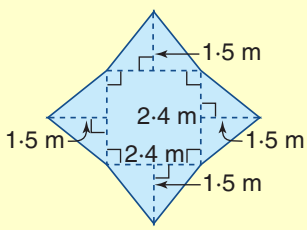


7 Find the area of each of the following composite figures.

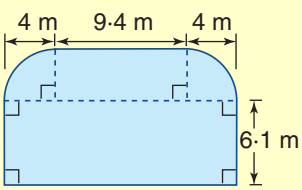
a



b

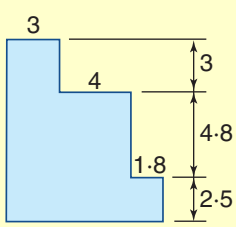


c

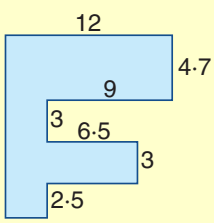


8 Calculate the area of the following figures. All angles are right angles and all measurements are in metres.

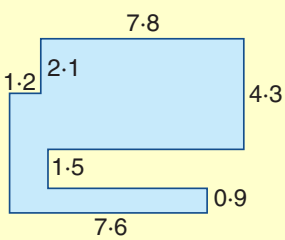
a



b

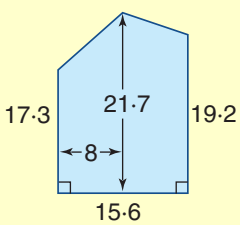


c

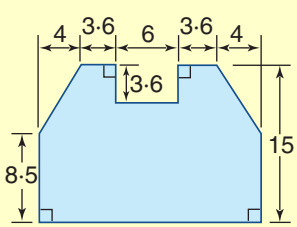


9 Calculate the area of the following figures. All measurements are in metres.

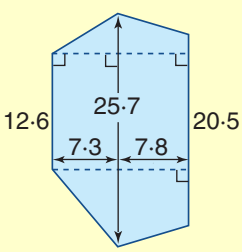
a



b

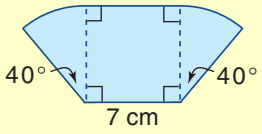


c

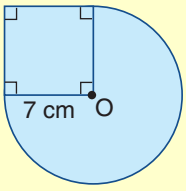


10 Find the area of these figures correct to the nearest cm^2.

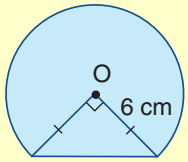
a



b



c



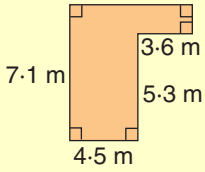
11 A bricklayer is asked to build a wall 10·5 m long and 1·6 m high. The wall is rectangular in shape.

a Find the area of the wall.

b If it is known that there are approximately 40 bricks per square metre, how many bricks are there in the wall?

c Find how much the bricks will cost if they are priced at $250 per thousand.

12 A floor is as shown in the diagram. Find the area of this floor and the cost of covering it with cork tiles if the cost of the tiles is $40 per m^2.

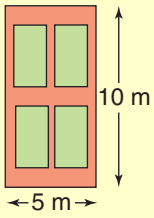


13 The diagram shows four garden beds, each 3·4 m long and 1·8 m wide, surrounded by paving bricks. Find:

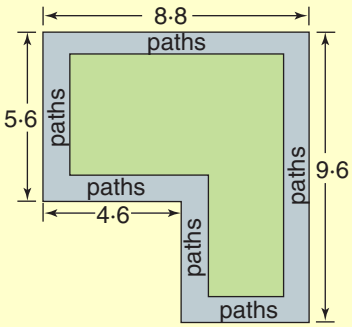
a the area of the garden beds

b the paved area

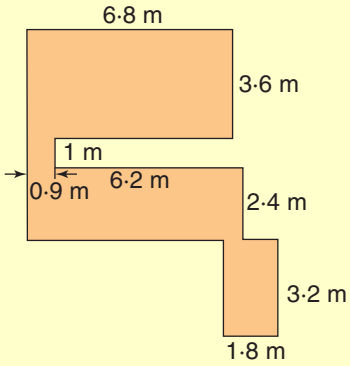
c the number of bricks needed to pave the area (rounding up to the next hundred) if a brick is 225 mm by 112 mm



14 Find the cost of concreting the paths in the diagram if concreting costs $50 per m^2. All paths are 0·9 m wide and all measurements are in metres.



15 The diagram below is the floor plan of a kitchen. This floor is to be covered with cork tiles. Before the cork tiles are laid, however, it is suggested that the floor be covered by sheets of hardboard underlay. These sheets are rectangular in shape and are 120 cm by 90 cm.



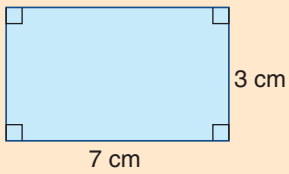
Find:
- **a** the area of the floor in square metres
- **b** the area of one sheet of hardboard
- **c** the number of sheets required to cover the floor (only whole sheets can be purchased)
- **d** the cost of the sheets of hardboard if the sheets cost $4.80 each
- **e** the cost of tiling the floor with cork at a cost of $35 per m^2

- Sectors are used to define the landing areas for shot put and discus.

12:05 | Surface Area of a Prism

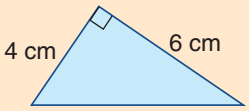
prep quiz 12:05

1



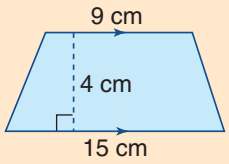
Area = . . .

2



Area = . . .

3



Area = . . .

For a cube:

4 How many faces are there?

5 What shape are the faces?

For a rectangular prism:

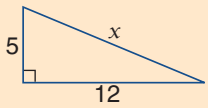
6 How many faces are there?

7 What shape are the faces?

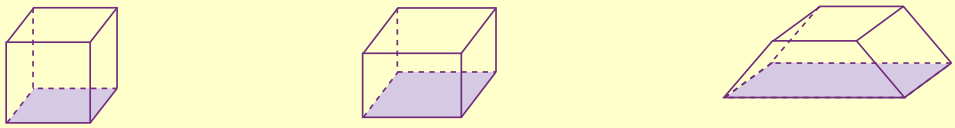
8 Which faces are congruent?

9 How many faces has a triangular prism?

10 Use Pythagoras' theorem to find x in this triangle.



If we look at solid shapes such as those pictured below, we can see that the faces of these solids are plane shapes.



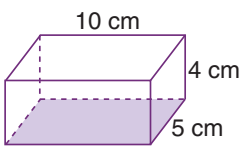
The surface area of a solid is the sum of the areas of its faces.

To calculate the surface area of the solid, you must know the number of faces and the shapes of the faces.

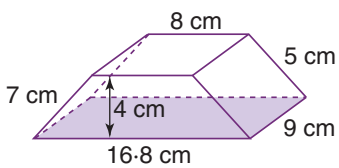
worked examples

Find the surface area of each of the following solids.

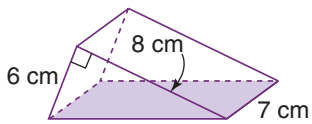
1



2



3



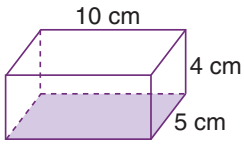
4 Find the surface area of a rectangular prism that is 6·5 m long, 2·4 m wide and 1·8 m high, if the prism is:

a closed **b** open

continued ➜➜➜

Solutions

1



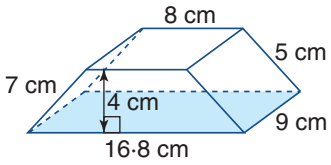
Area of top and bottom
$= (10 \times 5) + (10 \times 5)$
$= 50 + 50$
$= 100\text{ cm}^2$

Area of sides $= (5 \times 4) + (5 \times 4)$
$= 20 + 20$
$= 40\text{ cm}^2$

Area of front and back
$= (10 \times 4) + (10 \times 4)$
$= 40 + 40$
$= 80\text{ cm}^2$

$\therefore$ Surface area $= 100 + 40 + 80$
$= 220\text{ cm}^2$

2



Area of front $= \frac{1}{2}h(a + b)$
$= \frac{1}{2} \times 4 \times (16{\cdot}8 + 8)$
$= 2 \times 24{\cdot}8$
$= 49{\cdot}6\text{ cm}^2$

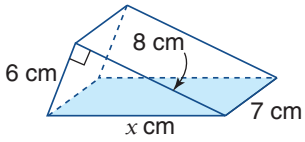
$\therefore$ Area of back $= 49{\cdot}6\text{ cm}^2$

Area of sides $= (9 \times 5) + (9 \times 7)$
$= 45 + 63$
$= 108\text{ cm}^2$

Area of top and bottom
$= (8 \times 9) + (16{\cdot}8 \times 9)$
$= 72 + 151{\cdot}2$
$= 223{\cdot}2\text{ cm}^2$

$\therefore$ Surface area $= 49{\cdot}6 + 49{\cdot}6 + 108 + 223{\cdot}2$
$= 430{\cdot}4\text{ cm}^2$

3 Calculate the width of the bottom using Pythagoras' theorem.



Now $x^2 = 6^2 + 8^2$
$\therefore x^2 = 100$
$\therefore x = 10$

Area of front $= \frac{6 \times 8}{2}$
$= 24\text{ cm}^2$

$\therefore$ Area of back $= 24\text{ cm}^2$

Area of sides $= (8 \times 7) + (6 \times 7)$
$= 56 + 42$
$= 98\text{ cm}^2$

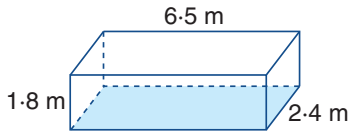
Area of bottom $= 10 \times 7$
$= 70\text{ cm}^2$

$\therefore$ Surface area $= 24 + 24 + 98 + 70$
Area $= 216\text{ cm}^2$

■ *Tips on Finding Surface Area*

1. Make a sketch of the solid. Find all necessary dimensions.
2. Calculate the area of the faces. Be systematic. Do front and back, top and bottom and sides.
3. Sum the area of the faces.
4. Check to make sure no face has been left out.

4 **a** Surface area of closed prism
$= 2 \times (6{\cdot}5 \times 2{\cdot}4) + 2 \times (6{\cdot}5 \times 1{\cdot}8) + 2 \times (2{\cdot}4 \times 1{\cdot}8)$
(top and bottom) + (front and back) + (sides)
$= 31{\cdot}2 + 23{\cdot}4 + 8{\cdot}64$
$= 63{\cdot}24\ m^2$



b Surface area of open prism
$= 2 \times (6{\cdot}5 \times 1{\cdot}8) + 2 \times (2{\cdot}4 \times 1{\cdot}8) + (6{\cdot}5 \times 2{\cdot}4)$
(front and back) + (sides) + (bottom)
$= 23{\cdot}4 + 8{\cdot}64 + 15{\cdot}6$
$= 47{\cdot}64\ m^2$

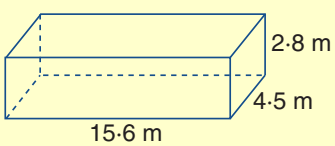
***Closed* prisms have a top.**
***Open* prisms have no top.**

Exercise 12:05

1 A cube has six square faces. Use this fact to find the surface area of a cube with a side length of:
- **a** 5 cm
- **b** 12·6 cm
- **c** 0·84 m

2 Copy and complete the following for the rectangular prism on the right.

Area of top and bottom = ...
Area of sides = ...
Area of front and back = ...
Total surface area = ...

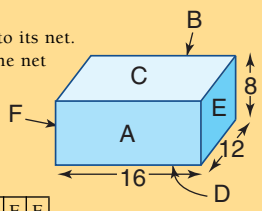


Foundation Worksheet 12:05

Surface area of prisms

1 **a** Match each solid to its net.
b Find the area of the net of the prism.

2 Complete the table shown for the rectangular prism shown.



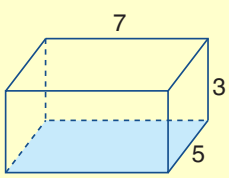
a

Face	A	B	C	D	E	F
Area						

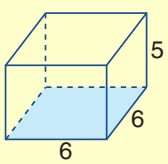
3 Find the surface area of each triangular prism.

3 Find the surface area of each of the following rectangular prisms. All measurements are in centimetres.

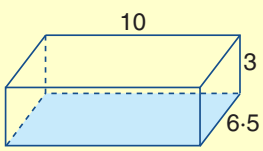
a



b

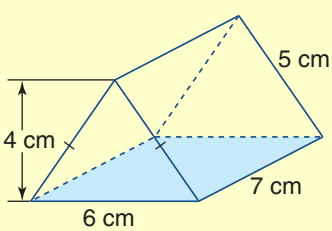


c

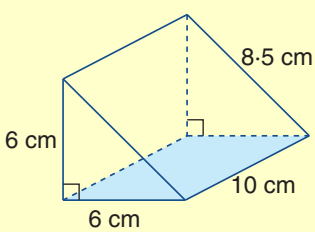


4 Find the surface area of each of these triangular prisms.

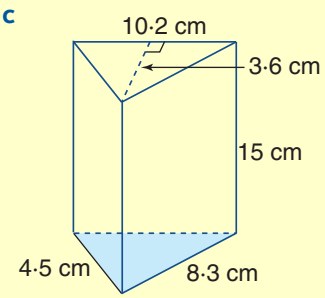
a



b

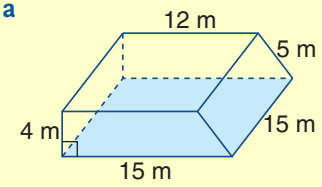


c

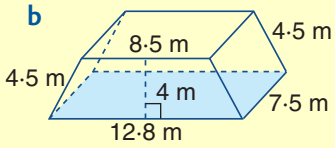


5 Find the surface area of each of the following trapezoidal prisms.

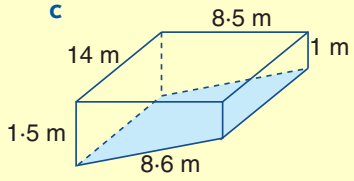
a



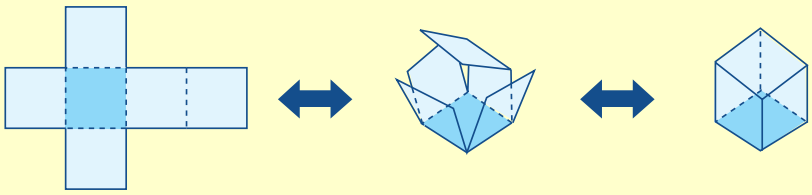
b



c



6



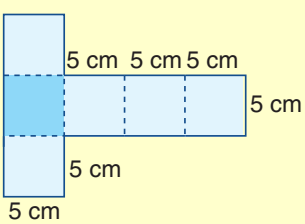
If the figure above left was traced on paper, cut out, and folded along the dotted lines, it would form the cube shown above right.

The figure above left is called the *net* of the solid. The area of the net is equal to the surface area of the solid.

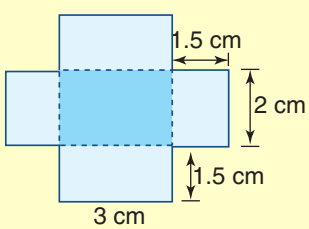
For each of the following nets find:

- **i** its area
- **ii** the name of the solid that it produces when folded along the dotted lines
- **iii** if the prism is open or closed

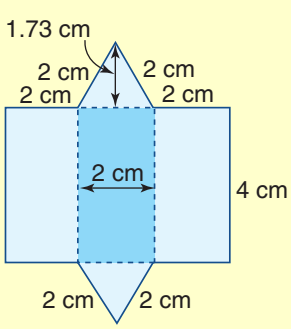
a



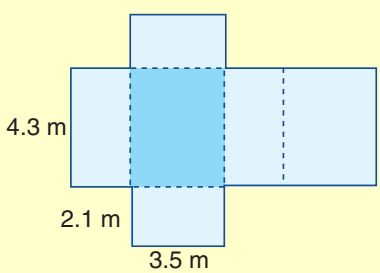
b



c

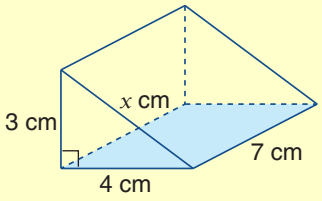


d

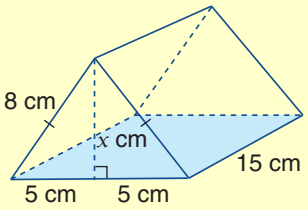


7 In each of the following questions, use Pythagoras' theorem to calculate the unknown length, x, correct to 2 decimal places, and then calculate the surface area.

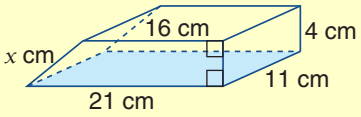
a



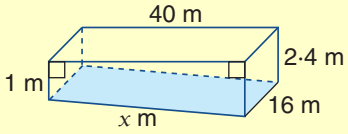
b



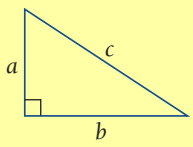
c



d

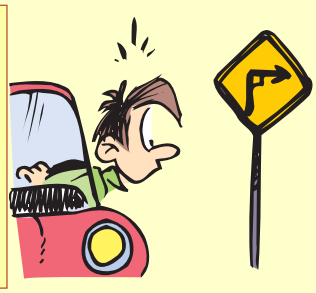


Caution



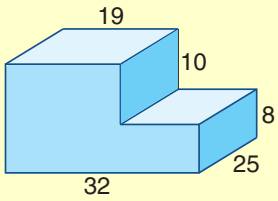
$c^2 = a^2 + b^2$

Right-angled triangles!

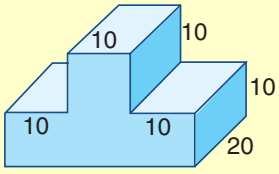


8 Find the surface area of the following prisms. All measurements are in centimetres.

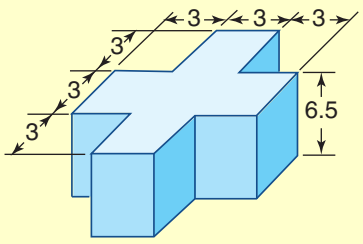
a



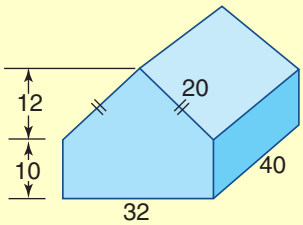
b



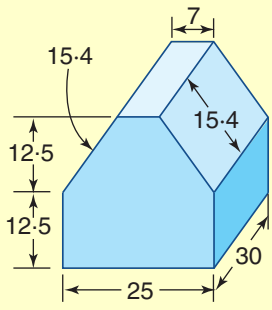
c



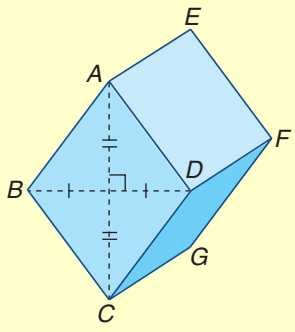
d



e



f



$AC = 4.8$
$BD = 3{\cdot}6$
$AE = 5{\cdot}2$

fun spot

12:05A

Fun Spot 12:05A | How did the boy know he had an affinity with the sea?

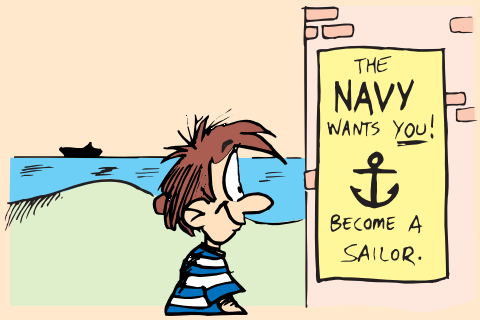
Answer each question and put the letter for that question in the box above the correct answer.

A $7^2 + 6^2$

E $\sqrt{5^2 + 12^2}$

H $9^2 - 4^2$

T $\sqrt{13^2 - 5^2}$



Complete the following.

W 3·5 m = . . . cm

E 0·7 cm = . . . mm

N 0·07 L = . . . mL

C 1 t = . . . g

I 1 cm^2 = . . . mm^2

A 20 000 m^2 = . . . ha

H 8700 kg = . . . t

T 0·5 min = . . . s

D 1 g = . . . mg

E 2 cm^3 = . . . mm^3

Write the basic numeral for:

A $1{\cdot}6 \times 10^2$

E $1{\cdot}6 \times 10^{-2}$

H 7×10^{-3}

W 7×10^3

Figures A, B and C have been formed by cutting rectangular pieces from a rectangle which is 30 cm long and 18 cm wide.

Find the perimeter of:

O Figure A

T Figure B

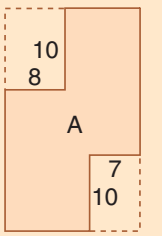
W Figure C

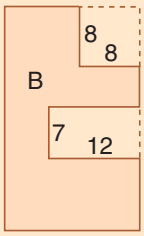
Find the area of:

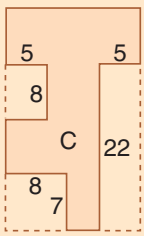
E Figure A

S Figure B

V Figure C



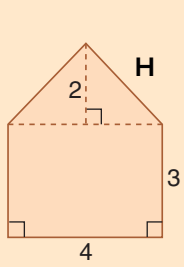


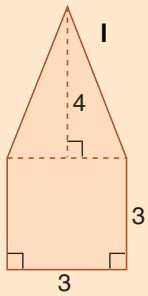


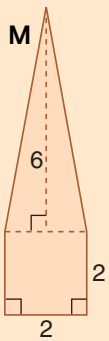
106	8·7	13	18	16	7	392	85	350	12	65	390

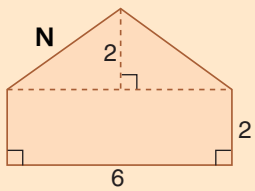
96	10^6	0·016	2	70	15	30	7000	14	334	2000	10^3	160	120	0·007	10^2	10

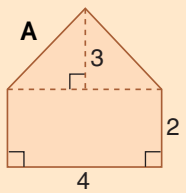
Calculate the area of each pentagon below.









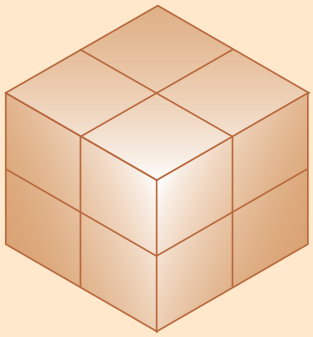


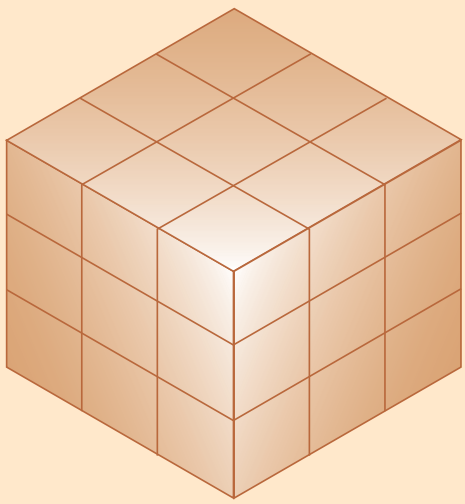
Fun Spot 12:05B | Let's play with blocks

Eight blocks have been stacked together here to form a cube. If the outside of the cube were painted, how many sides of each block would be painted?

How many blocks make up the second cube? If this cube were painted, how many blocks would have 3 sides, 2 sides, 1 side or even no sides painted?

What would be the result of painting a cube which had four blocks along each edge?





- Rubik's cube — a famous mathematical puzzle.

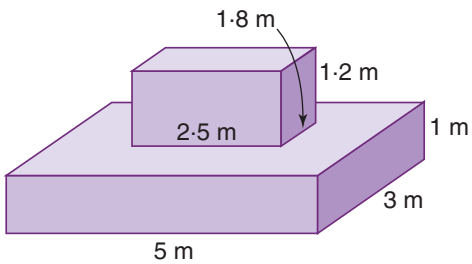
12:06 | Surface Area of Composite Solids

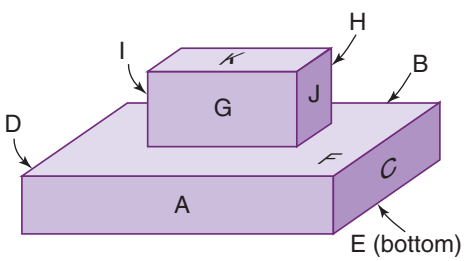
The surface area of composite solids is calculated in much the same way as for prisms.

Examine the shape and:
- check the number and type of the surfaces
- calculate the area of each surface
- develop a system for checking that all surfaces have been counted, especially the ones that cannot be seen in the diagram

worked example

Calculate the surface area of the following solids.





Solution

The solid consists of two rectangular prisms.
It has 11 surfaces, (A to K) as shown.
All are rectangles except for F.

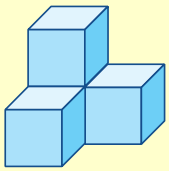
Surface	A	B	C	D	E	F	G	H	I	J	K
Area (m^2)	5	5	3	3	15	10·5	3	3	2·16	2·16	4·5

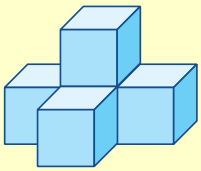
Note: Area of F = $(5 \times 3) - (2{\cdot}5 \times 1{\cdot}8)$
$= 10{\cdot}5$

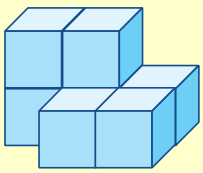
$\therefore$ Surface area = sum of areas of all surfaces A to K
$= 56{\cdot}32\ m^2$

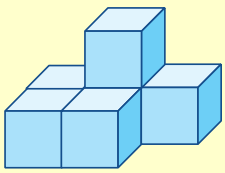
Exercise 12:06

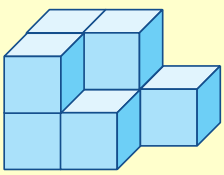
1 Each of the following solids has been built from 1 cm cubes. What is the surface area of each solid?

a 

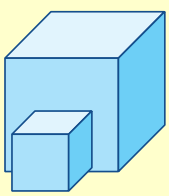
b 

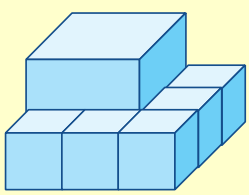
c 

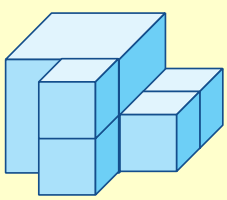
d 

e 

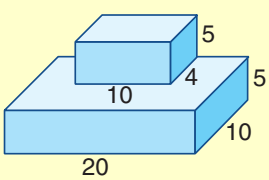
2 The following solids have been built from a 2 cm cube and 1 cm cubes. Calculate the surface area of each solid?

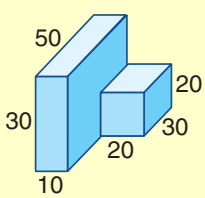
a 

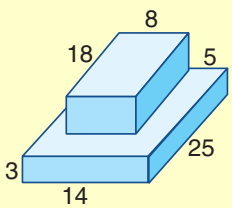
b 

c 

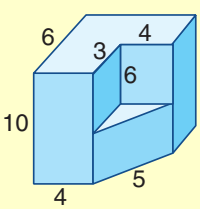
3 Calculate the surface area of the following solids. (All measurements are in centimetres.)

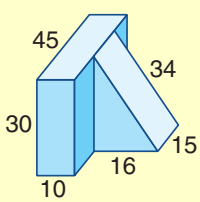
a 

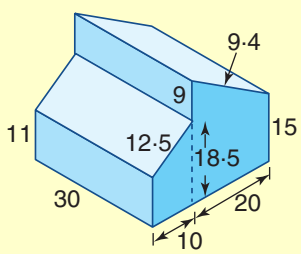
b 

c 

4 Calculate the surface area of the following solids. (All measurements are in centimetres.)

a 

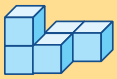
b 

c 

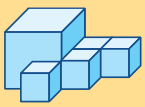
Foundation Worksheet 12:06

Surface area of composite solids

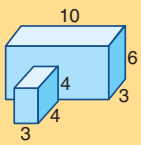
1 Find the surface area of these solids which have been built from 1 cm cubes.

a 

2 Cubes with sides of either 1 cm or 2 cm have been used to build these solids. Find the surface area of each solid.

a 

2 Find the surface area of the following composite solids.

a 

Investigation 12:06 | Perimeter and area

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

- Is there a connection between the area of a figure and its perimeter? If figure A has a greater perimeter than figure B does this mean it also has a greater area? Students often ask questions about the relationship between area and perimeter.
- One such question is:
  'Can rectangles with the same perimeter have different areas and, if so, which rectangle has the largest area?'
  We investigate that question here.

Investigation

Think of all the rectangles that have a perimeter of 20 cm.

1 Give the dimensions of four rectangles that have a perimeter of 20 cm.

2 Calculate the area of each of your rectangles.

3 Complete the table below. (Let rectangle *A* be the one with the smallest area and rectangle *D* the one with the largest area.)

	Length	*Width*	*Area*
Rectangle A			
Rectangle B			
Rectangle C			
Rectangle D			

4 Can you predict the area of the largest rectangle with a perimeter of 20 cm?

5 Test your theory with rectangles of perimeter 36 cm and 48 cm.

- Composite solids of many types are present in these buildings. How would you describe them?

Assessment Grid for Investigation 12:06 | Perimeter and area

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

		Assessment Criteria (B, C, D) for this investigation		Achieved ✓
Criterion B Investigating Patterns	a	An organised approach has not been used. The student has experienced difficulty doing routine calculations.	1	
			2	
	b	An organised approach has been attempted. The working out demonstrates a basic understanding of what is required.	3	
			4	
	c	An organised approach has been used and the patterns discovered have been described in words or symbols, with conclusions drawn to help predict answers in 5.	5	
			6	
	d	The correct patterns have been effectively recognised and explained clearly using words and symbols, and used to make sensible predictions in 5.	7	
			8	
Criterion C Communication	a	No working out is shown and presentation is poor with little or no use of diagrams or symbols.	1	
			2	
	b	Working out is shown with some explanation. Presentation is good with some structure to the work.	3	
			4	
	c	Work is well structured showing a clear progression and is well communicated using diagrams, words and symbols.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt has been made to explain the method used and to check the results.	1	
			2	
	b	The method used is justified and the results have been checked for reasonableness with some success.	3	
			4	
	c	A thorough, reasoned justification has been given for the method used and the accuracy and reasonableness of all results have been checked thoroughly, particularly for number 5.	5	
			6	

12:07 | Practical Applications of Surface Area

As we live in a three-dimensional world, we commonly deal with solid objects which are in fact prisms or cylinders.

The classrooms we work in are nearly all rectangular prisms, while water tanks and swimming pools are often cylindrical. Because we readily come into contact with many of these objects, a knowledge of surface area can be extremely useful. Again, as with area, tradesmen, such as painters and tilers, require a sound knowledge of surface area to enable them to carry out their work.

Exercise 12:07

1 A room is a rectangular prism in shape. It is 4·52 m long, 2·96 m wide and 2·41 m high. Find the area of the four walls.

2 A cylindrical water tank (with a top) has a radius of 2·1 m and a height of 3·2 m. The tank is to be painted inside and outside with a rust preventive paint. Find:

a the total area that is to be painted (correct to 1 decimal place)

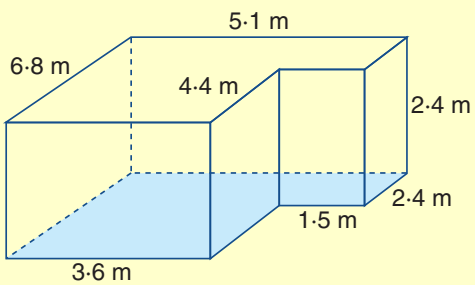
b the cost of the painting if it costs \$10.25 per m^2

3 The diagram represents a lounge room in a house. The walls and ceiling are to be painted. Find:

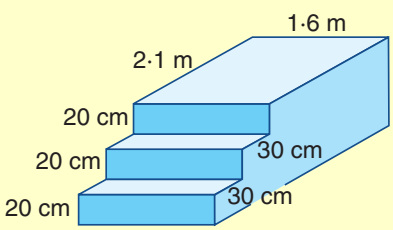
a the area of the walls

b the area of the ceiling

c the number of complete litres of paint that need to be bought if 1 litre of paint covers 16 m^2



4

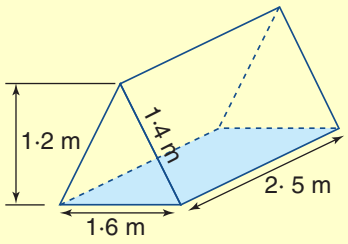


The steps and patio are to be covered with pebblecrete. Find:

a the area to be covered

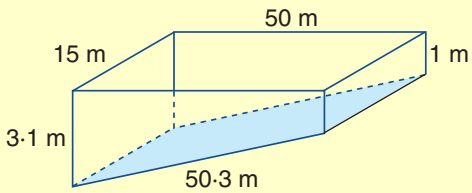
b the cost of pebblecreting, if it costs \$15 per m^2

5 A tent has the shape of a triangular prism with dimensions as shown in the diagram.

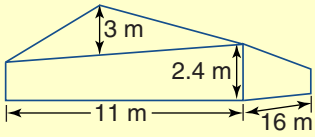


- **a** Find the area of material needed to make this tent. (Include the floor area.)
- **b** If the material comes in rolls which are 3·7 m wide, what length of material must be purchased so that the tent can be made without any joins except those at the edges? (*Hint:* Consider the net of the solid.)
- **c** If special joining tape is needed to strengthen each join, what length of tape will be needed?

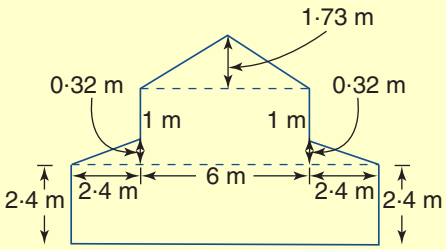
6 A swimming pool has the shape of a trapezoidal prism as shown in the diagram. If this pool is to be tiled, find the cost of tiling it at $45 per square metre.



7 A marquee is in the shape of a pentagonal prism. Use the dimensions shown to calculate the surface area. (There is no floor.)



8 A barn is made from aluminium. Calculate the area of metal used in its construction. (Assume it has a total window area of 8·8 m^2 and a length of 7.2 m.)



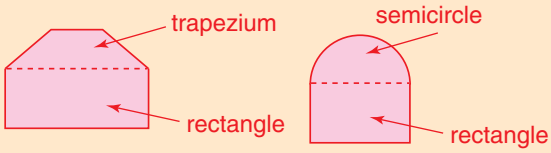
End view of barn

mathematical terms 12

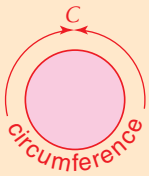
Mathematical terms 12

area
- The amount of space inside a two-dimensional shape.
- Units of area:
  square millimetre (mm^2)
  square centimetre (cm^2)
  square metre (m^2)
  hectare (ha)
  square kilometre (km^2)
- Formulae are used to calculate the area of the common plane figures.

composite figure
- A figure that is formed by joining simple figures.



circumference
- The length of a circle's boundary.
- The circumference is calculated using either the formula:
  $C = \pi D$ or $C = 2\pi r$



hectare
- An area of 10 000 m^2.
- A square with a side of 100 m.

perimeter
- The length of a plane figure's boundary.

sector
- A part of a circle bounded by two radii and an arc.

surface area
- The sum of the areas of the faces (or surfaces) of a three-dimensional figure (or solid).

Technology Applications Drag and Drops

Mathematical terms 12

Diagnostic Test 12: | Perimeter, Area and Surface Area

- Each part of this test has similar items that test a certain question type.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

diagnostic test 12

Section

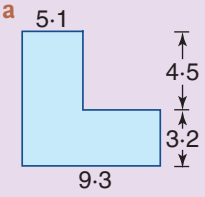
1 Find the circumference of each of the following circles. Give your answer correct to 3 significant figures. — 12:01

a diameter = 14 m **b** diameter = 7·6 cm

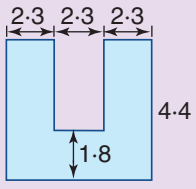
c radius = 25 mm **d** radius = 1·45 m

2 Find the perimeter of each of the following figures. Lengths are in metres. — 12:01

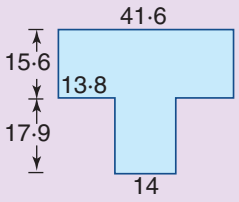
a



b

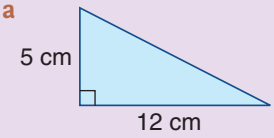


c

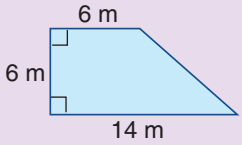


3 Find the perimeter of the following figures. — 12:01

a



b



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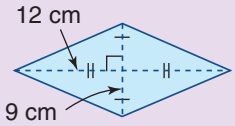
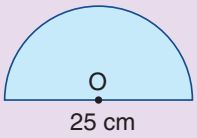
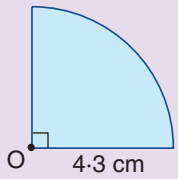


4 Find the perimeter of each of the following figures (correct to 2 decimal places). — 12:02

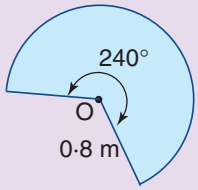
a



b

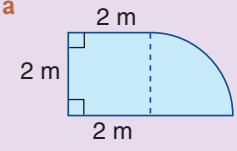


c

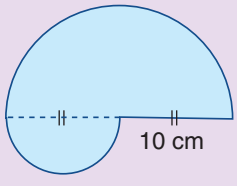


5 Find the perimeter of each of the following composite figures. (Where it is necessary, answer correct to 2 decimal places.) — 12:02

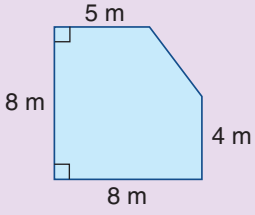
a



b



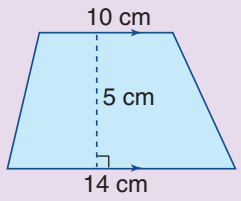
c



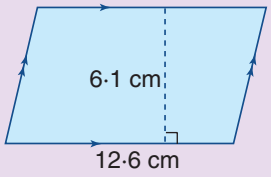
Section

6 Calculate the area of the following figures. 12:03

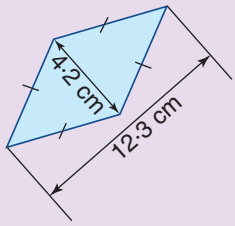
a



b

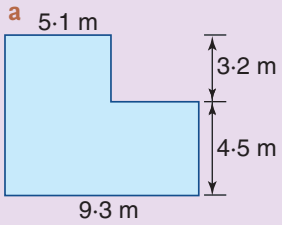


c

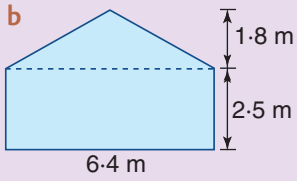


7 Calculate the area of the following figures. 12:03

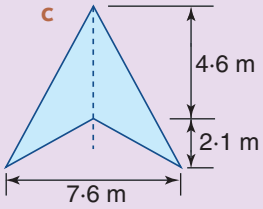
a



b



c

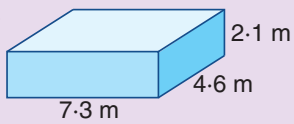


8 Calculate the area of the sectors in question **4** (correct to 2 decimal places). 12:04

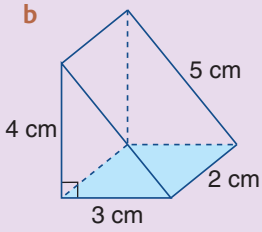
9 Calculate the area of the composite figures in question **5** (correct to 2 decimal places). 12:04

10 Calculate the surface area of the following prisms. 12:05

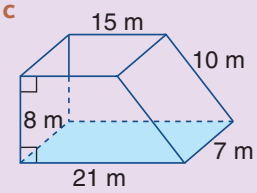
a



b

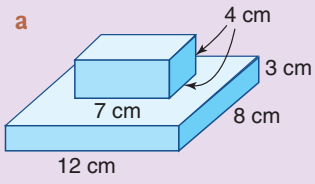


c

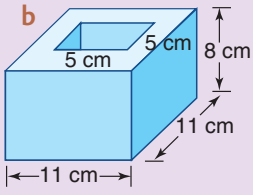


11 Calculate the surface area of these solids. 12:06

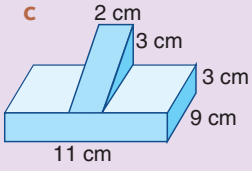
a



b



c



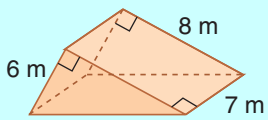
Chapter 12 | Revision Assignment

assignment 12A

1 A garden is rectangular in shape. It is 4·8 m long and 2·5 m wide. It is to have an edge placed around it to retain the soil. If the edge is to be made from rolls of logs wired together in 3 m lengths, how many of these lengths must be bought to do the edging?

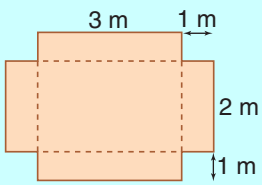
2 The minute hand of a clock is 9·5 cm long. How far will the tip of the hand travel in one day (to the nearest m)?

3 a



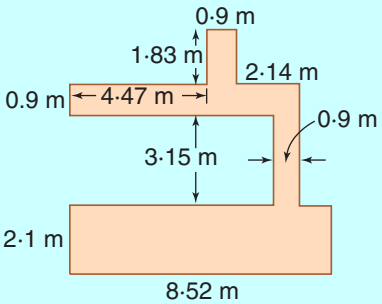
Find the surface area of this prism.

b

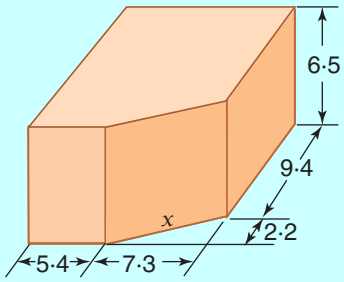


This is the net of an open rectangular prism. Find its surface area.

4 A concreter charges \$35 per m^2 to lay paths. What would be the cost of laying the paths in the diagram?



5

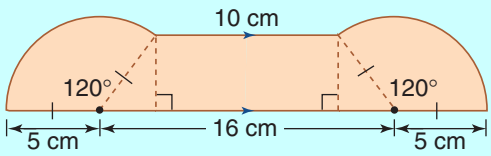


a Find the value of x correct to 1 decimal place.

b Calculate the area of the cross-section of the prism.

c Calculate the surface area of the pentagonal prism.

6



a Calculate the perimeter of the figure correct to 1 decimal place.

b Calculate the area of the figure correct to 1 decimal place.

Technology Applications Activities — Areas of rectangles, parallelograms, triangles, trapezia and circles

Technology Applications Drag and Drops
1 Perimeter 1
2 Perimeter 2
3 Area of sectors and composite figures
4 Surface area of a prism

Chapter Review Questions

assignment
12B

Chapter 12 | Working Mathematically

1 Use ID Card 6 on page xviii to give the correct mathematical term for:

a 13 b 14 c 15 d 16 e 18
f 19 g 20 h 9 i 10 j 11

2 What geometric shape is this saucepan hanger based on? Why do you think this shape was used?

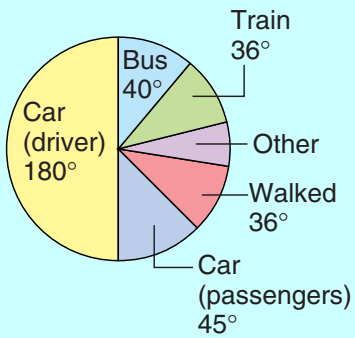
3 Heather is 7 years younger than Rachel. Ester is six times as old as Heather. Kuan is 7 years older than Ester. If Kuan is 43, how old is Rachel?

4 Every male bee has only one parent, a female. Every female bee has two parents, a male and a female. In the 8th generation back, how many ancestors has a male bee? (Assume that no ancestor occurs more than once.)

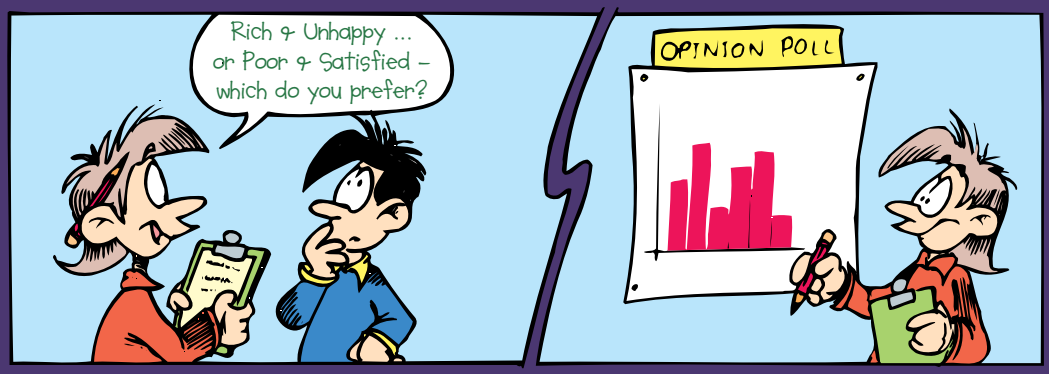
5 Four different playing cards are dealt into two piles: left first, then right, then left, and then right. The left pile is then placed on top of the right pile. How many times must this process be repeated before the cards return to their original positions? How many times would the process need to be repeated if there had been eight cards?

6 This sector graph shows the method of travelling to work for all persons.

a What percentage of the workforce caught a train to work?
b What percentage of the workforce was driven to work by car, bus or train?
c What is the size of the sector angle for 'other' means of transport? Do not use a protractor.
d What percentage of the workforce used a car to get to work?



13 Statistics



Chapter Contents

Learning Outcomes

Students will be able to:

- Construct, read and interpret graphs, tables, charts and statistical information.
- Collect statistical information by observation or census.
- Analyse data using measures of central tendency and range.
- Group data to aid analysis and construct frequency histograms.

Areas of Interaction

Approaches to Learning, Community and Service, Environment, Homo Faber

13:01 | Frequency

prep quiz 13:01

Huong asked students in her class to indicate how many pets they had. This resulted in the following data:

1	3	2	2	4	1	5	2	1	1
6	4	1	2	5	2	1	4	1	2

For this data, what is the:

1 number of outcomes? **2** mode?
3 number of children who had three pets?
4 number of children who had more than two pets?

This frequency distribution table shows the number of cars owned by the families in Huong's class. Complete the table and find how many families:

5 had no car **6** had two cars
7 had fewer than two cars
8 there were altogether.

Outcome	*Tally*	*Frequency*
0	||||	
1	~~||||~~ ||||	
2	~~||||~~ ||||	
3	|	

Total:

Frequency diagrams

A frequency distribution table is very good for collating and organising data. But, when analysing the data, it is often more desirable to have the information presented in the form of a diagram or graph.

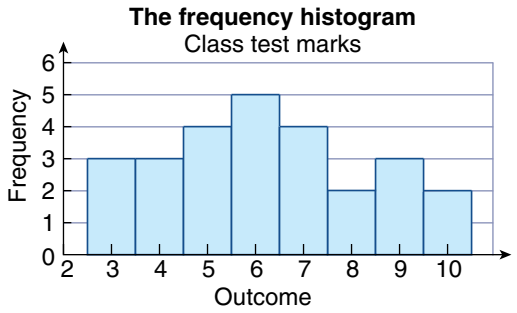
worked example

For a class of 26 students the following marks out of 10 were obtained in a test.

5	6	6	4	9	3	5	10	8	9	7	3	6
7	5	6	5	4	3	6	7	10	9	8	7	4

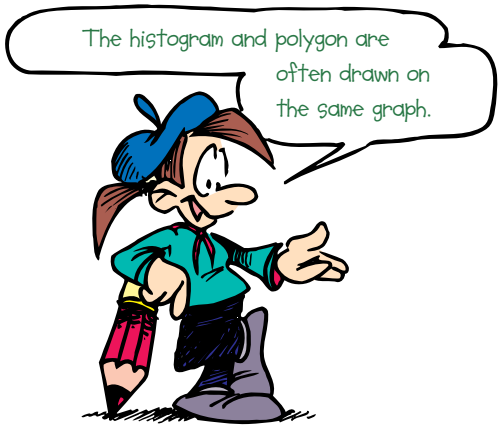
If this information is organised in a frequency distribution table, it looks like this.

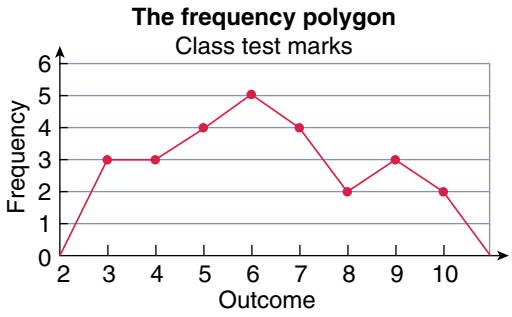
Outcome (marks)	*Tally*	*Frequency*
3	|||	3
4	|||	3
5	||||	4
6	~~||||~~	5
7	||||	4
8	||	2
9	|||	3
10	||	2
	Total:	26

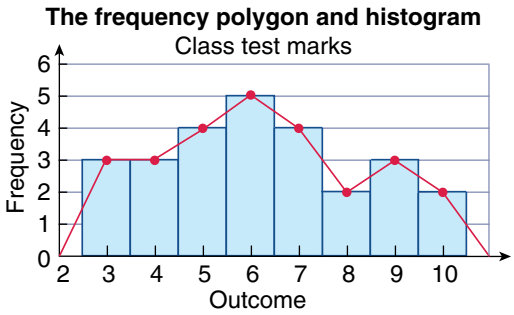


- The frequency histogram is a column graph.
- The graph has a title and the axes are labelled.
- The first column begins one-half of a column width in from the vertical axis.

- The frequency polygon is a line graph.
- The graph has a title and the axes are labelled.
- The first non-zero dot is one unit in from the vertical axis.
- The dots showing the data are joined by straight lines and joined to the horizontal axis as shown.







Exercise 13:01

Foundation Worksheet 13:01

Frequency and cumulative frequency

1 Arrange these scores into a frequency distribution table.

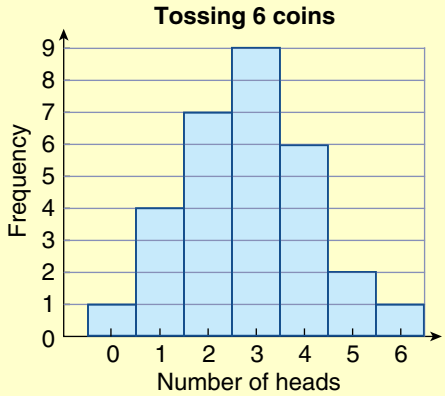
a 5,4,3,4,5,5,4,3,5,6,7,4,3,5,5,5,5,4,3,6

2 For the scores above find:

a how many scores of 3 there were

b how many scores were above 5

1 Six coins were tossed 30 times and the number of heads recorded. The results are shown in this frequency histogram.



a How many times were 4 heads thrown?

b What was the most frequent result?

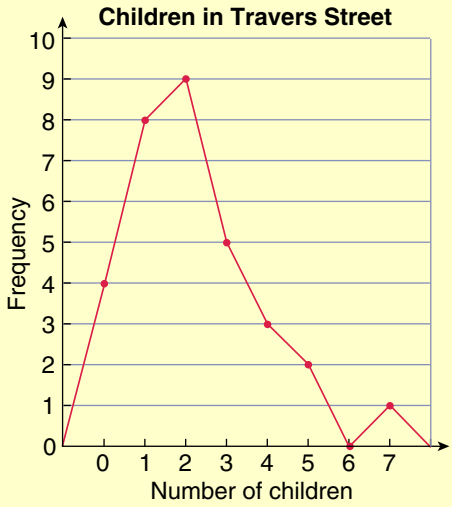
c What result occurred 4 times?

d If, for a particular throw, there were 2 heads, how many tails were there?

e On how many occasions did 5 tails result?

2 In Travers Street, each family was surveyed to determine its number of children. The results are shown in this frequency polygon.

a How many families had no children?
b What was the most common number of children?
c How many families had 6 children?
d How many families had 2 children or less?
e How many families were surveyed altogether?
f How many children were there altogether?



3 Draw the frequency histogram for the following sets of data.

a

Outcome (x)	*f*
0	3
1	8
2	11
3	17
4	9
5	2

Total:

b

Outcome (x)	*f*
9	1
10	13
11	22
12	
13	21
14	13

Total: 100

c

x	*f*
0	1
1	0
2	3
3	8
4	14
5	20
6	31
7	32
8	28
9	11
10	5

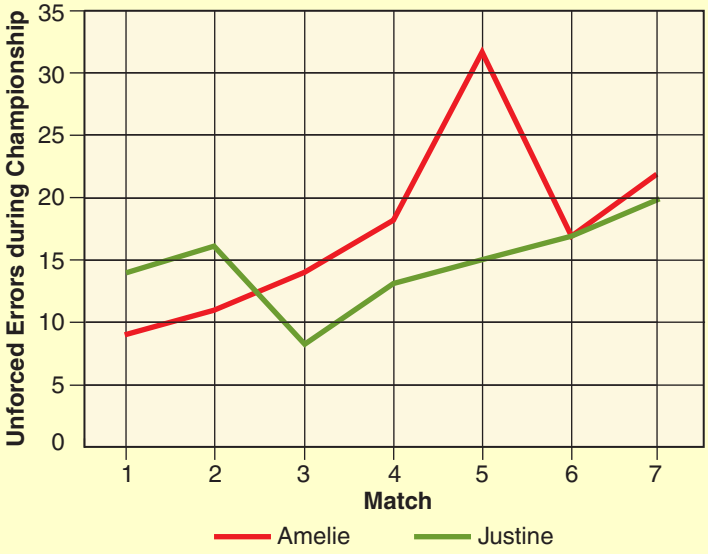
Total:

d

x	*f*
15	4
20	
25	3
30	9
35	7
40	10
45	15
50	8
55	10
60	2
65	4

Total: 80

4 The ladies' final of the Wimbledon Tennis Championship in 2006 was between Amelie Mauresmo and Justine Henin. The unforced erros they made in each match of the championship are shown in the frequency polygon.



How many unforced errors did Amelie make:

a in the first match? **b** in the first 3 matches?

c altogether? **d** in the final?

How many unforced errors did Justine make?

e in the first match? **f** in the first 3 matches?

g altogether? **h** in the final?

Who do you think won the championship? Give reasons for your answer.

5 In winning the 1993 Ford Australian Open men's singles, Jim Courier played 7 matches. The number of aces served in each is shown in this frequency distribution table.

a Draw, on the one set of axes, a frequency histogram and a frequency polygon using this data.

b What could have been the reason why more aces were served in the earlier matches than in the later ones?

Match number	*Number of aces*
1	8
2	6
3	9
4	5
5	4
6	5
7	3
Total:	40

6 The trees in each backyard of Eggleton Street were counted and the number recorded. The data is shown below.

0	4	8	7	2	1	3	1	4	2
12	6	5	1	4	3	5	7	4	6
2	3	9	8	3	4	2	2	5	1
6	3	1	4	5	5	2	3	10	4
7	5	4	6	2	1	0	3	5	5

a What are the highest and lowest scores in this data?
b Organise this data into a frequency distribution table.
c Draw a frequency histogram and a frequency polygon on one set of axes.

7 Sharon organises her family's football tipping competition. Each week Dad, Sharon, Adam and Bron have to pick the results of the 7 rugby matches played. The table below shows the results for rounds 1 to 5.

	Sharon		*Adam*		*Dad*		*Bron*	
	Score	*Prog. Total*	*Score*	*Prog. Total*	*Score*	*Prog. Total*	*Score*	*Prog. Total*
Round 1	4	4	6	6	4		4	4
Round 2	3	7	3	9	4		3	7
Round 3	6	13	4	13	6		5	12
Round 4	4	17	4	17	2		3	15
Round 5	5	22	4	21	5		4	19

a 'Prog. Total' is short for 'Progressive Total'. It tells the number of strokes altogether after that round. Complete Dad's Prog. Total column.
b Who was leading the competition at the end of:
i round 1? **ii** round 3? **iii** round 5?
c What has been the highest score achieved in a round? How many times has this happened?
d Who has had the lowest score in a round?
e In round 6 the scores were: Sharon 6, Adam 6, Dad 7, Bron 5. Use these results to add the next line in the table.

13:02 | Cumulative frequency

Cumulative frequency diagrams

A further column that may be attached to the frequency distribution table is the 'cumulative frequency' or c.f. column. This column gives the progressive total of the outcomes.

The cumulative frequency of an outcome gives the number of outcomes equal to, or less than, that particular outcome.

worked example

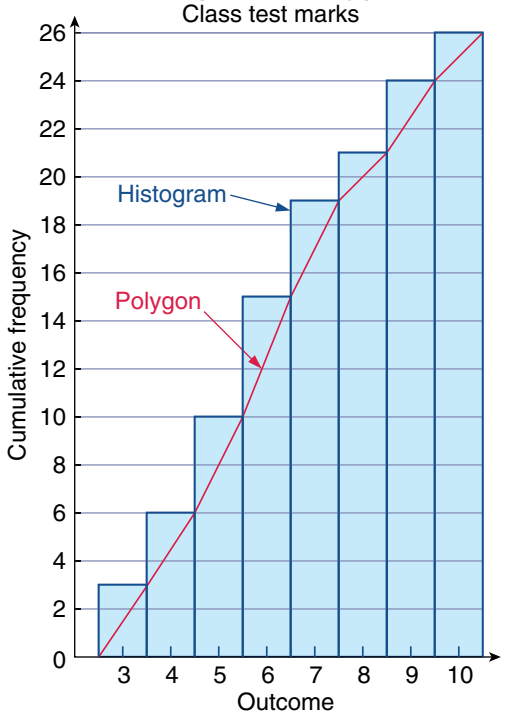
Outcome (x)	*Tally*	*Frequency* (f)	*Cumulative frequency*
3	|||	3	3
4	|||	3	6
5	||||	4	10
6	||||	5	15
7	||||	4	19
8	||	2	21
9	|||	3	24
10	||	2	26
	Total:	26	

- 15 students scored 6 or less. Since 4 students scored 7, the cumulative frequency of 7 is 15 + 4 or 19.
- The last figure in the c.f. column must be equal to the sum of the frequencies, as all students are on or below the highest outcome.

- The histogram progressively steps upwards to the right.
- The polygon is obtained by joining the top right-hand corner of each column. (Why is it drawn this way?)
- Imagine that the column before the '3' column has zero height.

■ Another name for the cumulative frequency polygon is the 'ogive'.

The cumulative frequency histogram and polygon
Class test marks



Exercise 13:02

1 Complete the cumulative frequency columns in these tables.

a

Outcome (x)	*f*	*c.f.*
0	3	3
1	8	11
2	11	22
3	17	
4	9	
5	2	

Total:

b

Outcome (x)	*f*	*c.f.*
9	1	
10	13	
11	22	
12		
13	21	
14	13	

Total: 100

c

x	*f*	*c.f.*
0	1	
1	0	
2	3	
3	8	
4	14	
5	20	
6	31	
7	32	
8	28	
9	11	
10	5	

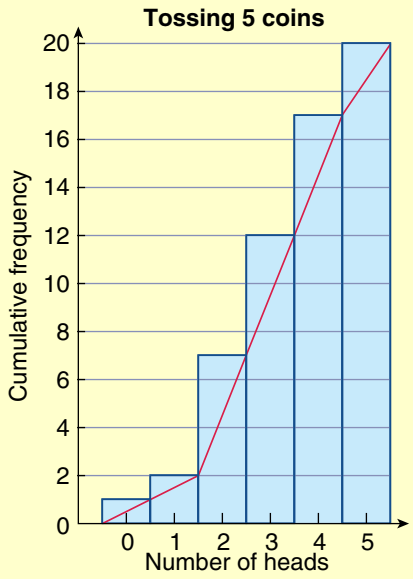
Total:

d

x	*f*	*c.f.*
15	4	
20		
25	3	
30	9	
35	7	
40	10	
45	15	
50	8	
55	10	
60	2	
65	4	

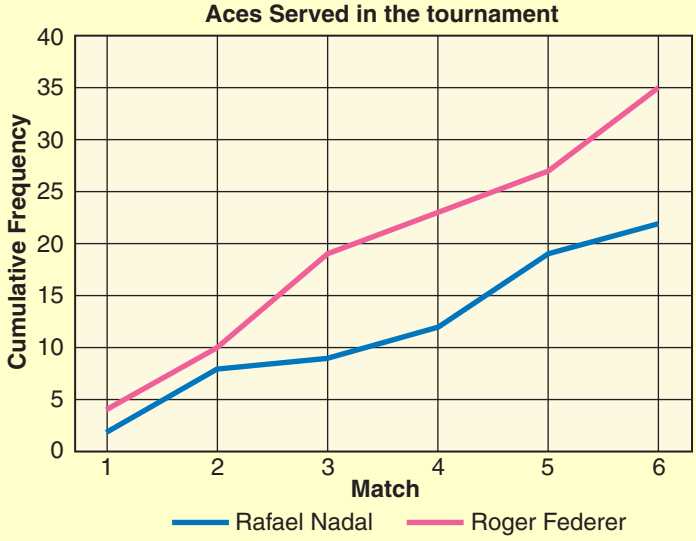
Total: 80

2 Five coins were tossed many times and the number of heads recorded. The cumulative frequency for each number of heads was calculated and these graphs drawn.



- **a** How many times did zero heads occur?
- **b** How many times did 1 or less heads occur?
- **c** How many times did 2 or less heads occur?
- **d** How many times did 3 or less heads occur?
- **e** Think about your answers to **c** and **d**. How many times did 3 heads occur?
- **f** How many times were the 5 coins tossed?

3 The final of the men's tennis final of the 2006 French Open was between Rafael Nadal and Roger Federer.
The aces each player served in each of the last six matches was recorded and graphed in this cumulative frequency polygon.



How many aces did Rafael Nadal make:

- **a** in the first match shown?
- **b** in the first 3 matches shown?
- **c** altogether
- **d** in the final?

How many aces did Roger Federer make:

- **e** in the first match shown?
- **f** in the first 3 matches shown?
- **g** altogether
- **h** in the final?

Is there enough information to tell you who won the final?

4 Kazuyoshi Tatsunami plays baseball for the Chunichi Dragons for Japan. The number of home runs he scored from 1999 to 2005 is sown in the frequency distribution table.

a Complete the cumulative frequency column
b Draw, on one set of axes, a cumulative frequency histogram and a cumulative frequency polygon.
c Give a possible reason why he scored so many more home runs in 2002

Year	*Home runs*	*c.f.*
1999	4	
2000	9	
2001	9	
2002	16	
2003	13	
2004	5	
2005	9	

5 Hendrik conducted a survey of the number of people living in every apartment in his building. His results are below:

2	2	0	4	3	3	1	3	3	1
2	4	5	4	5	3	3	4	5	2
3	5	1	2	1	3	3	5	3	3
3	2	5	6	3	3	4	3	2	5
2	4	4	4	2	4	6	2	0	4

a What are the highest and lowest scores in this data?
b Organise the data into a frequency distribution table including a cumulative frequency column.
c Draw a frequency histogram and a frequency polygon on one set of axes.
d Draw a cumulative frequency histogram and a cumulative frequency polygon on one set of axes.
e How many apartments are in Hendrik's building?
f How many people live in Hendrik's building?
g How many apartments in Hendrik's building are vacant?

6 Ella did a survey by asking all the students in her Homeroom class how many children there were in their family. These are her results and include her family.

1	2	2	4	3	2	1	3	3	1
2	4	5	4	2	3	2	4	2	2

a What are the highest and lowest scores in this data?
b Organise the data into a frequency distribution table including a cumulative frequency column.
c Draw a cumulative frequency histogram and a cumulative frequency polygon on one set of axes.
d How many students are in Ella's Homeroom class?
e If all the children from all the families in Ella's Homeroom class came to the school's musical production, how many children would that be?
f Why do you think there are no zeros in this data?

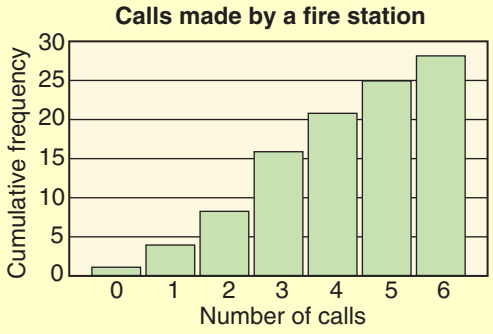
7 Dan drives a bus in his city. As part of an evaluation of how people travel by bus, he was asked to record how many people boarded his bus at each stop in a peak hour run. The results are below:

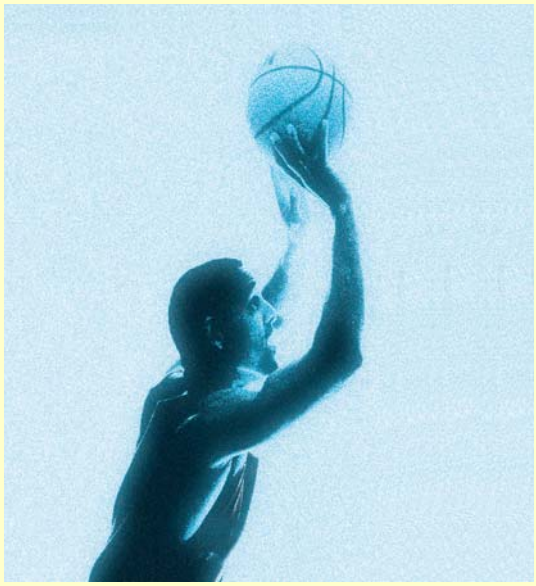
5	3	2	3	3	1	6	4	2	4
2	4	1	4	0	5	3	3	3	3
0	3	0	8	0	4	2	0	5	1

a What are the highest and lowest scores in this data?
b Organise the data into a frequency distribution table including a cumulative frequency column.
c Draw a frequency histogram and a cumulative frequency polygon on different axes.
d How many stops were there on this route?
e How many passengers boarded the bus altogether?
f If Dan's bus can carry only 45 passengers at a time, how can you explain this?

8 The number of calls made by a fire station in the city was recorded for a period of days. The results are shown in the cumulative frequency histogram.

a Make a frequency distribution table from this data.
b Over how many days were the number of calls recorded?
c How many calls did the fire state make altogether?
d What was the most number of calls made in a day during this period?

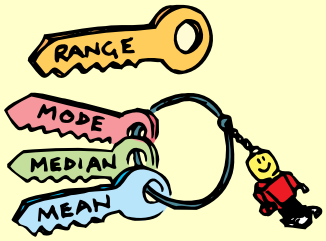




• Statistics are used in many sports to measure performance.

13:03 | Analysing Data (1)

After data has been collected, certain 'key' numbers can be calculated that give us further information about the data being examined.



- The **range** is a measure of the spread of the scores.
- The **mean**, **median** and **mode** are all measures that try to summarise or average the scores. There are three averages because all have some disadvantage in certain situations.

The *range* = highest score − lowest score.
The *mode* is the outcome that occurs the *most*.
The *median* is the middle score for an odd number of scores.
The *median* is the average of the middle two scores for an even number of scores.
The *mean* is the arithmetic average.

$$\text{mean} = \frac{\text{sum of scores}}{\text{total number of scores}} = \left[\frac{\text{sum of } fx \text{ column}}{\text{sum of } f \text{ column}}\right]$$

worked examples

Find the range, mode, median and mean of each set of scores.

1 4 4 4 12 9 6 10

2 15 36 40 23 18 46 21 28 32 36

Solutions

1 **Range** = highest score − lowest score
= 12 − 4
= 8

Mode = outcome occurring most
= 4

Median = middle score
= 6

$$\textbf{Mean} = \frac{\text{sum of scores}}{\text{total number of scores}}$$
$$= \frac{4+4+4+12+9+6+10}{7}$$
$$= 7$$

2 **Range** = highest score − lowest score
= 46 − 15
= 31

Mode = outcome occurring most
= 36

Median = average of two middle scores
$$= \frac{28+32}{2}$$
= 30

$$\textbf{Mean} = \frac{295}{10}$$
= 29·5

Discussion

- The mean of a set of five test marks is 7. What might the scores have been?
- The range of a set of eight scores is 3 and the mode is 11. What could the scores be?
- The salaries of the eight people working in my company are $170 000, $27 000, $26 000, $33 000, $26 000, $31 000, $40 000 and $28 000. Which of the mode, median and mean provides the most accurate indication of the salary of a normal employee?

Exercise 13:03

Foundation Worksheet 13:03

Mean, median and mode

1 For the following sets of scores find the mean.
- **a** Batsman's scores: 47, 58, 20, 0, 15, 48, 72
- **b** Golf hole scores: 4, 5, 6, 4, 4, 5, 4, 4, 3, 5

2 For the scores in question 1 what is:
- **a** the median? **b** the mode?

3 Which of the mean, median or mode is of most use in summarising the:
- **a** batsman's scores?
- **b** golf hole scores?

1 Determine the range, mode, mean and median of each set of scores.

- **a** 5, 7, 5, 8, 3
- **b** 3, 2, 2, 5, 4, 2, 3, 7, 4
- **c** 9, 8, 6, 6, 8, 7, 5
- **d** 5, 8, 7, 9, 8, 5, 5, 7
- **e** 15, 17, 23, 17, 19, 20, 21
- **f** 12, 10, 10, 12, 9, 13, 12
- **g** 31, 29, 25, 32, 29, 29
- **h** 51, 53, 53, 50, 51, 49, 47
- **i** 23, 25, 21, 22, 21, 26, 21
- **j** 97, 88, 93, 89, 93, 91, 92, 88
- **k** 5·1, 5·3, 4·9, 5·0, 5·1, 5·3, 5·1
- **l** 102, 99, 101, 102, 98, 103, 100, 99

2 **a** Cliff is a basketballer. In 10 games he scored the following number of points. Use the mean to calculate his average points per game.

17, 28, 22, 33, 17, 15, 14, 27, 25, 20.

b Rob is a ten-pin bowler. He has to bowl 9 games to get his bowling average (mean). What is his average if he has the following scores.

180, 188, 193, 175, 210, 188, 199, 215, 201.

c In swimming, at least three times have to be taken for record attempts. The swimmer's official time is the median of all the times. What would a swimmer's time be if the times were:

i 32·68, 32·64, 32·70 **ii** 32·68, 32·64, 32·70, 32·64

3 **a** Barbara's bowling average after 9 games is 178. If her next three games are 190, 164 and 216 what is her new average (mean) based on the 12 games she has bowled?

b Rob bowls in a competition where 4 games are bowled in each night of competition. His average (mean) is 187. In his first two games he scores 191 and 163. What must he total in the last two games if he wants his average to stay at 187?

4 The selling prices of ten houses are listed below.

$263 000	$235 000	$195 000	$175 000	$3 865 000
$204 000	$190 000	$245 000	$200 000	$ 175 000

Which measure of central tendency would best reflect these sales: the mode, the median or the mean? Give reasons for your choice.

5 The size of every dress sold in Milan last year was recorded. Which measure of central tendency would be most useful to a dress manufacturer: the mode, the median or the mean? Give reasons for your answer.

6 A student wants to use a measure of central tendency to show how he has performed in eight maths tests. His marks are listed here.

56, 56, 58, 58, 85, 97, 56, 55

Which should he use: the mode, the median or the mean? Give reasons for your answer.

7 Alan has the following scores in 10 rounds of golf.

78 79 82 77 78

95 82 79 79 82

Calculate the mode, mean and median of these scores. Which is the best measure of Alan's performance?

8 **a** Calculate the mean for each of the following sets of marks. How many marks in each set are larger than the mean?

i 75, 75, 75, 75, 70 **ii** 75, 75, 75, 75, 80

b In a test the mean was 60. Does this mean there were as many scores over 60 as there were below 60? Give reasons for your answer.

9 In a gymnastics competition, each judge is asked to calculate a deduction. The mean of the two middle deductions is called the 'Judges' Deduction'. The gymnast's score is calculated by subtracting the Judges' Deduction from 8.8. What would a gymnast's score be if four judges calculated deductions of 0·62, 0·54, 0·37 and 0·62?

10 In a diving competition the five judges gave scores of 8, 7·5, 8·5, 9 and 8.

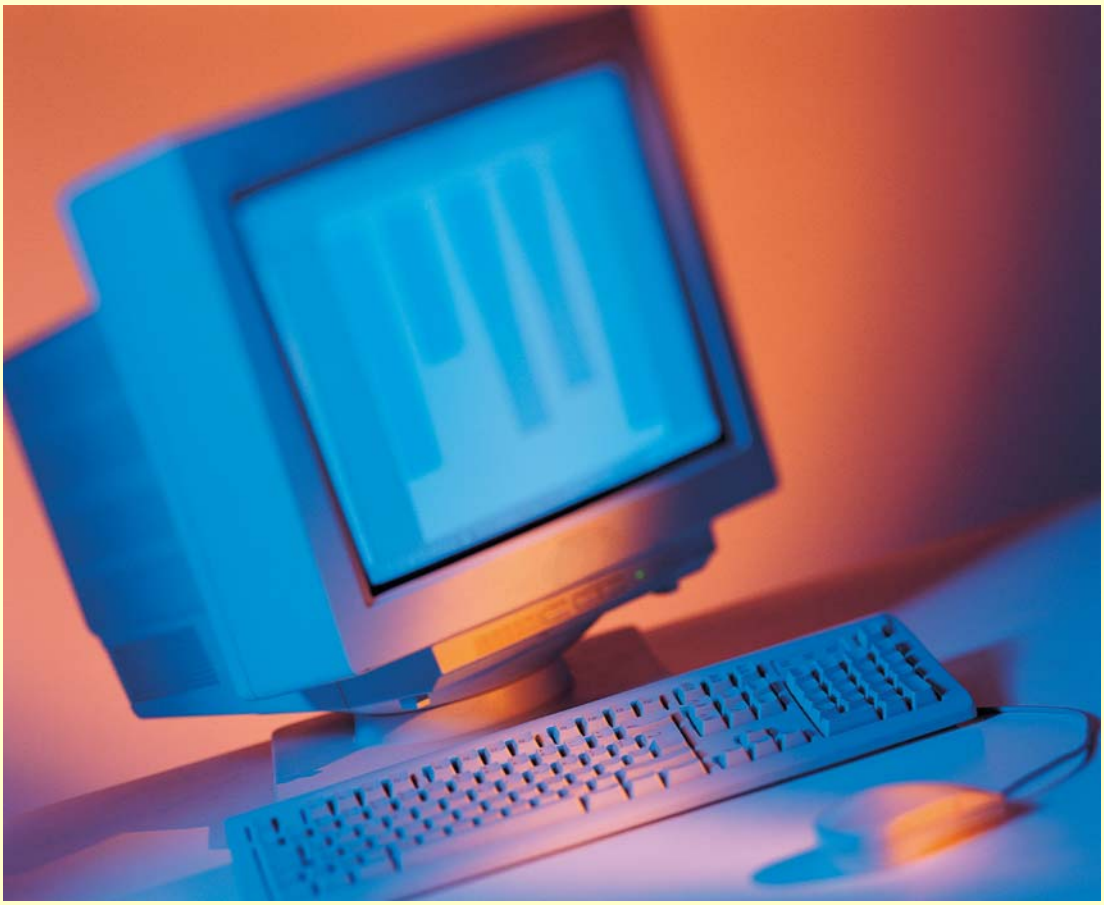
a Delete the highest and lowest scores and find the mean of the remaining three scores. Multiply this by the Degree of Difficulty of 2·8 to get the score for the dive.

b Repeat the process in **a** to find the score for a dive with a Degree of Difficulty of 3·2 if the judges' scores are 6, 6·5, 6, 7, 7·5.

Practical Activity 13:03 | Codebreaking and statistics

Codebreakers use statistics to help decipher codes. It is based on the fact that certain letters are more common than others. What letter of the alphabet appears most often? What is the most common vowel? What consonant occurs most often?

1 Write down what you think the answers are to the three questions above.
2 Use the statement above to do an alphabetic analysis. Were your answers in **1** supported by the statistics?



- As well as deciphering codes, mathematicians are often employed to devise security codes to prevent access by unauthorised users. In particular, cryptographers are employed to stop computer hackers from accessing computer records.
- On one side we have mathematicians trying to break codes, and on the other side we have mathematicians trying to design codes that cannot be broken.

13:04 | Analysing Data (2)

Often data has already been organised into a frequency distribution table or a frequency or cumulative frequency diagram.

This allows the mean and median to be calculated in other ways.

■ An outcome is a possible value for the data.

Finding the mean from a frequency distribution table

- To calculate the mean from a frequency distribution table, another column is added: the *frequency* × *outcome* column.
- It is appropriate at this point to mention that an outcome is normally given the notation x and frequency the abbreviation f. Therefore the *frequency* × *outcome* column is given the shortened title of the fx column.
- The sum of the fx column then gives the total of all the scores, and the sum of the frequency column gives the total number of scores. Here the Greek letter Σ (sigma) can be used to indicate 'the sum of' and the symbol $\bar{x}$ (x bar) to signify the mean. So a compact definition for the mean would be $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$.

worked example

Outcome (x)		*Frequency* (f)		fx
0	×	1	=	0
1	×	5	=	5
2	×	7	=	14
3	×	9	=	27
4	×	5	=	20
5	×	3	=	15
Totals		30		81
		Σf		Σfx

Note how each number in the fx column is obtained by multiplying the outcome and frequency together.

To calculate the mean, $\Sigma fx = 81$, $\Sigma f = 30$

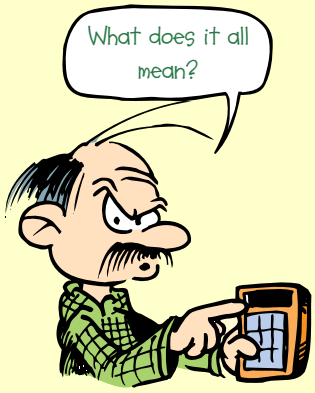
So $\bar{x} = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{81}{30}$

$= 2{\cdot}7$

We have found the sum of the scores (81) by first finding the sum of the 0s, 1s, 2s, 3s, 4s and 5s.

Mean $(\bar{x}) = \dfrac{\Sigma fx}{\Sigma f}$, **ie** $\dfrac{\text{sum of all the scores}}{\text{total number of the scores}}$

Using a calculator to calculate the mean



To calculate the **mean** when dealing with a small set of scores, you may simply add the scores together and divide the total by the number of scores.

For larger numbers of scores you should use your calculator.

Check to see if your calculator has statistical function keys, and if so, switch your calculator to the **statistics mode** (indicated by the letters SD on some calculators).

The calculator is now ready to accept a set of scores into its memory and perform various calculations with them.

At this point, identify the following keys on your calculator and note their meanings.

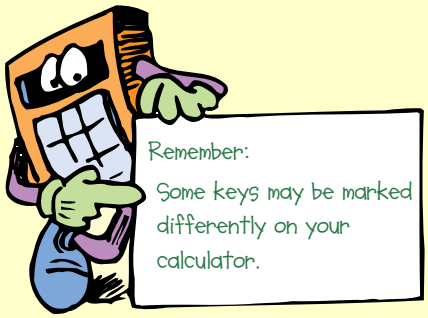
x or data 'score button', which must be pressed after entering each score. (This may be on the M+ button.)

n indicates the number of scores entered.

Σx gives the sum of the scores entered.

$\bar{x}$ gives the mean of the scores entered.

SAC 'statistics all clear'. The second function button may have to be pressed before the clear button to clear the statistics information from the calculator.



Entering the scores one at a time

Now, if the set of scores 5, 9, 7, 3, 8 is entered into your calculator, using the method appropriate to your calculator, the following results should be obtained.

n gives 5 (the number of scores)

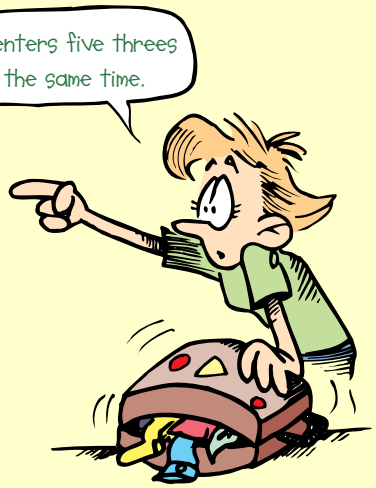
Σx gives 32 (the sum of the scores)

$\bar{x}$ gives 6·4 (the mean)

Entering the scores from a frequency table

The calculator can also be used to calculate the mean from a frequency table.
Examine the example below.

Outcome x	*Frequency* f
3	5
4	3
5	7
6	8
7	4
8	2
Total:	29



Press:

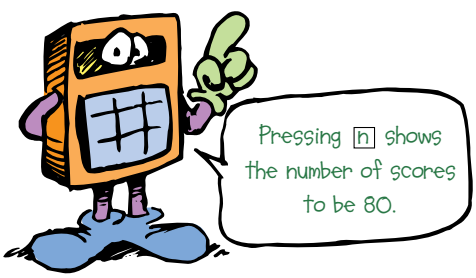
3 × 5 x
4 × 3 x
5 × 7 x
6 × 8 x
7 × 4 x
8 × 2 x

Then press $\bar{x}$ to obtain the mean, 5·3 (to 1 dec. pl.).

worked example

Find the mean of the scores entered in the frequency distribution table.

Outcome	48	49	50	51	52	53
Frequency	9	15	23	19	10	4



Press:

48 × 9 x
49 × 15 x
50 × 23 x
51 × 19 x
52 × 10 x
53 × 4 x

Then press $\bar{x}$ which shows the mean to be 50·225.

Finding the median from a frequency distribution table

The cumulative frequency can be used to find the median of a set of scores.

worked examples

1

Outcome x	*Frequency* f	*Cumulative frequency*
3	5	5
4	3	8
5	7	15
6	8	23
7	4	27
8	2	29

The middle score is the 15th score (14 above it and 14 below it).

The 15th score is a 5.

Hence the median = 5.

2

Outcome (x)	f	*c.f.*
5	2	2
6	4	6
7	3	9
8	7	16
9	5	21
10	1	22

Here there is an even number of scores, ie 22; so the middle two scores are the 11th and 12th scores.

From the *c.f.* column it can be seen that each of these scores is 8.

$\therefore$ median = 8.

3

Outcome (x)	f	*c.f.*
5	6	6
6	9	15
7	5	20
8	4	24
9	3	27
10	3	30

Here there is also an even number of scores, ie 30. So the middle two scores are the 15th and 16th scores.

In this example the 15th score is a 6, whereas the 16th score is a 7. So the median is the average of these two scores.

$\therefore$ median = 6·5 (or $6\frac{1}{2}$).

Finding the median from an ogive or polygon

The cumulative frequency polygon or ogive which was met earlier in the chapter can be used to find the median.

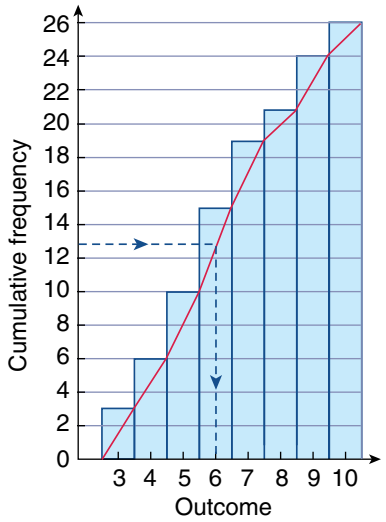
Note that the method used is different from that used for the table.

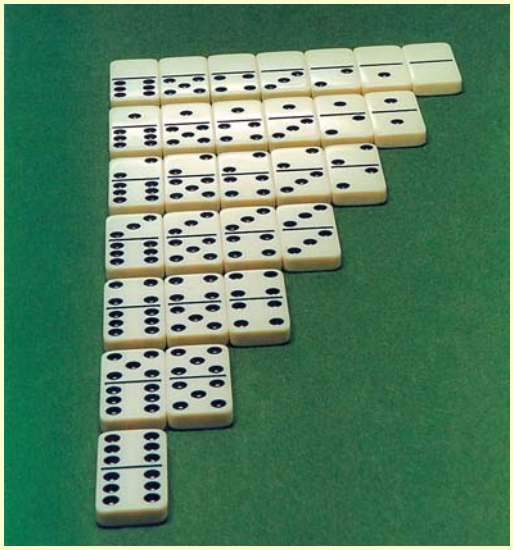
worked example

To find the median follow these steps.

- Find the halfway point ($\frac{1}{2} \times 26 = 13$).
- Draw a horizontal line from this point to the ogive.
- Then draw a vertical line to meet the horizontal axis.
- This meets the horizontal axis within the '6' column.

  $\therefore$ The median is 6.





- The photo shows a set of double-six dominoes. Each domino consists of two squares with a number of dots from 0 to 6.
- A set of double-six dominoes goes from double blank (0, 0) to double six (6, 6) with each pair occurring only once. So a (3, 4) is the same as a (4, 3).
- How many dots are there on this double-six set of dominoes?
- There are 28 dominoes in a double-six set. How many dominoes would there be in a double-eight set?

Exercise 13:04

1 Use your calculator to evaluate the mean ($\bar{x}$) for each set of 20 scores.

a 6, 9, 8, 7, 8
6, 5, 4, 9, 8
3, 9, 8, 7, 6
5, 8, 7, 8, 4

b 12, 11, 9, 13, 14
15, 13, 11, 10, 16
15, 9, 10, 13, 15
16, 12, 11, 10, 9

c 38, 41, 56, 29, 41
48, 25, 50, 49, 36
27, 32, 55, 45, 33
44, 29, 30, 47, 52

d 77, 82, 46, 62, 91
51, 93, 49, 75, 81
90, 86, 51, 42, 39
45, 59, 47, 83, 72

e 201, 532, 462, 191, 256
89, 169, 346, 291, 343
251, 507, 417, 392, 219
121, 197, 283, 400, 92

f 7·6, 5·2, 3·9, 4·7, 6·0
3·2, 7·9, 4·2, 2·6, 5·9
5·2, 4·1, 3·3, 2·9, 6·2
4·9, 3·0, 6·3, 4·9, 5·7

2 Use your calculator as shown in the examples, to evaluate the mean for the scores in the tables below.

a

Outcome	*Frequency*
5	3
6	5
7	9
8	7
9	2

b

Outcome	*Frequency*
11	5
12	8
13	11
14	10
15	8
16	2

c

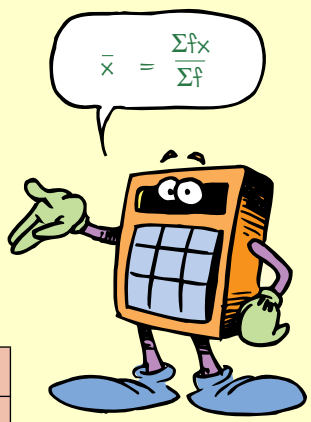
Outcome	*Frequency*
28	6
29	9
30	13
31	18
32	15
33	11
34	5

d

Outcome	22	24	26	28
Frequency	9	15	13	8

e

Outcome	5·0	5·2	5·4	5·6	5·8	6·0
Frequency	7	11	16	21	15	10



3 For each part of question **2** find the range, mode and median.

4 Two dice were thrown and the number of times that each total appeared was recorded.

Outcome	2	3	4	5	6	7	8	9	10	11	12
Frequency	3	6	8	11	14	17	15	12	7	5	2

a How many times were the dice thrown?
b What is the modal score?
c Calculate the mean for this data.

Foundation Worksheet 13:04

Mean and median

1 Calculate the mean, median and mode for each of the following.
a 3,3,3,4,4,4,4,4,4,5,5,5,5,5,5,5,5,6,6,6,7

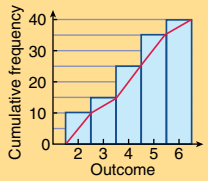
2 Calculate the mean, median and mode from the following frequency distribution tables.

a

Outcome	*f*	*c.f.*
0	5	
1	21	
2	37	
3	10	
4	4	

3 Use the following cumulative frequency diagrams to find the median.

a



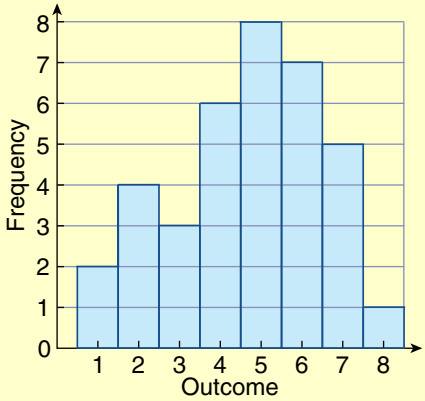
5 A goalkicker for a football team kicked the following number of goals in his twenty-four games this season.

2	3	0	2	2	1	1	4
6	0	1	3	3	2	1	4
5	1	5	2	0	2	4	1

Complete a frequency distribution table and determine the range, mode and mean for this set of scores.

6 Use the histogram on the right to find:

a the range of scores
b the mode of the scores
c the median of the scores
d the mean of the scores by completing a frequency distribution table



7 The scores of seventy students for a spelling test are recorded below. There were 20 words tested.

19	13	18	14	15	20	19	19	17	12
10	14	8	12	11	17	18	16	16	15
14	18	14	13	19	20	10	13	11	17
16	15	13	19	12	17	17	20	10	11
14	13	15	16	19	12	12	20	10	17
11	13	18	19	20	18	18	11	12	15
16	15	14	14	17	16	19	18	17	13

A set of scores may have more than one mode.

Determine the range for this set of scores and collate them in a frequency distribution table. Hence find the mode and calculate the mean.

Also construct a frequency histogram and polygon for this distribution.

8 Complete this table and find the:

a range
b mode
c mean
d median

for this set of scores.

x	*Tally*	*f*	*fx*	*c.f.*
3	||			
4	~~||||~~			
5	||||			
6	~~||||~~ ||			
7	~~||||~~ |			
8	||||			
9	||			
Totals:				

9 Two dice were thrown one hundred times and the total showing on the two upper faces was recorded to obtain this set of scores.

5	7	6	12	10	2	4	5	7	9
7	6	4	3	5	8	6	3	5	6
5	8	7	9	6	8	9	4	8	7
8	4	8	4	8	7	6	7	10	5
9	5	6	5	2	9	5	9	11	10
6	7	7	7	10	6	11	10	7	8
8	3	9	3	5	8	7	12	10	9
7	8	7	5	6	4	5	8	9	11
10	6	9	6	7	8	9	10	11	3
6	4	7	2	4	8	8	4	6	7

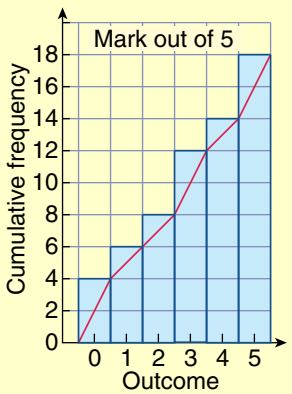
a By completing a frequency distribution table, determine the mode and mean of the set of scores.

b Complete a cumulative frequency column and use it to find the median.

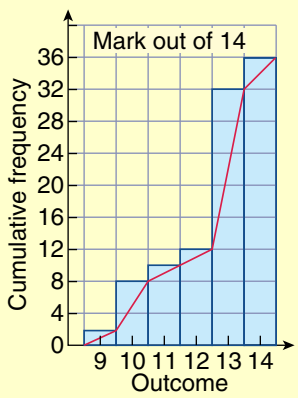
c Look at the cumulative frequency column to determine how many scores were less than the median.

10 Use the ogive to find the median from each graph.

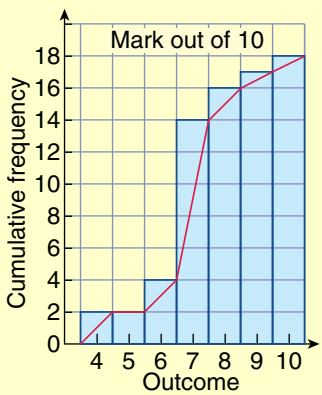
a



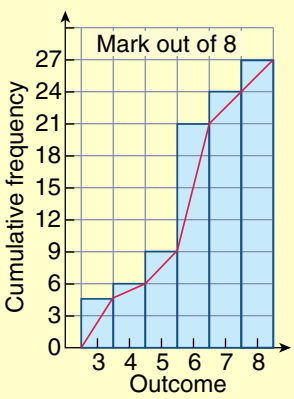
b



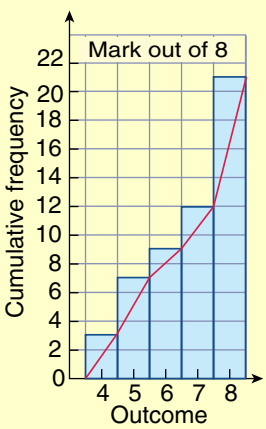
c



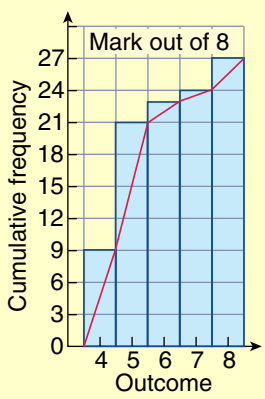
d



e



f



fun spot

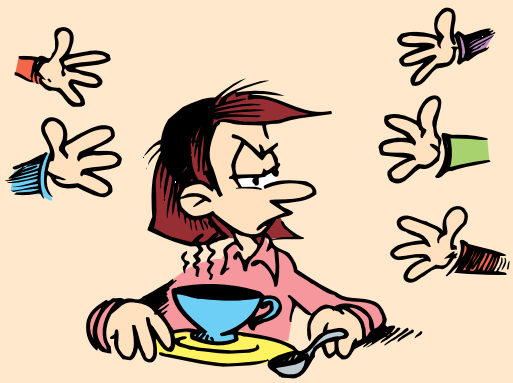
13:04

Fun Spot 13:04 | Which hand should you use to stir tea?

Work out each part and put the letter for that part in the box above the correct answer.

What is the probability of:

A throwing a 7 on one dice?
E throwing a sum of less than 14 on two dice?
E throwing a number less than six on one dice?
E selecting a club from a standard pack of cards?
H Selecting a 4 from a standard pack?
I selecting a picture card from a standard pack?
N selecting a King or a heart from a standard pack?
N getting an even number when a dice is thrown?



Use this frequency distribution table to find:

O the total frequency
O the frequency of 3
P the range of the scores
R the mode of the scores
S the median of the scores
S the cumulative frequency of 1
T the cumulative frequency of 4
U the mean correct to one decimal place.

x	f	$c.f.$
0	1	
1	7	
2	6	
3	3	
4	9	
5	1	

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{13}$	26	$\frac{1}{13}$	$\frac{5}{6}$	4	$2.\dot{5}$	2	1	0	8	5	3	27	$\frac{4}{13}$

Investigation 13:04 | Adding and averaging

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

B10				=SUM(B2:B8)												
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	*DAY*	*TAKINGS*		*DAY*	*TAKINGS*		*DAY*	*TAKINGS*		*DAY*	*TAKINGS*		*DAY*	*TAKINGS*	
2	27-Oct	$2,490		3-Nov	$3,260		10-Nov	$2,800		17-Nov	$2,570		24-Nov	$2,170	
3	28-Oct	$4,360		4-Nov	$4,040		11-Nov	$4,690		18-Nov	$2,920		25-Nov	$3,640	
4	29-Oct	$1,440		5-Nov	$1,420		12-Nov	$1,520		19-Nov	$2,360		26-Nov	$1,420	
5	30-Oct	$1,660		6-Nov	$1,960		13-Nov	$1,340		20-Nov	$1,100		27-Nov	$1,350	
6	31-Oct	$1,370		7-Nov	$1,180		14-Nov	$1,900		21-Nov	$1,170		28-Nov	$1,480	
7	1-Nov	$1,430		8-Nov	$1,230		15-Nov	$1,440		22-Nov	$1,700		29-Nov	$1,350	
8	2-Nov	$1,860		9-Nov	$1,510		16-Nov	$1,660		23-Nov	$1,550		30-Nov	$1,440	
9															
10		$14,610													
11															
12															

The calculating power of a spreadsheet makes it an extremely useful statistical tool.

Part of a spreadsheet is shown above. It shows the daily takings for the Lazy Lizard Café for the 5-week period from 27th October until the 30th November.

Note the formula **=SUM(B2:B8)** has been added in cell B10. This gives the weekly takings by adding the numbers in cells B2 to B8.

Questions

1. Add a formula to give the weekly takings for the other 4 weeks.
2. Write a formula that could give you the total of the 5 weeks sales.
3. Each row of the spreadsheet gives the takings for the same day of the week. For instance in row 2 the days are all Saturdays while in row 3 the days are Sundays.

The formula **=AVERAGE(B2,E2,H2,K2,N2)** will calculate the mean of the numbers in cells B2, E2, H2, K2 and N2. Typing this formula into cell O2 will give the average sales for Saturday.

Copy this formula into cells O3 to O8. Use the results to find the average sales for each day of the week from Saturday to Friday.

Which day of the week has the highest average takings?

Which day has the lowest average takings?

Why would cafe managers want to know which days of the week were the busiest?

Assessment Grid for Investigation 13:04 | Adding and averaging

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (C, D) for this investigation				Achieved ✓
Criterion C Communication	a	Little or no working out is shown and there is no explanation for what has been done.	1	
			2	
	b	Working out is shown and there is some interpretation of the results.	3	
			4	
	c	Working out is shown and the results are fully interpreted in the responses to the answers.	5	
			6	
Criterion D Reflection in Mathematics	a	There has been some attempt to explain what has been done and to check the results obtained in the spreadsheet.	1	
			2	
	b	Explanation of work done is given and results obtained have been checked with some success.	3	
			4	
	c	Detailed responses to the questions are given, relevant to the results, and the student has described further practical applications of spreadsheets.	5	
			6	

13:05 | Grouped Data

In cases where there are a large number of possible outcomes it is usually more convenient to group the outcomes into classes.

For example, if test marks out of 100 were being tallied, the range could be as large as 100, so a more convenient frequency distribution table might be constructed using class intervals of 10 marks, ie 1–10, 11–20, 21–30, etc.

Class could also be used where the data is 'continuous' rather than 'discrete'.

Continuous data arises from measurements that can assume any value within a class, eg heights of people or rainfall figures. In **discrete data**, each score is an exact value, usually an integer, eg throws of a dice or marks in a test.

The class size should be chosen so that you are using between 5 and 10 classes.

worked example

The percentage results for sixty students in an examination were:

78	63	89	55	92	74	62	69	43	90	91	83	49	37	58
73	78	65	62	87	95	77	69	82	71	60	61	53	59	42
43	33	98	88	73	82	75	63	67	59	57	48	50	51	66
73	68	46	69	70	91	83	62	47	39	63	67	74	52	78

The frequency distribution table for this set of data would look like this.

Class	*Class centre (c.c.)*	*Tally*	*Frequency (f)*	*f* × c.c.	*c.f.*
29–37	33	||	2	66	2
38–46	42	𝍸 |	5	210	7
47–55	51	𝍸 |||	8	408	15
56–64	60	𝍸 𝍸 ||	12	720	27
65–73	69	𝍸 𝍸 ||||	14	966	41
74–82	78	𝍸 ||||	9	702	50
83–91	87	𝍸 ||	7	609	57
92–100	96	|||	3	288	60
		Totals:	60	3969	

Note:

- The *class centre* (c.c.) column has been added.
- The *frequency* × *score* column has been changed to the *frequency* × *class centre* (*f* × c.c.) column.

For the measures of central tendency:

1 The *mode* becomes the '*modal class*'.
In the example above, the modal class is 65–73.

2 The *mean* is estimated by using the class centre as a representative figure for each class. So the mean is given by:

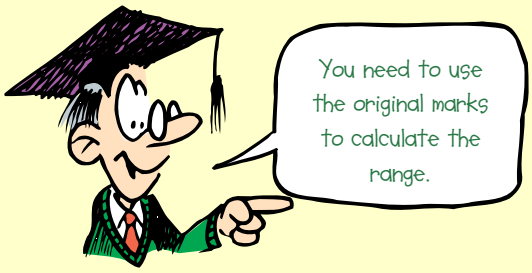
$$\bar{x} = \frac{\sum(f \times \text{c.c.})}{\sum f}$$

In the example above, $\bar{x} = \frac{3969}{60} = 66{\cdot}15$

3 The *median* becomes the *median class*.
In the above example there are 60 scores, so the middle score is the average of the 30th and 31st scores. Both lie in the 65–73 class, so the median class is 65–73.

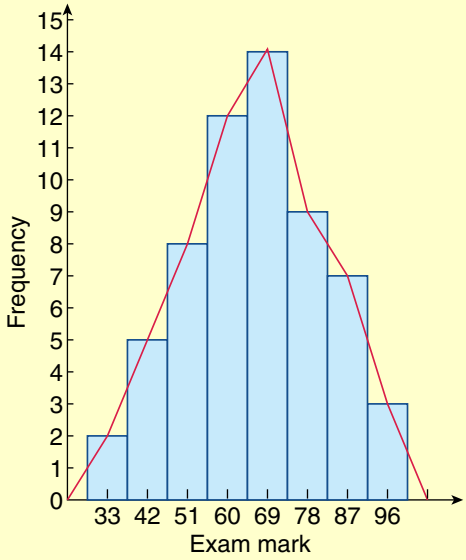
When constructing frequency diagrams for grouped data, the only point to note is that the columns are indicated on the horizontal axis by the class centres. The diagrams for the worked example above would look like these.

- The modal class 65–73 is represented by the class centre 69.
- The **frequency polygon** can be drawn by joining the midpoints of the tops of columns. To complete the polygon, assume that the classes on either side of the columns have zero members.

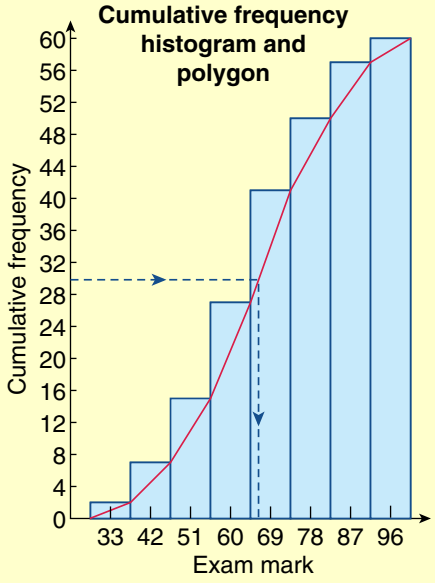


- The **cumulative frequency polygon** can be drawn by joining the top right corners of each column.
- There are 60 scores altogether so to find the median class we come across from 30 until we meet the polygon and then down to the horizontal axis.
- Clearly the median class is 65–73.
- An estimate of the median mark can be read from the horizontal axis, ie 67.

Frequency histogram and polygon



Cumulative frequency histogram and polygon



■ The dotted line shows the median class.

For grouped data:
- **Outcomes are grouped into classes.**
- **The modal class is the class with the highest frequency.**
- **The mean ($\bar{x}$) is calculated using the class centres.**
- **The median class is the class containing the middle score.**

Exercise 13:05

1 What would the class centre be for these classes?

a 6–10 **b** 15–21 **c** 31–39 **d** 5–8

2 Complete this frequency distribution table and find the modal class, mean and median class.

Class	*c.c.*	*Tally*	*Frequency*	$f \times c.c.$	*c.f.*
1–5		|||			
6–10		𝍸 ||			
11–15		𝍸 𝍸			
16–20		𝍸 |||			
21–25		||			
			$\Sigma f =$	$\Sigma(f \times c.c.) =$	

3 The heights of a group of forty students were measured to the nearest centimetre and recorded.

160	149	153	143	179	159	152	161	169	157
167	163	148	175	173	150	175	160	170	162
145	158	163	164	172	178	169	171	153	165
152	161	170	174	166	165	158	157	152	148

a Complete a grouped frequency distribution table with class intervals of 5, beginning with 141–145, 146–150, etc.
b What is the highest score? What is the lowest score? Therefore what is the range?
c Determine the modal class.
d Estimate the value of $\bar{x}$.
e Draw a frequency histogram for the data.
f Draw a cumulative frequency histogram and polygon. Find the median class.
g Why do you think the data was grouped using a class interval of 5 instead of 10?

4 The number of cans of drink sold by a shop each day was as follows. The highest and lowest scores are circled.

30	28	42	21	(54)	47	36	41	29	39
37	22	(18)	25	26	43	50	29	37	38
23	29	30	19	28	20	40	47	51	28
33	35	31	27	42	26	44	46	36	36
53	50	29	20	32	41	36	25	27	19
51	46	37	42	27	28	31	47	36	35
29	32	41	36	32	41	35	33	28	40

a Tabulate these results using classes of 16–22, 23–29, 30–36, 37–43, 44–50, 51–57.

b For how many weeks was the survey conducted?

c What was the mean number of cans sold?

d Construct a cumulative frequency histogram and ogive and find the median class.

■ An 'ogive' is a cumulative frequency polygon.

5 The exam results for Year 9 students were as follows.

69	32	75	51	67	36	88	3	71	7
95	81	40	99	20	51	16	67	75	20
82	62	28	15	87	76	27	46	32	13
14	49	17	10	28	83	25	55	24	73
41	22	37	39	25	16	37	72	21	49
37	90	34	14	81	52	69	66	89	66
8	84	29	14	69	28	75	36	7	50
40	51	62	13	64	24	47	47	86	77
50	56	47	42	39	97	16	65	62	45
52	89	71	43	61	98	74	63	12	10
96	46	52	90	15	59	67	69	54	74
28	55	70	43	56	80	73	63	26	21
86	41	49	44	69	30	93	21	28	72
16	31	52	88	53	56	27	32	10	35
34	68	56	88	62	82	5	34	41	49
71	95	92	86	76	48	85	59	25	10

a Using classes of 1–10, 11–20, etc, construct a frequency distribution table.

b Determine the:

i range **ii** modal class **iii** mean

c Construct a cumulative frequency histogram and ogive. From your graph, determine the median class.

d What percentage of students obtained:

i more than 80?

ii 20 or less?

iii more than 50?

Mathematical terms 13

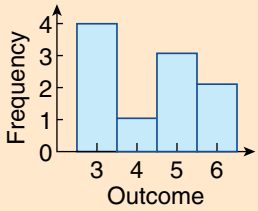
statistics
- The collection, organisation and interpretation of numerical data.

frequency
- The number of times an outcome occurs in the data.
  eg For the data 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 the outcome 5 has a frequency of 3.

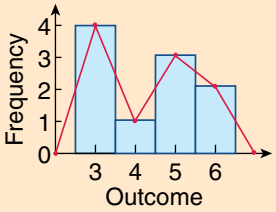
cumulative frequency
- The number of scores less than or equal to a particular outcome.
  eg For the data 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 the cumulative frequency of 5 is 8 (there are 8 scores of 5 or less).

frequency histogram
- A type of column graph showing the outcomes and their frequencies.

eg



frequency polygon
- A line graph formed by joining the midpoints of the top of each column. To complete the polygon the outcomes immediately above and below those present are used. The heights of these columns is zero.



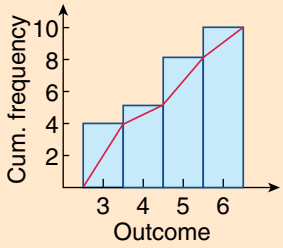
outcome
- A possible value of the data.

frequency distribution table
- A table that shows all the possible outcomes and their frequencies. (It usually is extended by adding other columns such as the cumulative frequency.)

eg

Outcome	*Frequency*	*Cumulative frequency*
3	4	4
4	1	5
5	3	8
6	2	10

cumulative frequency histogram (and polygon)
- These show the outcomes and their cumulative frequencies.



grouped data
- The organisation of data into groups or classes.

class interval
- The size of the groups into which the data is organised.
  eg 1–5 (5 scores); 11–20 (10 scores).

class centre
- The middle outcome of a class.
  eg The class 1–5 has a class centre of 3.

ogive
- This is another name for the cumulative frequency polygon.

mean

- The number obtained by 'evening out' all the scores until they are equal.
  eg If the scores 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 were 'evened out' the number obtained would be 4·3.
- To obtain the mean use the formula:

$$\text{Mean} = \frac{\text{sum of scores}}{\text{total number of scores}}$$

median

- The middle score for an odd number of scores or the mean of the middle two scores for an even number of scores.

mode (modal class)

- The outcome or class that contains the most scores.

range

- The difference between the highest and lowest scores.

median class

- In grouped data the class that contains the median.

Technology Applications Drag and Drops

Mathematical terms 13

- Cricket is one of many sports that uses statistics extensively. Both batsmen and bowlers have an average to measure their overall performance.

Diagnostic Test 13: | Statistics

- These questions reflect the important skills introduced in this chapter.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

diagnostic test 13

Section

1 The students of class 8M were given a reading test and rated from 0 (a non-reader) to 5 (an excellent reader). The results are given below. — 13:01

4	1	0	2	3	3	3	2	2	1
0	2	2	4	3	5	3	2	1	3
2	0	3	1	3	4	5	1	0	2

Outcome (x)	*Tally*	f	*c.f.*
0			
1			
2			
3			
4			
5			
Total:			

a Complete this frequency distribution table.

b What is the frequency of 5?

c How many students were given a rating less than 4?

d On the same diagram, draw the frequency histogram and the frequency polygon.

e On the same diagram draw the cumulative frequency histogram and the cumulative frequency polygon. — 13:02

f What is the range of these scores? — 13:03

g Find the mode, median and mean for these scores.

2 Use your calculator to evaluate the mean for the scores in: — 13:04

a

Outcome	*Freq.*
27	18
28	50
29	23
30	9

b

Outcome	4·1	4·2	4·3	4·4	4·5	4·6	4·7
Freq.	7	11	16	8	12	7	3

(Give your answer correct to two decimal places.)

3 Find the median for each of the following. — 13:04

a

Outcome (x)	*Cum. Freq.*
4	3
5	7
6	10
7	16
8	23

b

Outcome (x)	*Cum. Freq.*
11	7
12	15
13	33
14	53
15	62

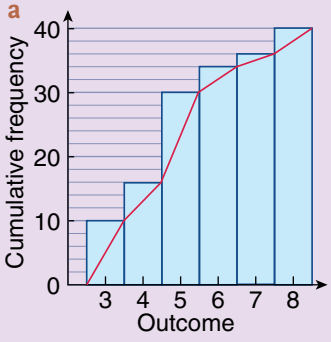
c

Outcome (x)	*Cum. Freq.*
1	24
2	37
3	44
4	47
5	50

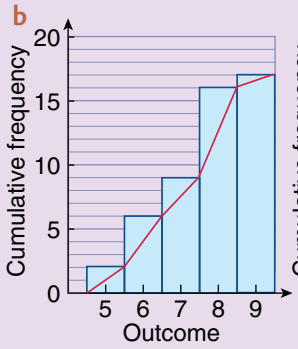
Section 13:04

4 Use the following ogives to calculate the median.

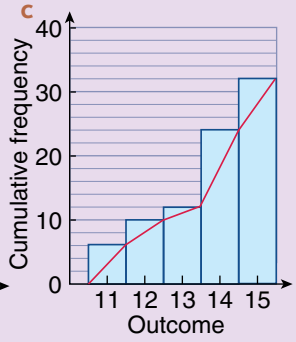
a



b



c



13:05

5 Complete each frequency distribution table and find the modal class, mean and median class.

a

Class	*c.c.*	*Tally*	*f*	*f* × *c.c.*	*c.f.*
10–19		𝍸 ||			
20–29		𝍸 𝍸			
30–39		𝍸 𝍸 ||			
40–49		|||			
50–59		|			

b

Class	*c.c.*	*f*	*f* × *c.c.*	*c.f.*
1–5		11		
6–10		18		
11–15		22		
16–20		10		
21–25		7		

c

Class	*c.c.*	*f*	*f* × *c.c.*	*c.f.*
150–154		3		
155–159		6		
160–164		16		
165–169		9		
170–174		6		

Chapter 13 | Revision Assignment

assignment 13A

1 In a large manufacturing company 100 samples are taken each day and tested. Over 40 days the following number of products were found to be faulty.

3	5	4	4	6	7	3	2	1	2
2	3	4	5	5	3	4	2	2	2
1	4	2	3	5	1	1	3	3	4
4	2	3	5	4	5	3	4	3	1

a Organise this data into a frequency distribution table.

b Calculate the mode, mean and median.

2 A group of Grade 8 students sat for a college reading test and the results were entered in this stem and leaf display.

a How many scored in the fifties?

b What was the mode of the results?

c How many students scored less than 50?

d How many students were tested?

e What was the median of the results?

f If 65 and above is considered to be a passing grade, how many passed?

g Make a grouped frequency table for this data using classes of 30–39, 40–49, . . .

Test Results, Grade 8
(7/3 represents 73)

Stem	*Leaf*
3	182
4	63313
5	4115
6	2284771
7	63465514
8	2667331
9	299100

3

Class	*Class centre (c.c.)*	*Frequency (f)*	*f × c.c.*
1–5		3	
6–10		7	
11–15		12	
16–20		8	
21–25		6	
26–30		5	

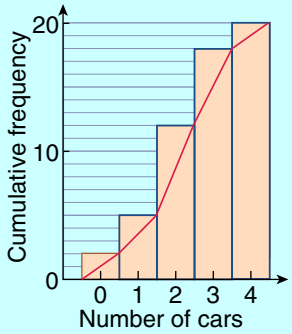
a Use the table to calculate:
i the modal class ii the mean

b Construct a frequency polygon.

c Is it possible to work out the range from this table?

d What is the median class?

4 The cumulative frequency diagram shows the results of a survey into the number of cars per household.



a How many households had no cars?

b How many households had at least 2 cars?

c What is the median number of cars per household.

5 A survey of families was conducted to find out how many children were in the family. Use the information in the table to answer the following questions.

No of children	*Frequency*
1	14
2	38
3	22
4	8
5	0
6	2

a How many families were surveyed?

b How many families had 1 or 2 children?

c Draw a frequency histogram to illustrate the data.

6 After the Grade 8 semester exam the maths staff organised the marks into a grouped frequency distribution. The results are shown in the table.

Class	*Class centre (c.c.)*	*Freq. (f)*	*Cum. Freq.*	*f × c.c.*
10–19		2		
20–29		9		
30–39		10		
40–49		8		
50–59		16		
60–69		20		
70–79		13		
80–89		14		
90–99		7		
	Totals:	99		

a Copy and complete the grouped frequency distribution table.
Use it to find:
i the modal class **ii** the mean

b Construct an ogive and use it to find the median class.

Technology Applications Activities

Sunburnt country

Chapter Review Questions

- Weather bureaus around the world keep statistics on many aspects of weather. Average rainfall and average temperature are often quoted in weather broadcasts.

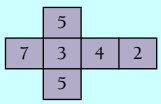
Chapter 13 | Working Mathematically

1 Give the names of the shapes found in this leadlight window.

2 Use ID Card 6 on page xviii to identify:

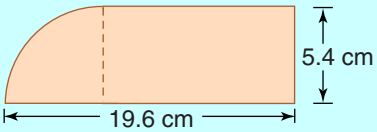
a	1	**b**	2	**c**	3	**d**	4	**e**	13
f	14	**g**	15	**h**	16	**i**	21	**j**	24

3 When this net is folded to form a cube, three faces meet at each vertex. The three numbers on faces meeting at each vertex are multiplied together. What is the smallest product possible?

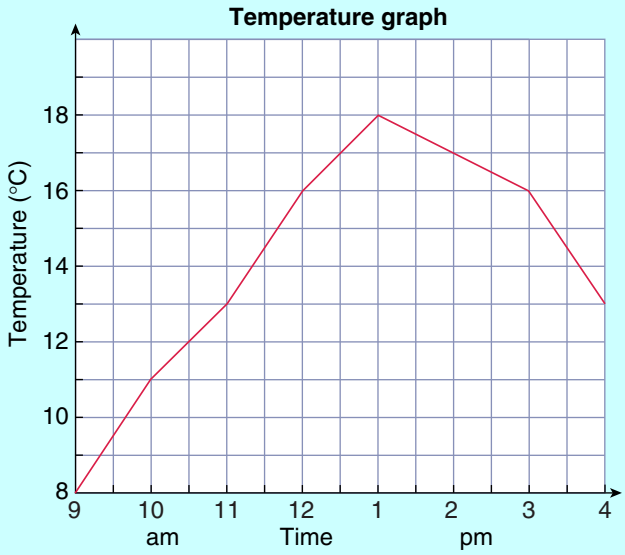


4 In a tennis match, 4 players were always on the court. Altogether there were 12 players (6 from each team). Each of the players was on the court for the same length of time. How many minutes did each player spend on the court if the match took 360 minutes to play?

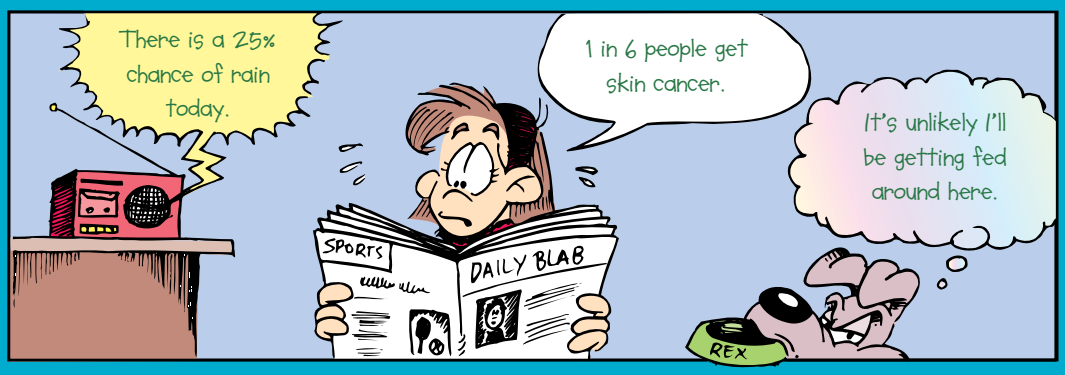
5 Find the perimeter and area of this figure.



6 **a** What was the temperature at 9:00 am?
b At approximately what time in the morning did the temperature reach 14°C?
c What was the approximate temperature at 2:30 pm?
d For how long was the temperature above 16°C?
e In what hourly periods did the greatest increase in temperature occur?



14 Probability



Chapter Contents

Learning Outcomes

Students will be able to determine relative frequencies and theoretical probabilities.

Areas of Interaction

Approaches to Learning, Environment, Homo Faber

14:01 | Describing Your Chances

Language

We need to know the language of chance or probability.

certain:	will always happen
very likely:	will usually happen
impossible:	will never happen
very unlikely:	usually won't happen
even chance:	is just as likely to happen as it is not to happen
at random:	to choose so that each outcome is just as likely to happen
probability:	the calculated chance of something happening

prep quiz 14:01

For each of these events, estimate its probability of happening to be:
A an even chance, **B** less than an even chance, or **C** more than an even chance.

1. If I toss a coin I will get a *head*.
2. The next person I see driving a car will be over 20.
3. If I take a card at random from a standard pack, it will be a club.
4. If I throw a dice, I will get an even number.
5. The next child born in the hospital will be female.
6. I will see a thylacine on the way home from school.
7. If I choose a class in Year 9, at least 3 of the students will have a computer in their home.
8. If I throw two dice, their sum will be greater than 9.
9. If I throw two coins, I will have either two heads or two tails.
10. In my next maths test, I will score over 80%.

worked example

Heather is going to throw a dice.

a List all of the possible outcomes.

b Use the language at the beginning of this section to describe the chance that:
- **i** Heather will throw a four
- **ii** Heather will throw a number less than four
- **iii** Heather will throw a seven
- **iv** Heather will throw a number less than seven.

Solution

a If a dice is thrown the possible outcomes are: 1, 2, 3, 4, 5, and 6.

b **i** Since only one number out of six is a four, the chance of throwing a four would be unlikely.

ii Since three of the six numbers are less than four, there is an even chance that this will happen.

iii There is no seven on a dice, so throwing a seven is impossible.

iv All six numbers that can be thrown are less than seven, so it is certain that the number thrown will be less than seven.

Subjective probability

- Subjective probabilities are probabilities based on opinions or judgements. All of the facts may not be known and we make the best prediction we can.
  eg 'There is a high probability of rain tomorrow.'
  I would have more confidence in this judgement if it were given by a meteorologist than if it were given by my son.
- Assessments of the probability of success of Germany or Italy as they play each other in the next World Cup will vary greatly according to the bias of the person asked. A more accurate assessment of the chance of success for eachteam would come from an unbiased observer who has studied the form of each team.
- Remember that the most likely outcome will not always occur.

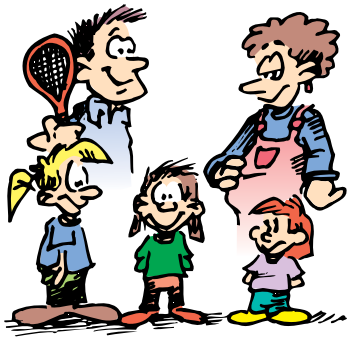
worked example

Rhonda and Alan had three girls. Rhonda was expecting their fourth child.

'I hope this child is a boy,' said Alan.

'After three girls, it's almost sure to be a boy,' said a friend.

Is the friend's reasoning correct?



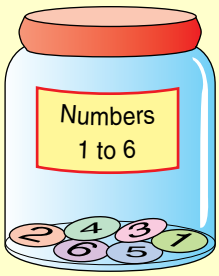
Solution

- For each birth, the chance of a boy will always be $\frac{1}{2}$ irrespective of the number of girls previously born.
  So the friend's reasoning is not correct.
- There is a chance however that the opposite is true.
  Some males may have a larger percentage of sperm of one sex. The probability of having a girl may be much higher than $\frac{1}{2}$. Only medical tests could reveal this.

Exercise 14:01

1 The numbers 1 to 6 were written on six counters and placed in a jar. Two of these counters are to be chosen at random.

a List all possible outcomes of this experiment.

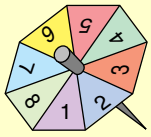


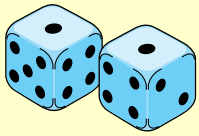
Choose a label from those on the right to answer each question below.

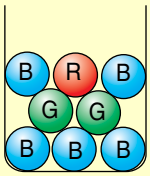
For the experiment of the six counters in the jar, what is the chance that:

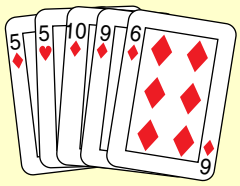
b the numbers 0 and 1 are picked?
c the numbers 5 and 6 are picked?
d two numbers less than 10 are picked?
e the first number selected will be even?
f the sum of the numbers will be odd?

impossible
not likely
even chance
likely
certain









2 Using the diagrams above, list all possible outcomes if:

a the spinner is spun once
b the two dice are thrown and the numbers on the upper faces added
c two balls are drawn from the container at random
d two cards are chosen at random

3 The spinner above is spun once. Choose the label to the right that best describes the probability that the number will be:

a 1 **b** not 8 **c** more than 5
d odd **e** 0 **f** less than 10

4 The dice above are thrown. Choose the label that best describes the probability that the sum of the top numbers will be:

a 12 **b** less than 12 **c** less than 6
d even **e** 1 **f** less than 15

5 A coloured ball is drawn at random from the container above. Which label best describes the probability that the ball is:

a blue? **b** green?
c red? **d** coloured?

Write the events in parts **a** to **d** in order, from least likely to most likely.

Chance

impossible
very unlikely
unlikely
even chance
likely
very likely
certain

6 Two cards are drawn at random from the five above. Which label best describes the probability that the cards:

a are both fives?
b have a sum equal to 15?
c have a sum greater than 9?
d are both larger than 9?

Write the events in parts **a** to **d** in order, from least likely to most likely.

7 Sri Lanka is playing cricket against England at R Premadasa Stadium, Colombo.

a From the list below, select the five factors that you think would most influence the outcome of the match. Give a reason for each choice.

- The colour of the uniforms
- Who wins the toss
- The recent form of the players
- The number of tickets sold
- The country where it is played
- The condition of the wicket
- The weather
- The choice of umpires
- The length of the grass
- The ability of the captains

b Who do you think would win? Give a reason for your answer.

c If Sri Lanka has never lost in Colombo, is it possible that Sri Lanka will lose?

d Are the only possible outcomes: Sri Lanka wins or Sri Lanka loses? Explain your answer.

8 Is each statement reasonable or not? Discuss the reasoning in each case.

a On my last four attempts, I have failed to lift this weight. I'm sure to succeed this time.

b My diamond ring is in one of these three jewellery boxes. I have searched two and it is not in them. It must be in the third one.

c We almost pushed the car out of the bog that time. Now that Chub is here to help, I'm sure we will get it out this time.

d The first three puppies were female. The next is sure to be male.

e I've turned 48 cards of this pack and have turned only one ace. The next card I turn is sure to be an ace.

f I've just heard that a fire storm has passed through our street. I fear our home has been destroyed.

g I'm glad it's raining today because that means it won't rain for tennis tomorrow.

9 Discuss the contradictory nature of these statements.

a The horse was a certainty beaten.

b I was so close to winning the lottery. My number was 908 146. The winning number was 808 146.

c The winner came from an impossible position.

10 Comment critically on the statement: 'Since there are 26 letters in the English alphabet, the probability that a person's name starts with X is 1 in 26'.

Investigation 14:01 | Throwing dice

14:01

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation

Throw two dice and record the result. Carry out this experiment 50 times.

1 Use your results to complete the table below.
Note: One result may be entered against several outcomes.

2 Choose an expression from those in the list to the right that best describes the probability for each outcome.

Outcome	*Tally*	*Frequency*
12		
2		
less than 12		
less than 6		
even		
1		
less than 10		

Probability
impossible
very unlikely
unlikely
even chance
likely
very likely
certain

3 Compare your findings with the answers to Exercise 14:01, question 4.

14:02 | Experimental Probability

14:02

A dice was rolled several times and the following results obtained: 5, 1, 2, 5, 6, 5, 2, 4, 6, 3

1 How many times was the dice rolled?
2 How many times was the result a 5?
3 What fraction of the throws were 5s?
4 What fraction of the throws were 2s?
5 What fraction of the throws were odd numbers?
6 A coin was tossed 10 times, resulting in 4 heads. How many tails were there?
7 A coin was tossed several times and $\frac{2}{5}$ of the results were tails.
What fraction of the tosses were heads?

Simplify: **8** $\frac{4}{12}$ **9** $\frac{16}{20}$ **10** $\frac{24}{50}$

One way of determining the chance of something happening is by observing what occurs in a sample 'experiment'.

If simple equipment such as coins, dice, spinners, cards or random numbers are used to represent real events, then the 'experiment' is called a **simulation**.

Experimental probability formula:

The experimental probability of an event $=$ $\frac{\text{number of times this event occurred}}{\text{total number in sample}}$

Assessment Grid for Investigation 14:01 | Throwing dice

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	An organised approach has not been attempted.	1	
			2	
	b	An organised approach has been attempted and there has been some interpretation of the results.	3	
			4	
	c	An organised approach has been used and some results and patterns have been described.	5	
			6	
	d	The correct results have been justified using a successful strategy, with a full comparison made in question 3.	7	
			8	
Criterion C Communication	a	Little or no working out is shown and presentation is poor.	1	
			2	
	b	Working out is shown and results have been recorded in a systematic way. Some interpretation of the results is given.	3	
			4	
	c	Working out is shown and results have been recorded in a systematic way. Results are interpreted fully using symbols and/or words.	5	
			6	
Criterion D Reflection in Mathematics	a	Some attempt has been made to describe the results obtained.	1	
			2	
	b	The results obtained are described and an attempt has been made to check them against theoretical probability.	3	
			4	
	c	Detailed justification of the results has been given, as well as a thorough comparison of theoretical and experimental probability.	5	
			6	

More and more statistics are being collected (empirical evidence) from which predictions can be made. Probabilities based on this evidence are used to determine the cost of insurance, life expectancy and the likelihood of events occurring. These estimates are often called empirical probabilities and are a type of experimental probability. If Australia had beaten England at the SCG four of the last five times they have played there, then it would be highly likely that Australia will win next time.

Experimental probabilities are usually based on an examination of a **sample** or **trial run** of the activity under examination.

worked examples

1 A farmer collects 10 eggs and finds that 2 of them are bad. If he chose another egg what is the chance of getting another bad one?

2 The contents of 20 matchboxes were examined and the results recorded.

Number of matches	48	49	50	51	52	53
Number of boxes	1	5	8	3	2	1

If the contents of a similar box of matches were counted, what would be the experimental probability that it would contain 50 matches or more?

Solutions

1 Since 2 of the first 10 eggs were bad, it seems that $\frac{2}{10}$, or $\frac{1}{5}$ of the farmer's eggs might be bad.
So, *if* the first 10 eggs were truly representative of all the farmer's eggs, *then* the chance of picking another bad one is $\frac{1}{5}$, or 1 out of 5.

■ The probability of an event occurring in an experiment is the same as its 'relative frequency'.

2 In the sample, 14 of the 20 boxes had 50 or more matches.

$$\text{Experimental probability} = \frac{\text{number of times this event occurred}}{\text{total number in sample}}$$
$$= \frac{14}{20}$$

In boxes like these we would expect the chance of choosing one with 50 or more matches to be $\frac{14}{20}$ or $\frac{7}{10}$.

Exercise 14:02

Foundation Worksheet 14:02
Experimental probability
1 In a class of 24, 12 were blonde, 10 were brunette, 2 were redheads. Find the probability that one student chosen at random will be:
a blonde **b** not brunette

1 The first 100 vehicles to pass a checkpoint gave the results in the table. If these figures truly represent the traffic at any time past this checkpoint, determine the experimental probability that the next vehicle will be:

a a car **b** a motor cycle
c a bus **d** *not* a car
e *not* a car or truck

Type of vehicle	*Frequency*
Cars	70
Trucks	15
Motor cycles	10
Buses	5

2 We examined the contents of 37 packets of coloured sweets. The average number of each colour in a packet is shown in the table. One sweet is taken at random from a new packet. Use these results to determine:

a which colour is most likely to be picked
b which colour has the least chance of being picked
c the probability that it is red

Colour	*Frequency*
Brown	10
Green	5
Red	6
Yellow	3

Total = 24

3 A factory tested a sample of 100 light bulbs and 5 were found to be faulty. From these results, what is the probability of buying a faulty light bulb? What is the probability of buying a good bulb?

4 Farmer Fowler recorded the number of eggs his chickens laid each day, for 6 weeks. The results are shown in the table. If these results are typical for Farmer Fowler's chickens at any time of the year find, as a fraction, the probability that on any particular day the number of eggs laid is:

a 2
b 4
c 2 or more

Convert each of your answers to a percentage correct to the nearest whole per cent.

No. of eggs	*No. of days*
0	8
1	13
2	10
3	6
4	3
5	2

Total = 42

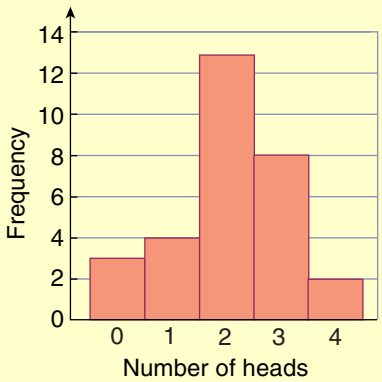
5 A survey of 100 households was taken to determine how many used certain washing powders. Based on these results, what is the probability of a household chosen at random:

a using Foam brand?
b using Supersoap?
c not using Pow?
d not using any of these four brands?

Brand	*Number*
Foam	18
Suds-O	27
Supersoap	20
Pow	15

6 Jenny tossed four coins 30 times and the number of heads was recorded each time. The histogram shows the results.

a From this experiment what is the probability that when four coins are thrown there will be:
  i no heads?
  ii two heads?
  iii at least three heads?
b If this experiment were to be repeated, would you expect the same results?



7 A dice was thrown 50 times and the results were recorded. Using the results in the table:

a What is the experimental probability of throwing:

i a six?

ii a two?

iii an odd number?

b Would you expect to get seven ones every time a dice is thrown 50 times?

Number shown	*Frequency*
1	7
2	5
3	5
4	10
5	9
6	14

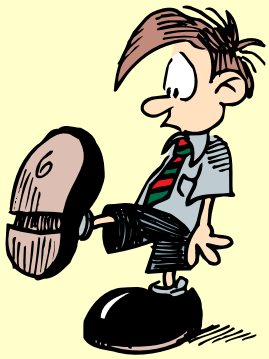
8 $\frac{4}{20}$ of Grade 8 students have a shoe size greater than $10\frac{1}{2}$, but $\frac{19}{20}$ have a shoe size less than $11\frac{1}{2}$. What is the chance of a Grade 8 student having a shoe size:

a less than or equal to $10\frac{1}{2}$?

b $11\frac{1}{2}$ or larger?

c between $10\frac{1}{2}$ and $11\frac{1}{2}$?

(*Note:* This is every size not in **a** and **b**.)



9 **a** Order the following events, from least likely to most likely.

A There will be a hail-storm tomorrow.

B The next person to visit my home will be male.

C My next maths teacher will be over 60 years of age.

D I will see the principal of our school next week.

E At least one member of our class will be married within 10 years.

b Estimate the probability of each event listed in part **a**, giving each on the scale 0 to 1.

10 Explain the meaning of a probability of:

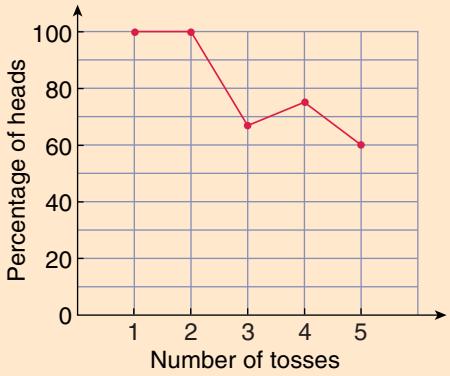
a 0 **b** $\frac{1}{2}$ **c** 1

d 0·5 **e** 100% **f** 50%

Reading mathematics 14:02 | Tossing a coin

When tossing a coin we assume that the probability of getting a *head* is $\frac{1}{2}$ or 50%, but is this true?

Luke tossed a coin five times and graphed the percentage of *heads* after each toss.
He tossed: *head*, *head*, *tail*, *head*, *tail*.
His graph is shown below.



- Percentage of *heads* after 3 tosses
  $= \frac{2}{3} \times 100\%$
  $= 66\frac{2}{3}\%$
- Percentage of *heads* after 4 tosses
  $= \frac{3}{4} \times 100\%$
  $= 75\%$

1 Toss a coin ten times and graph the percentage of *heads* after each toss. Did the percentage get closer to 50% as the number of tosses increased?
2 Would this experiment be a reasonable simulation for the gender of babies born in a local hospital?
3 If you repeat this investigation, would you obtain the same graph? In what way would the second graph resemble the first?

- Is it reasonable to assume that the probability is $\frac{1}{2}$? Explain.
- Does any bias exist in the design of this experiment?

- I often play my son Luke at chess. How could I estimate my chances of beating him in our next game?

14:03 | Theoretical Probability

prep quiz

14:03

Simplify:

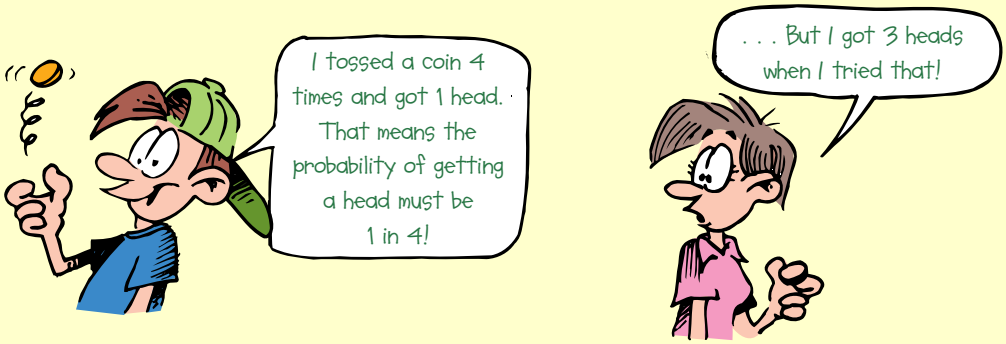
1 $\frac{21}{24}$ **2** $\frac{34}{51}$ **3** $\frac{39}{91}$ **4** $1 - \frac{13}{16}$ **5** $1 - \frac{27}{30}$

A bag contains three white, five red and four black marbles. What fraction are:

6 white? **7** black? **8** not black?

9 What fraction of the letters of the alphabet are vowels?

10 What fraction of integers from 1 to 50 inclusive are prime?

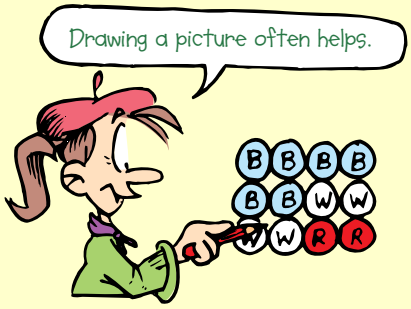


Performing an experiment will not always give a consistent result, or even a result we may think is most likely to occur.

In many cases we can work out the expected or theoretical probability of an event by considering the possible outcomes. For example, when tossing a coin there are two possible outcomes, a head or a tail.

Since there is only one head, the probability of throwing a head would be 1 out of 2, ie $\frac{1}{2}$.

■ When calculating the probability of an event we shall assume that each possible outcome is equally likely, ie no two-headed coins or loaded dice.



worked examples

1 If a dice is rolled, what is the probability of getting:
   a a six? **b** an odd number? **c** a number less than seven?

2 In a bag there are six blue marbles, four white marbles and two red marbles. What is the probability of choosing at random:
   a a blue marble? **b** a blue or white marble? **c** a pink marble?

Solutions

1 The possible outcomes when rolling a dice are 1, 2, 3, 4, 5, 6. So the number of possible outcomes is 6.

a The number of sixes on a dice is 1. So the probability of throwing a six is 1 out of 6, or $\frac{1}{6}$. This can be written as:
$P(6) = \frac{1}{6}$

b The number of odd numbers on a dice is 3. So the probability of throwing an odd number is 3 out of 6.
$P(\text{odd no.}) = \frac{3}{6}$
$= \frac{1}{2}$

c Since all six numbers on a dice are less than seven, the probability of throwing a number less than seven is:
$P(\text{no.} < 7) = \frac{6}{6}$
$= 1$

■ The probability of an event *certain* to happen is 1.
P(sure thing) = 1

2 The total number of marbles in the bag is twelve. So the number of possible outcomes is 12.

a Number of blue marbles is six.
$\therefore P(\text{blue marble}) = \frac{6}{12}$
$= \frac{1}{2}$

b Number of blue or white marbles is ten.
$\therefore P(\text{blue or white}) = \frac{10}{12}$
$= \frac{5}{6}$

c Number of pink marbles is zero.
$\therefore P(\text{pink}) = \frac{0}{12}$
$= 0$

■ The probability of an event that *cannot happen* is 0.
P(impossibility) = 0

If each possible outcome is equally likely, then:

$$\textbf{probability of an event, } P(E) = \frac{n(E)}{n(S)}$$

where $n(E)$ = number of ways the event can occur
$n(S)$ = number of ways all events can occur
(S is used to represent the sample space, which is the set of possible outcomes.)
The probability of any event occurring must lie in the range $0 \leqslant P(E) \leqslant 1$.

It must be pointed out that the probabilities of each possible event must add up to 1. As a consequence of this, if the probability of an event occurring is P(E), then the probability of E *not* occurring is 1 – P(E).

■ P(E') = 1 – P(E) where P(E') is the probability of E not occurring.

E' is set notation for the 'complement' of E, ie those outcomes outside of E. For example:

- The complementary event for getting an even number after rolling a dice is getting an odd number.
- The complementary event for drawing a red card from a deck of cards is drawing a black card.

Exercise 14:03

Foundation Worksheet 14:03

Theoretical probability

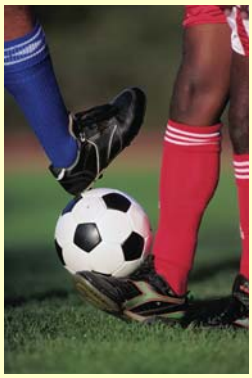
1 A coin is tossed.
Find the probability of:
a head **b** tail

2 Four cards are labelled A, B, C, D.
Find the probability of selecting the:
a A **b** B or C

1 A single dice is thrown. What is the probability of getting:
- **a** a one?
- **b** an even number?
- **c** a number less than 3?

2 Ten coloured discs are placed in a hat. Five are red, three are yellow and two are black. If one disc is drawn from the hat, what is the probability that the disc will be:
- **a** red? **b** black? **c** red or black?
- **d** not black? **e** blue? **f** red, yellow or black?

3 For each event given here, write the complementary event.
- **a** Getting an *odd number* after a dice is thrown.
- **b** Getting a *tail* when a coin is tossed.
- **c** Getting *a number less than 6* when a dice is thrown.
- **d** Drawing a *spade* from a standard deck of playing cards.
- **e** Seeing *red* displayed on a traffic light that is working.
- **f** *Winning a soccer match.*
- **g** Choosing a *vowel* from the letters of the alphabet.



- What is the complement of winning?

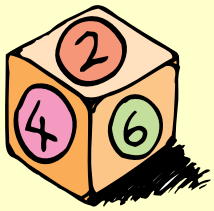
4 From a standard pack of 52 playing cards, a card is drawn at random. What is the probability that the card is:
- **a** the Ace of diamonds? **b** a King?
- **c** a red card? **d** a spade?
- **e** a black Jack? **f** a 7 or 8?
- **g** a picture card (Jack, Queen or King)?

5 The 26 letters of the alphabet are written on cards and placed in a box. If one card is picked at random from the box, what is the chance that the letter on it will be:

a X? **b** a vowel?
c M or N? **d** a letter in the word *MATHEMATICS*?

6 Stickers were placed on a dice so that the faces showed three 2s, two 4s and a 6. If the dice is now thrown, what is the probability that the upper surface will be:

a a 2? **b** a 4? **c** a 6?
d even? **e** odd? **f** less than 6?



7 Six students, John, Bob, Joan, Helen, Betty and Janet, each wrote their name on a piece of paper and put it in a hat. If one name was drawn at random, what is the probability that the name would:

a be Helen? **b** start with J?
c be a girl's name? **d** have four letters in it?

8 A letter is chosen at random from the word *HOKKAIDO*. What is the probability that the letter is:

a O? **b** a consonant?
c one of the first four letters of the English alphabet? **d** Z?

9 The numbers from 1 to 25 are written on cards. If one card is chosen at random, what is the probability that the number on the card will be:

a an odd number? **b** a multiple of 5?
c a factor of 24? **d** a number that contains the digit 2?

10 A five-digit number is to be formed using the digits 1, 2, 3, 4, 5. What is the probability that the number will:

a be odd? (It ends with 1, 3 or 5.) **b** be even? (It ends with 2 or 4.)
c start with 5?
d be greater than 30 000? (It starts with 3 or 4.)
e be divisible by 3? (*Hint:* Is the sum of the digits divisible by 3?)

11 If the probability of an event is $\frac{1}{5}$, how many times, on average, would you expect it to occur in 20 trials? Can you say for certain how many times it will occur?

Challenge worksheet 14:03 Probability: An unusual case

Mathematical terms 14

complementary event

- The *opposite* event that covers all possibilities.
  eg if the event is throwing a '6' the complementary event would be 'not throwing at 6'. Similarly the complementary event to selecting a girl would be selecting a boy.

experimental probability

- Determining the chance of an event occurring by observing what happens in a sample experiment.

$$\text{exp. prob.} = \frac{\text{no. of times event occurred}}{\text{total number in sample}}$$

mutually exclusive events

- Events that cannot occur at the same time.
  eg throwing an even number and throwing a three with a dice.

outcomes

- The possible results when calculating a probability.
  eg the outcomes when tossing a coin are heads or tails.

probability

- the calculated chance of an event happening.

random

- Without predetermination.
- To choose 'at random' means that each outcome is equally likely to occur.

sample

- The list of possible outcomes.
  eg when throwing a dice the sample would be 1, 2, 3, 4, 5, or 6.

simulation

- An experiment which uses simple equipment such as cards or coins to represent a real event.
  eg using heads or tails to represent the birth of boys or girls.

survey

- To gather data or information from which conclusions might be drawn.
  eg to count the number of trucks in a line of traffic or to question a group of students about their favourite music.

theoretical probability

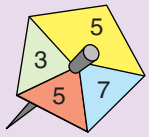
- The probability of an event that is determined by considering the possible outcomes.
  eg the probability of throwing a '6' with a dice is $\frac{1}{6}$, because there are 6 equally likely outcomes.

Diagnostic Test 14: | Probability

- These questions reflect the important skills introduced in this chapter.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

Section

1 List all possible outcomes of each experiment below. — 14:01



- **a** A dice is thrown.
- **b** A coin is tossed.
- **c** The spinner on the right is spun.

2 The points scored by our middle school basketball team in the last 44 games is shown in the table. — 14:02

My score	35–39	40–44	45–49	50–54	55–59
Frequency	3	20	14	5	2

We are about to play another game. What is the experimental probability that our score will be:

- **a** lower than 40?
- **b** higher than 39?
- **c** higher than 59?

Explain why the experimental probability that our score is higher than 59 is not the real probability.

3 In a bag there are 12 blue Lego blocks, eight white Lego blocks and four red Lego blocks all the same size. A block is chosen at random. What is the probability of choosing: — 14:03

- **a** a blue block?
- **b** a blue or a white block?
- **c** a pink block?
- **d** anything but a white block?

Chapter 14 | Revision Assignment

assignment 14A

1 A dice is rolled. What is the probability that the uppermost face will show:

- **a** a 1?
- **b** a 0?
- **c** a number less than 4?
- **d** an even number?
- **e** a number divisible by 3?

2 Presuming that the figures shown in the table are typical of the students in a certain international school, what is the probability of a person chosen at random from this town:

Eye colour	*Number*
Brown	110
Blue	70
Green	20

- **a** having blue eyes?
- **b** having green eyes?
- **c** *not* have blue eyes?

3 A toy box contains five red balls, four green balls and a yellow ball, all the same size. If a ball is chosen from the box at random, what is the chance of getting:

- **a** a green ball?
- **b** a yellow ball?
- **c** a green or yellow ball?
- **d** a blue ball?

4 A game is played by picking counters numbered 1 to 50 out of a bag. The table shows the results for picking various counters.

Counter	*Result*
1	Win $1
A prime number	Win 50c
A number ending with a 4 or a 6	No result
Any other number	Lose 50c

If Huong picks a counter at random, what is the probability that she will:

- **a** lose money?
- **b** neither win nor lose?
- **c** win money?
- **d** not lose money?

5 Liam threw three darts at a dartboard 20 times to gain the following scores.

56	74	53	85	52
57	53	91	37	82
120	100	73	45	55
42	75	169	40	76

- **a** Going on his past performances, what is the probability that Liam's next score will be:
  - **i** greater than 80? **ii** less than 70?
- **b** Liam did not score over 170. Does this mean that the probability of him scoring 170 or more is zero? Comment on your answer.

6 When two teams play football there are three possible results. Each team could win and the game could be drawn. Therefore the probability that a particular team wins is $\frac{1}{3}$. Is this statement correct? Justify your answer.

Technology Applications Drag and Drops
1 Rolling two dice
2 Pack of cards

Probability Technology Applications Activities

assignment
14B

Chapter 14 | Working Mathematically

1 Use ID Card 6 on page xviii to identify:

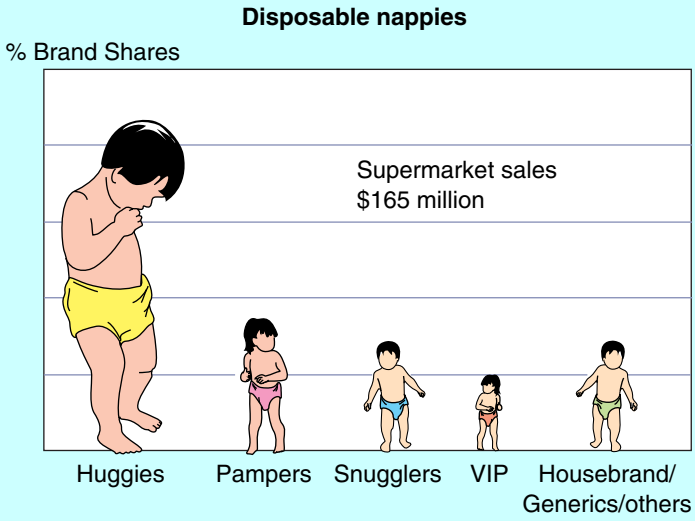
a 6	b 7	c 8	d 12
e 14	f 15	g 17	h 18
i 19	j 22		

2 Use the Algebra Card on page xxii to:
- a add column M to column N
- b subtract column M *from* column N
- c add columns N and O
- d find the value of the terms in J if $x = -2$

3 Through how many degrees does the spoke of a wheel turn in 5 minutes if the wheel is turning 700 revolutions per minute?

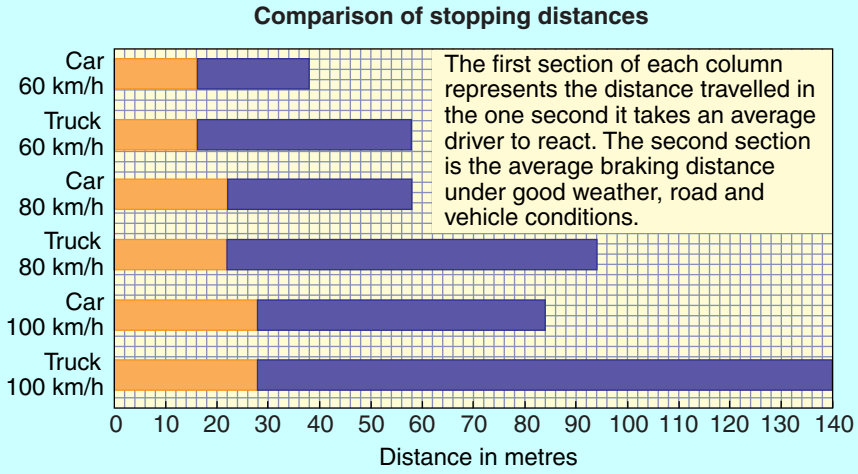
4 Indu is eight years younger than John, but John is twice her age. How old is Indu?

5 This graph appeared in 1992. The scale has been removed. 100% of sales are represented here.



Estimate the percentage of the sales belonging to Huggies if:
- a the *height* of each picture is the significant measure
- b the *area* of each picture is the significant measure
- c the *volume* of the child represented in each picture is the significant measure

6 The graph is a comparison of the stopping distances of a car and truck at speeds of 60 km/h, 80 km/h and 100 km/h.



The reaction distance is the distance travelled by the vehicle in the time it takes the driver to take his/her foot off the accelerator and apply the brakes. The braking distance is the distance travelled after the brakes are applied. Stopping distance = reaction distance + braking distance.

a What is the reaction distance at:
- **i** 60 km/h?
- **ii** 80 km/h?
- **iii** 100 km/h?

b What is the braking distance for:
- **i** a truck travelling at 80 km/h?
- **ii** a car travelling at 80 km/h?

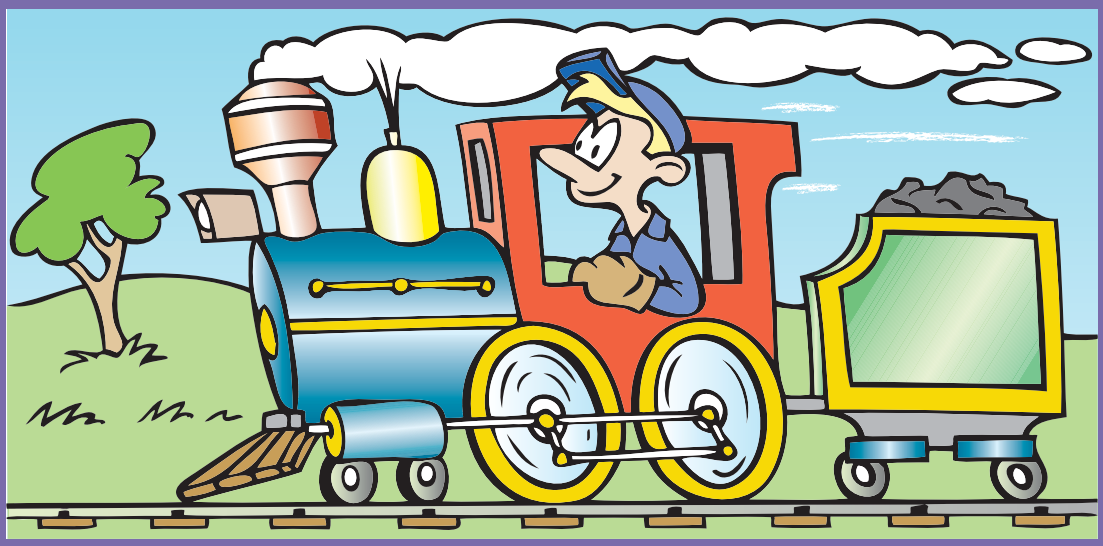
c What is the difference in stopping distances for a car and truck travelling at:
- **i** 60 km/h?
- **ii** 80 km/h?
- **iii** 100 km/h?

d If a truck is following a car, both travelling at 80 km/h and both drivers brake simultaneously, find:
- **i** the braking distance of the car
- **ii** the braking distance of the truck
- **iii** how far the truck must be behind the car to avoid a smash

Chapter Review Questions

15

Networks and Topology



Chapter Contents

Learning Outcomes

Students will be able to:

- Solve problems using networks.
- Understand what is meant by a Eulerian trail and a Hamiltonian path.
- Understand the basic concept of topology through the use of networks.

Areas of Interaction

Approaches to Learning, Homo Faber, Environment, Community and Service

15:01A | Graph Networks: A Review of the Basics

In Book 2 you were introduced to graphs and networks and the following definition:

A **graph** is a series of dots that may or may not be connected to each other by lines.

These dots we called **vertices** and the lines we called **edges**. The **degree** of a vertex is determined by how many edges join it to other vertices. The number of vertices a graph has is the **order** of the graph.

The examples below of how graphs are used came mainly from transportation networks. Many of these types of graphs are called **networks**.

The network, or graph, in Figure 1 represents the Hong Kong MTR train system. This is not exactly how the train lines run but is much easier to follow than the real map in Figure 2.

Figure 1 Hong Kong MTR Network.

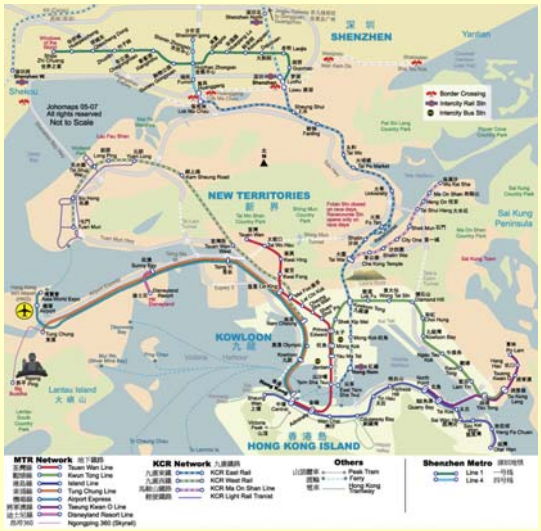


Figure 2 Map of Hong Kong showing MTR lines.

If a part of a graph is taken and drawn separately, it is know as a **subgraph**. For example, the Island Line of the Hong Kong MTR is shown in Figure 3. It is a subgraph of Figure 1.

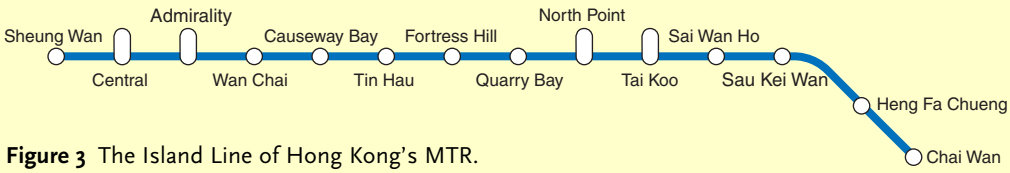
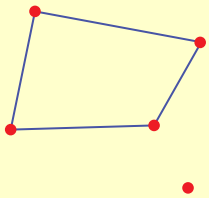


Figure 3 The Island Line of Hong Kong's MTR.

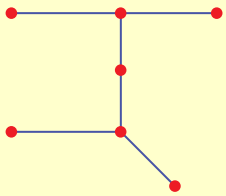
In a **connected graph**, it is possible to travel along edges from one vertex to any other vertex. All the examples shown so far are connected graphs. The graph in Figure 4 is a **disconnected graph**.

Figure 4 A disconnected graph



A **tree** is a graph that has only one way of getting from one vertex to another. Figure 3 is an example of a tree as is Figure 5.

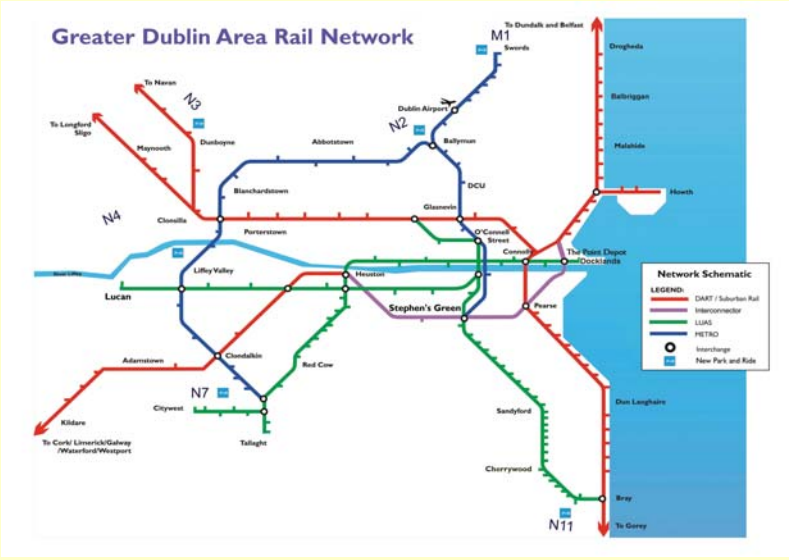
Figure 5 A tree



Exercise 15:01

1 **a** Draw another subgraph from Figure 1 other than the one given above.
b Does it matter whether it is the same shape as in the original graph?
c What is the graph called when it represents the same information but is a different shape?
d Why is the Hong Kong MTR network not an example of a tree?

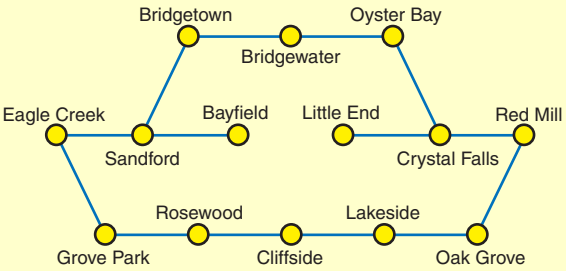
2 The Greater Dublin Rail Network is shown below.

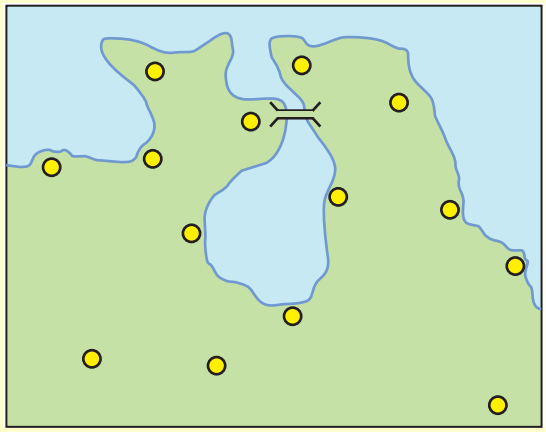


Source: http://www.fiannafail.ie/downloads/dublin_rail_network_rgb.jpg

a Which of the lines (red, blue, green or purple) is the only subgraph which is not a tree?
b How many of the vertices in this graph have a degree greater than 2? What is special about the stations these vertices represent?
c What do you think the arrows mean at the ends of the red graph?
d What does this make the red part of the network?
e Suggest why a tourist may have a problem using this particular graph.

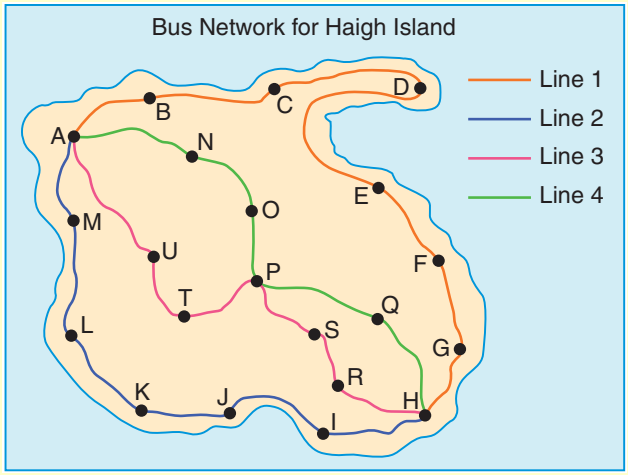
3 Below is a rail network for a small town as well as a map of the town with the actual positions of the stations.





a Try to match the vertices on the network with the stations on the map.
b What information does the map tell you that the network doesn't?
c What could be included on the network to supply this information?

4 A map of the bus network for an island is shown.



a Draw a simplified graph of this network.
b Complete this statement: 'The graph drawn in part (a) is called an _____ of the original map'.
c Which of the vertices do you think represent major towns on this island and why?
d What is the quickest way of travelling from D to P? How do you know that this is the quickest way?

5 A **complete graph** is one in which each vertex is joined to all the others. Draw a complete graph of order a 5 b 6

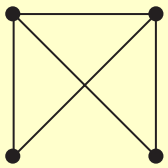
6 The diagram shows the BART subway in San Francisco.

a If this network was represented in graph form, what type of graph would best describe it? Why?
b Give a possible reason for it not being represented in graph form?
c Why do you think all the branches of the tree meet in the middle of the network?
d How many vertices have an odd degree?

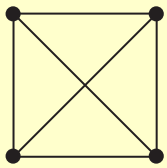
7 A graph has a **Eulerian trail** if a path can be found around it that uses each edge only once. It is not permitted to lift your pen from the page when tracing the path.

a Which of the following graphs have Eulerian trails?

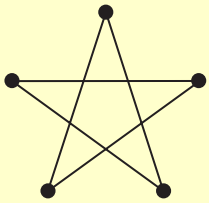
i



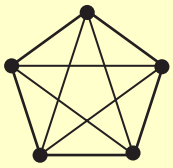
ii



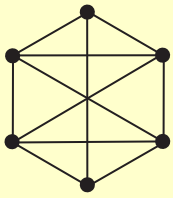
iii



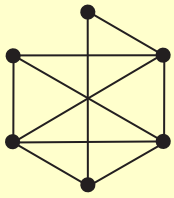
iv



v



vi



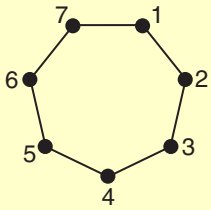
b What is special about the degrees of the vertices in graphs with Eulerian trails? (This was done in Investigation 16:03 in Book 2.)

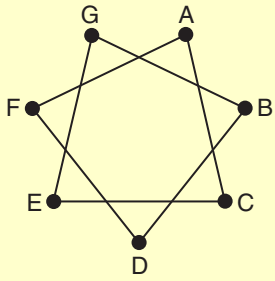
8 **a** Draw complete graphs with order 2, 4, and 6.

b How many of these graphs have Eulerian trails?

c If you continued drawing graphs with even order (8, 10 12 etc), do you think any of them would contain Eulerian trails? Why?

9 Below are two graphs of order 7.

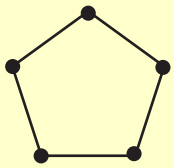


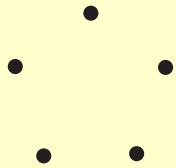


a Is it possible to draw each of these graphs without going over an edge twice or taking your pen off the page (are they Eulerian)?

b By tracing around the graphs, match up the vertices and complete the following:
1 → A, 2 → C, 3 → __, 4 → __, 5 → __, 6 → __, 7 → __.

c Graphs that can be matched like this are **isomorphic** (like in question 4). Complete the second graph below so that it is an isomorphism of the first one.





10 Use the graphs in question 6a to complete this table:

a

Graph	*Total of the degrees of all the vertices (v)*	*Number of edges (e)*
i		
ii		
iii		
iv		
v		
vi		

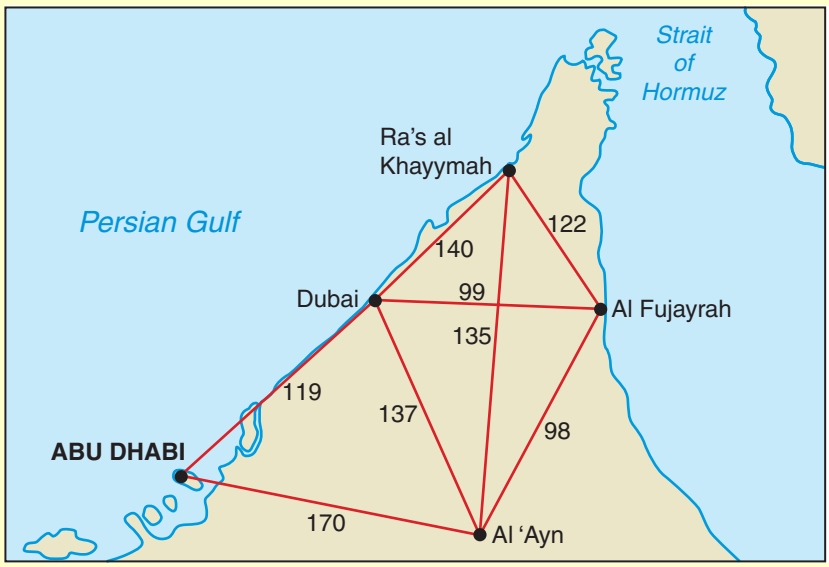
b What is the relationship between v and e? Write this in algebraic form.

c Can you explain why this is so?

15:01B | Weighted Graphs, Trees and Problem Solving: A Review of Network Applications

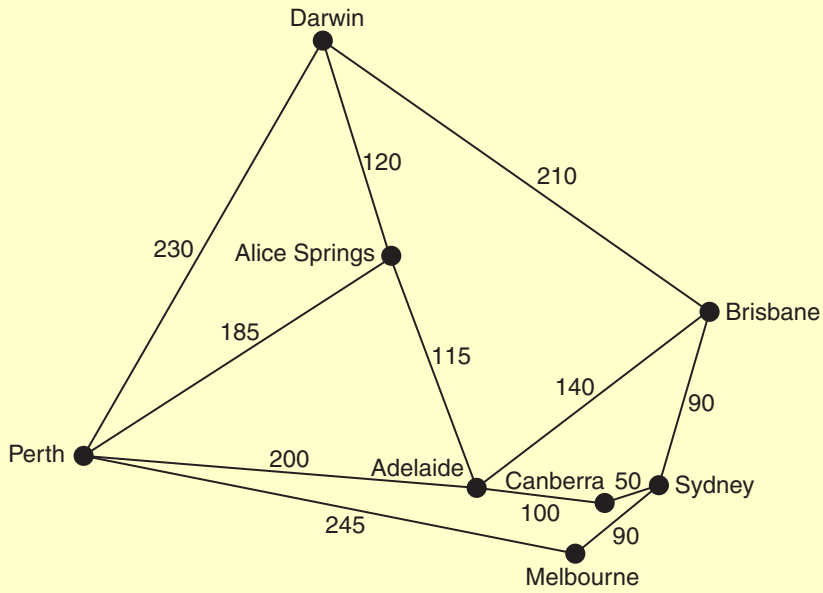
A **weighted graph** is one in which the edges have been given numbers. These numbers can represent many things as shown below.

This graph shows the distance between towns in the UAE.



Obviously the straight lines don't represent the actual roads, just the distances between the cities in kilometres.

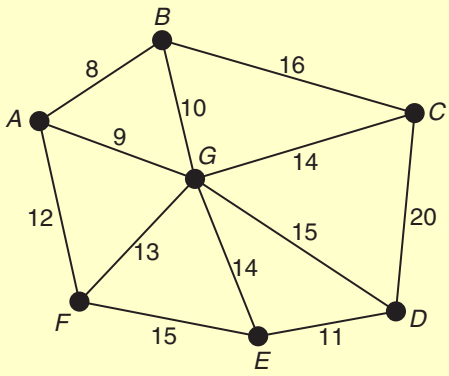
This weighted network shows the flying time in minutes between some Australian cities.

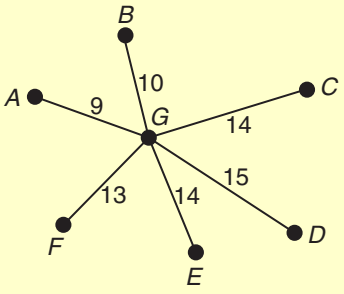


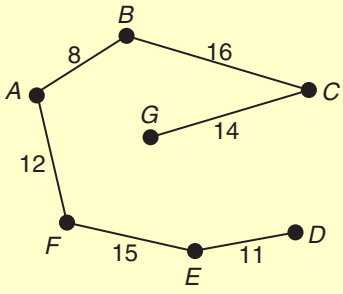
This network shows the cost in millions of dollars of supplying gas to farmhouses in a rural area.

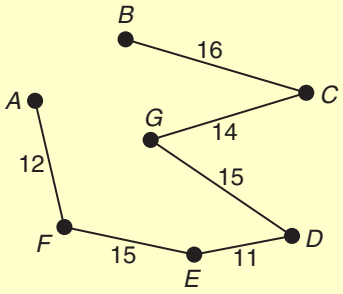
We can use weighted graphs to help find the smallest tree that joins all the vertices. This is called the **minimum spanning tree**.

In this case it would help find the lowest cost of connecting all the farmhouses with gas. There are lots of **spanning trees**. Some are shown below:



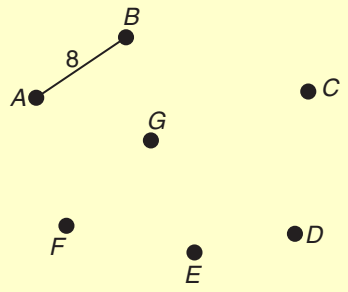




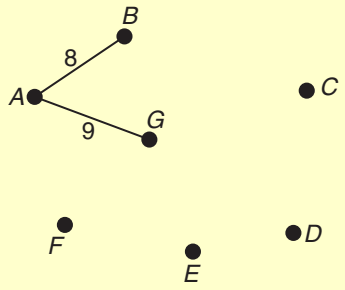


To find the **minimum spanning tree**, follow these steps:

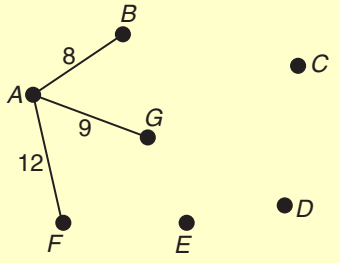
1 Start with any vertex, say *A*, and join it to the closest vertex; in this *B*.



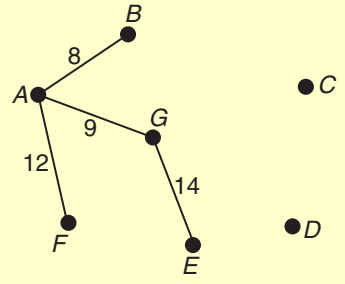
2 Now from the connected vertices, join the edges you have to the next closest vertex; in this case *G*.



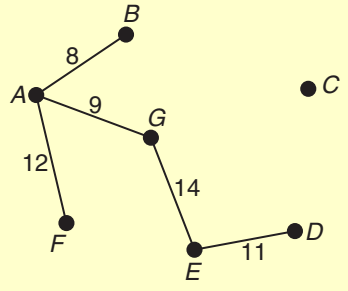
3 Now do the same from the new edges. the closest unconnected vertex from the edges you now have is *F*.



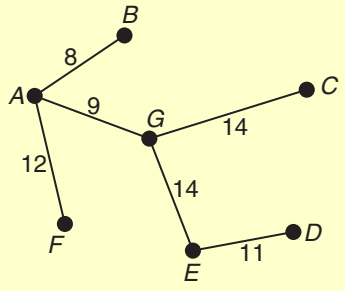
4 From the connected vertices you now have, the next closest is *E* from *G*.



5 The next closest is *D* from *G*.



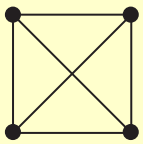
6 Now the closest is *C* from *G*.



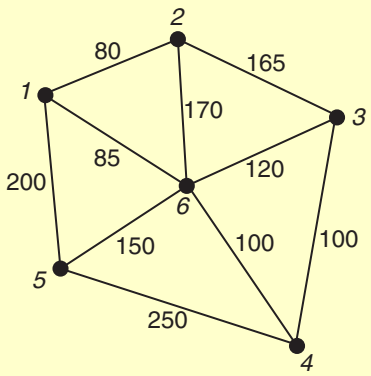
The network shown in number 5 above is the **minimum spanning tree** and it has a weight of 12 + 8 + 9 + 14 + 14 + 11 = 68 which represents the minimum cost in millions of dollars of connecting all the farmhouses together with gas.

Exercise 15:01B

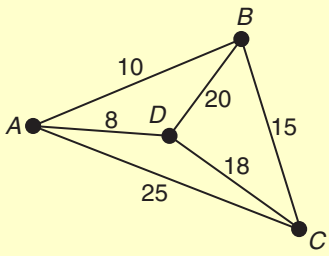
1 Find all the spanning trees of this complete network:



2 The network on the right shows the length, in metres, of paths between campsites that need to be paved. Find the *minimum* distance to be paved so that it is possible to walk to all the campsites on paved paths (the minimum spanning tree).

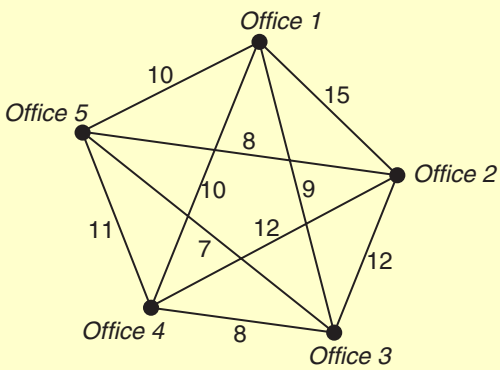


3 Three oil wells A, B and C and a central oil depot, D, are to be joined together with pipelines. The diagram shows a network with the cost, in millions of dollars, of connecting the wells and the depot together. It does not matter if a well is connected directly to the depot or through another oil well.



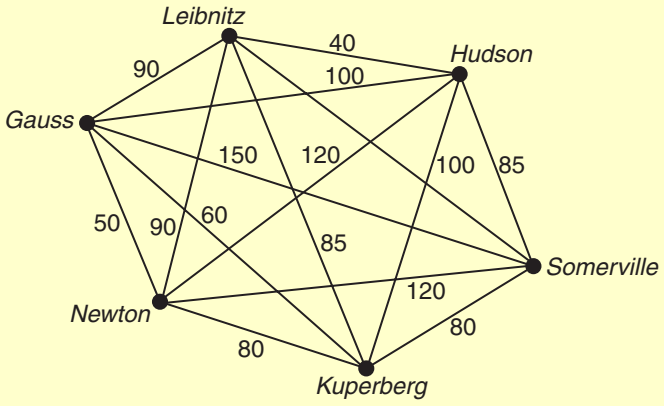
- **a** Find the minimum cost of joining all wells and the depot with a pipeline (the minimum spanning tree).
- **b** How much is saved by doing this, rather than connecting all three wells directly to the depot?

4 This network shows the cost, in thousands of dollars, of joining a company's offices with a dedicated computer network.



- **a** Find the minimum cost of joining these offices together (the minimum spanning tree).
- **b** If computer networks can go from one computer to another without any loss of efficiency, does this network have the same problem as the one in question 3?
- **c** If there is to be a hub (a centre for the network), in which office would you place it?

5 Sean is establishing a small airline to service six cities in a remote area. The flying time, in minutes, between the cities is shown in the network.



a Find the minimum spanning tree for this network.
b What does this tree represent?
c If this was your airline, which city would you make your home base? Why?
d What would be a possible problem of making this tree the only flights the airline makes?
e Based on this route network, how would you fly from Newton to Hudson? How does this differ from a direct flight if it was offered?

Investigation 15:01 | The travelling salesman

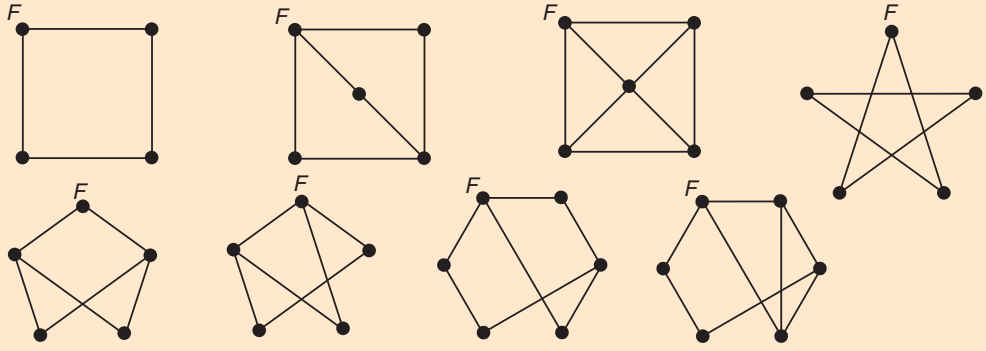
investigation 15:01

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

In this problem the salesman has to start at his factory (vertex *F*) and visit a number of shops (the other vertices) to sell his goods, and return to his factory (vertex *F*). He only wants to visit each shop once and return to the factory.

The object then is to start at *F* and visit every vertex once and return to *F*. This is possible in some graphs but not in others.

a In which of the following graphs is this possible?



b Does changing which vertex is labelled *F* make any difference?
c Suppose the salesman didn't have to return to *F*. Is it now possible for him to visit each vertex once only?
d Draw complete graphs of order 2, 3, 4, 5 and 6. Is it possible to visit every vertex on these graphs once only and return to the starting vertex?

If it is possible to visit each vertex once in a graph, it has what is called a **Hamiltonian path**. If the path returns to the original vertex it is a **Hamiltonian circuit**.

e Can you find anything common in the graphs which have Hamiltonian paths and circuits? Try to prove this by drawing more graphs of your own.

f What predictions can you make about complete graphs and Hamiltonian paths and circuits? Can you explain why?

Assessment Grid for Investigation 15:01 | The travelling salesman

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	The student has had difficulty finding paths through those graphs where paths exist.	1	
			2	
	b	An attempt has been made to find paths, draw complete graphs and answer parts (b) and (c).	3	
			4	
	c	Paths through the graphs have been found, satisfactory answers for parts (b) and (c) are given, and some prediction has been made for part (f).	5	
			6	
	d	The correct patterns have been found and generalised to predict solutions for parts (e) and (f), with full explanations and further examples given.	7	
			8	
Criterion C Communication	a	Very little working out and explanation is given. Presentation is poor.	1	
			2	
	b	Working out and explanations are given with some interpretation (eg why any generalisations or predictions work).	3	
			4	
	c	Predictions and any generalisations are well explained demonstrating interpretation of results.	5	
			6	
Criterion D Reflection in Mathematics	a	There is some explanation of how the paths were found, demonstrating the method used.	1	
			2	
	b	The reliability of generalisations and patterns have been checked to some extent, with some review of these results, or further examples.	3	
			4	
	c	All results have been justified and checked for reliability through further examples, and an evaluation of the generalisations and predictions, with possible limitations, has been given.	5	
			6	

15:02 | Directed Graphs

Some graphs have arrows on the edges showing which way to travel. These types of graphs are **directed graphs**.

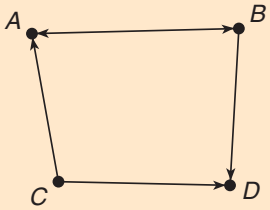
In this directed graph,

a On which edge can you travel in both directions?

b Is it possible to travel to vertex C from any other vertex?

c Is it possible to find a Eulerian path in this graph?

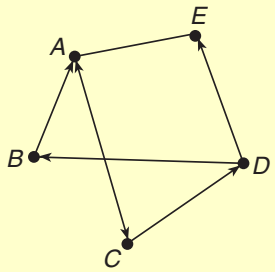
d Is there a Hamiltonian path in this graph?



prep quiz 15:02

Exercise 15:02

1 Consider the directed graph shown.



a In how many ways is it possible to travel from

- **i** D to A?
- **ii** C to A?
- **iii** B to E?
- **iv** B to D?

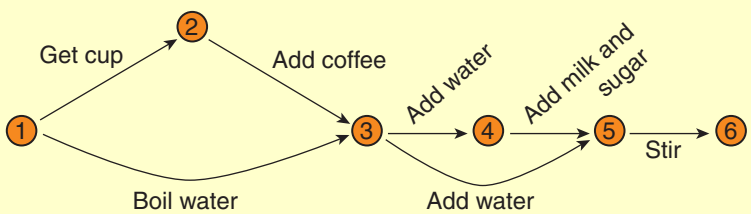
b Is there a Hamiltonian path in this graph? If so, describe it.

c Is there a Hamiltonian circuit? If so, describe it.

d Is there a Eulerian path in this graph? If so, describe it.

e From which point(s) is it impossible to move?

2 A directed network is shown giving directions on how to make a cup of instant coffee.



a Draw an alternative network to this, giving directions on how to make a cup of instant coffee.

b Why do the edges joining vertices 1 and 3 and joining vertices 3 and 5 skip vertices?

c The edges joining vertices 1 and 2 and joining vertices 2 and 3 go in both directions. When would this be useful?

d Why do the edges joining vertices 3, 4 and 5 only go in one direction?

e When would the edge joining vertex 3 to 4 be used rather than going from vertex 3 to 4 then 5?

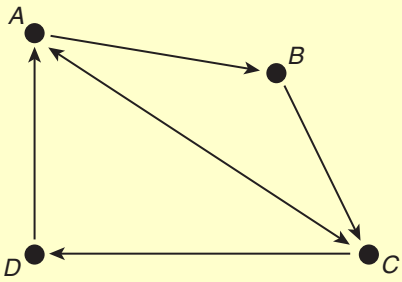
f Would it matter if the edge joining vertices 4 and 5 came before vertex 3?

g Can you see a potential problem with this network of how to make a cup of instant coffee?

h Draw a similar network giving directions on how to prepare a bowl of cornflakes for breakfast.

3 The table below is to be used to record how many ways it is possible to travel from each vertex to the other.

A	B	C	D



		To			
		A	B	C	D
From	A	0	1	1	0
	B				
	C				
	D				

So from vertex *A*:
There are 0 ways of getting to vertex *A*.
There is 1 way to get to vertex *B*.
There is 1 way to get to vertex *C*.
There is 0 ways to get to vertex *D*.

We will call this an ***incident table*** for the network.

a Complete the incident table for the other vertices.

b Complete a similar table for the directed graph in Question 1.

4 An incident table for a directed network is shown.

		To			
		A	B	C	D
From	A	0	0	0	0
	B	1	0	0	0
	C	0	1	0	1
	D	1	1	1	0

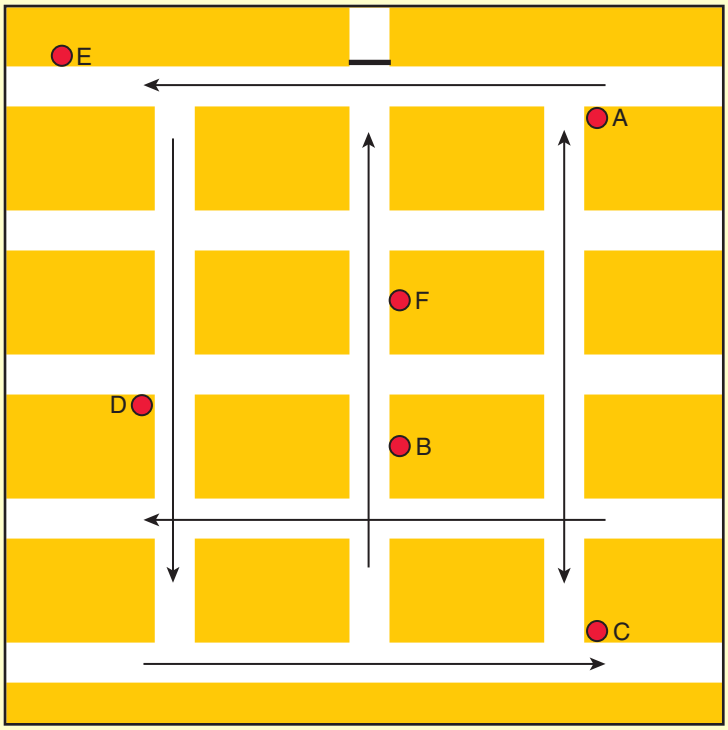
Draw a directed network to correspond to the table.

5 The street layout for a city centre is shown. The arrows show which way the traffic can go: some streets are two-way and others are one-way.

The red dots indicate tourist landmarks that a tour company needs to include on a tour of the city. The tour company wants to plan a route through the city centre that visits all the landmarks.

a Where must the tour end? Why?

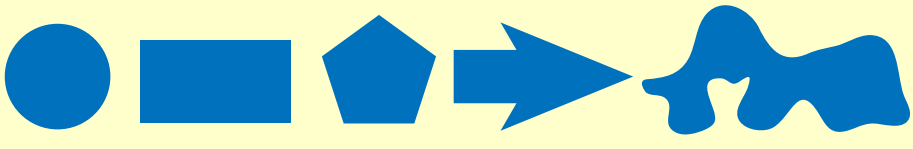
b If the tour must start at A, describe a route the bus could take.



15:03 | Topology

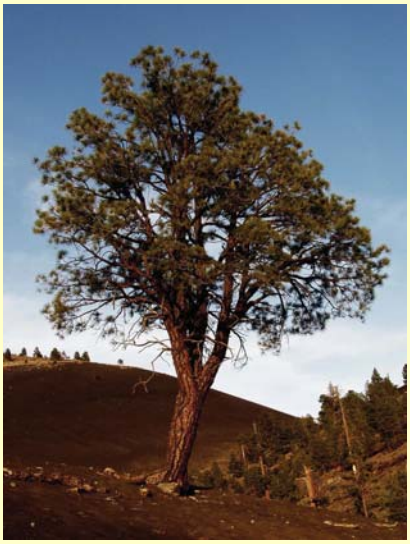
Topology is the study of surfaces and spaces. Surfaces and spaces, however, are no longer defined by their shape, but by what they have in common after the surface or space is twisted and bent from one shape to another.

All of the shapes below are *topologically equivalent*. This is because they all divide the page (space) into two regions: inside the shape and outside the shape. If the perimeter of the shape was made of string, you could make the string into any of these shapes.



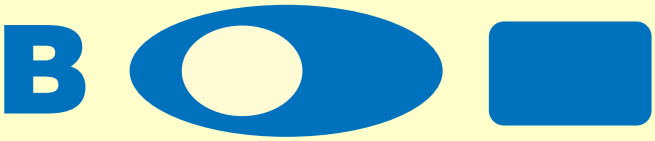
We can bend and twist a shape any way we want. As long as each point in the original shape has a corresponding point in the other, the shapes are topologically equivalent.

Below left is a photograph of a tree. Below right is the same photograph that has been twisted and bent. Although they don't look the same, every pixel in the original has a corresponding pixel in the other. These images are topologically equivalent.



As long as the shape is not torn, it can be twisted and bent just like it was made of rubber and it will be topologically equivalent to the original.

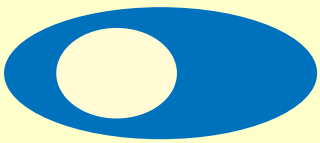
However, the following shapes are not topologically equivalent.



This is because it doesn't matter how much you bend and twist each of them, it is impossible to make one of the other shapes because of the holes. Each shape has divided the available space into a different number of regions.

Topologically speaking then, shapes are classified by the number of holes they have.

This shape has no holes and is the most basic.
It is a genus 0 shape. It is also called a *Jordan curve*.



This shape has one hole. It can be made into a Jordan curve with one cut.
It is a genus 1 shape.

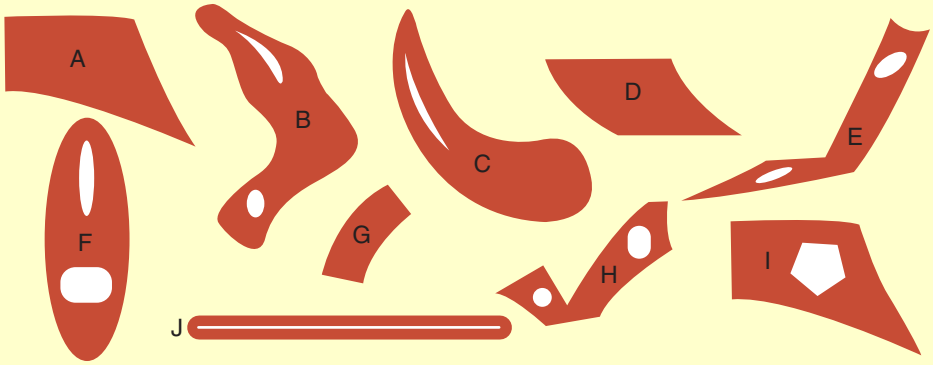
This shape has two holes. It can be made into a Jordan curve with two cuts.
It is a genus 2 shape.

Exercise 15:03

1 Classify the numbers 0 to 9 into genus 0, 1 or 2.

2 Classify the capital letters of the alphabet into genus 0, 1 or 2

3 Which of the following shapes are topologically equivalent?



4 Three-dimensional objects can be classed topologically in the same way. Which of the following everyday objects are topologically equivalent?


Trophy


Bottle opener

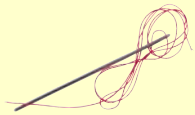

Dinner plate


Teacup


Doughnut


360° Protractor


Milk bottle


Sewing needle

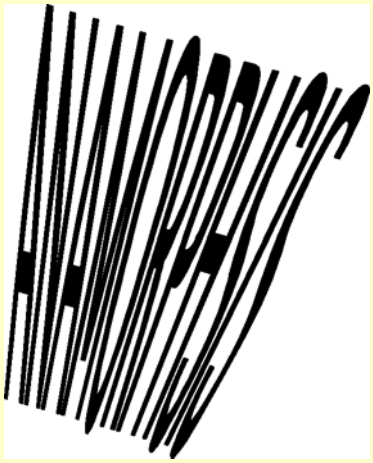

Dinner tray


Fork


Wooden spoon


Bowl

5 Sometimes at football matches words are written on the grass like this.



a What has happened to the letters?

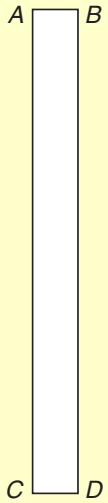
b Would these letters be topologically equivalent to the original letters?

c Why do you think the letters may be written like this on the grass?

d If you tilt the page away from you and look it at an angle, you should be able to read the word. What is it? What does it mean?

e You might see this sort of writing on the road on the way to school. Give an example.
Why do you think this is done?

6 The ends of the strip of paper like the one shown are glued together. Before being glued, however, a twist is put in the strip of paper so that *C* is joined to *B* and *D* is joined to *A*.



a How many sides does the strip have after it is glued?

b How many edges does it have?

Now make the strip as described.

c Put a pencil in the middle of the strip and draw a line along the strip until you get back to the start of your line. How many sides do you think it has now?

d Using a marker, begin marking the edge of the strip until you get back to the start. How many edges do you think it has now?

e Try to find out the name of this special strip.

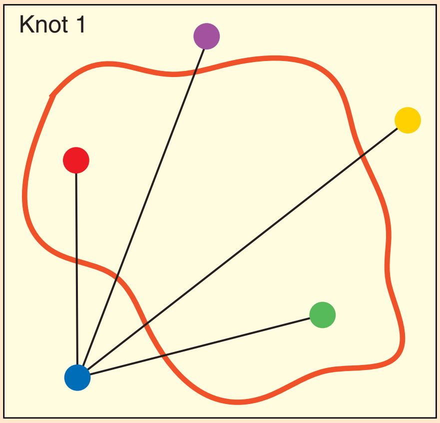
investigation
15:03

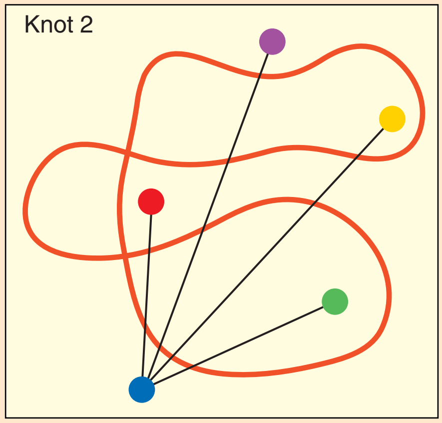
Investigation 15:03 | Knots

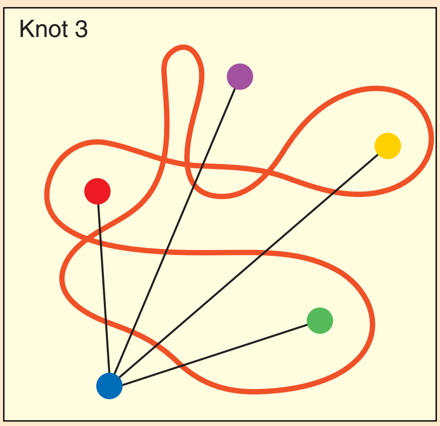
Please use the Assessment Grid on page 423 to help you understand what is required for this Investigation.

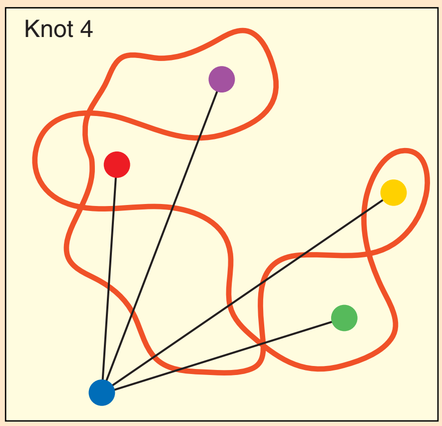
The diagrams show four tangles or knots. In each one a loop of string is shown and a number of coloured dots. Some of these dots are inside the loop and some are outside. In each diagram, the blue dot (B) is outside.

The aim of this investigation is to make a rule to help decide which dots are inside the loop and which are outside. To do this it is necessary to count how many times the boundary of the loop is crossed when travelling in a straight line from the blue dot (B) to the others. The first table is done.









From the blue dot to the	*Knot 1*	*No of crossings*	*Inside the loop*
	Red dot	1	Yes
	Purple dot	2	No
	Yellow dot	2	Yes
	Green dot	1	No

1 Complete the following tables for the other three knots.

From the blue dot to the	*Knot 1*	*No of crossings*	*Inside the loop*
	Red dot		
	Purple dot		
	Yellow dot		
	Green dot		

From the blue dot to the	*Knot 1*	*No of crossings*	*Inside the loop*
	Red dot		
	Purple dot		
	Yellow dot		
	Green dot		

From the blue dot to the	*Knot 1*	*No of crossings*	*Inside the loop*
	Red dot		
	Purple dot		
	Yellow dot		
	Green dot		

2 Compare your results with other people in your class. Discuss any differences between your results and those of others.

3 When you have decided on the correct answers, try to think of a rule which would help you decide whether a point is inside or outside a tangled knot.

4 Draw a tangled knot of your own and test your rule.

5 Test your rule on tangled knots that other people in you class have drawn.

6 Try to give a reason why this rule works

Assessment Grid for Investigation 15:03 | Knots

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	A systematic method has not been used, and the student has demonstrated difficulty in following instructions.	1	
			2	
	b	A systematic method has been adopted and an attempt has been made to describe any patterns discovered.	3	
			4	
	c	A systematic method has been used and the patterns identified have been described with some conclusions drawn.	5	
			6	
	d	The correct patterns have been recognised and justified clearly, and used to make further predictions.	7	
			8	
Criterion C Communication	a	Presentation is poor and only oral generalisations and reasons can be given.	1	
			2	
	b	Some explanations and reasons for the results are given in writing. Presentation is good.	3	
			4	
	c	The work is well structured and results are well communicated using symbols, diagrams and/or words.	5	
			6	
Criterion D Reflection in Mathematics	a	An attempt has been made to explain the method used and to check the results.	1	
			2	
	b	The method used is explained and results have been checked with some success.	3	
			4	
	c	A thorough justification and evaluation has been given for both the method used and generalisations made, with further examples given.	5	
			6	

Mathematical terms 15

degree
- of a vertex is determined by how many edges join it to other vertices.

disconnected graph
- A graph in which it is not possible to travel to all the vertices along edges.

directed graph
- A graph in which the edges have arrows indicating the direction in which they travel.

edge
- A line joining two vertices.

Eulerian trail
- A path around a graph that uses each edge exactly once.

graph
- A series of dots that may or may not be connected to each other by lines.

Hamiltonian path
- A path which visits each vertex only once. If this path returns to the original vertex then it is a **Hamiltonian circuit.**

incident table
- A table showing how many edges join vertices.

isomorphic graphs
- Graphs which are the same but look different.

network
- A type of graph usually representing real-life situations

minimum spanning tree
- A spanning tree of a weighted graph whose total weight is less than any other spanning tree.

order
- of a graph is determined by how many vertices it contains.

subgraph
- Part of a graph.

spanning tree
- A subgraph which includes all the vertices and is a tree.

topology
- A branch of Mathematics that studies surfaces and spaces and the properties of these as they are twisted, stretched and bent.

tree
- A graph in which there is only one edge joining any two vertices.

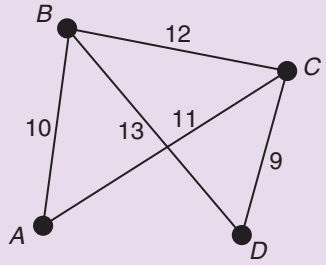
vertex
- A dot in a graph. If there is more than one vertex, they are called **vertices**.

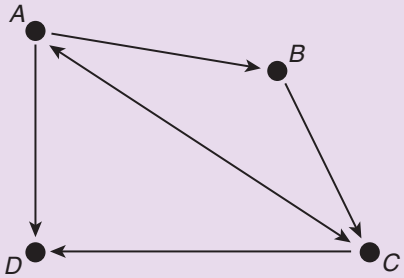
weighted graph
- A graph in which numbers are assigned to the edges. The total of these numbers is the weight of the graph.

Diagnostic Test 15: | Networks and Topology

- These questions reflect the important skills introduced in this chapter.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
Questions 1 to 7 refer to the graph on the right.	
1 What is the order of this graph?	15:01A
2 What is the weight of this graph?	15:01B
3 What is the degree of vertex B?	15:01A
4 Does this graph have **i** a Hamiltonian path? **ii** a Hamiltonian circuit? **iii** a Eulerian path?	15:04A & Investigation 15:01
5 Draw a subgraph of this graph.	15:01A
6 Draw a spanning tree for this graph.	15:01B
7 Draw the minimum spanning tree. What is its weight?	15:01B
Questions 8 and 9 refer to the directed graph.	
8 Complete an incident table for this graph.	15:02
9 Does this graph have **i** a Hamiltonian path? **ii** a Hamiltonian circuit? **iii** a Eulerian path?	15:01A & Investigation 15:01
10 For shapes or objects to be topologically equivalent, what must they have in common?	15:03





assignment 15

Chapter 15 | Working Mathematically

1 After swimming 18 lengths of a 50 m swimming pool, Anisha has completed 60% of her usual training. How far does she have left to swim?

2 A garden is in the shape of an L. To get the L shape, a 10 m square is cut from one corner of a 20 m square. What is the perimeter of the garden?

3 George receives a discount of 15% on the price of an mp3 player.
   a What percentage of the original price did he pay?
   b If he paid $297.50 for the player, what was the original price?

4 Wendy has done eight tests in Mathematics and her mean mark overall is 70%. After sitting one more test, her average rose to 72%. What percentage did she score on this test?

5 The interior angles of a triangle are in the ratio 3:7:8. Find the size of the largest angle.

6 In Jamie's class of 22 students, everyone plays either basketball or soccer or both. Fifteen students play soccer and 9 play basketball. If a student is picked at random, what is the probability that the student plays:
   a both basketball and soccer?
   b basketball but not soccer?

7 Tam has four books to put on a shelf in his room. In how many different ways can these books be arranged? (Hint: call the books ABCD)

8 In a money box there are only 10-, 20- and 50-cent pieces. The ratio of 10-cent pieces to 20-cent pieces is 3:4 and the ratio of 20-cent pieces to 50-cent pieces is 3:4. If there is $8.10 worth of 10-cent pieces, how much money is in the box?

assignment 15A

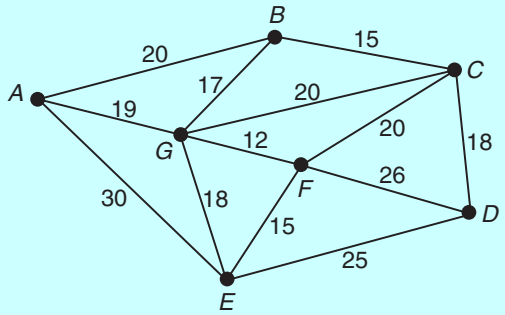
Chapter 15 | Revision Assignment

1 What is the order of this graph?

2 What is the degree of vertex A?

3 Does this graph have a Hamiltonian path?

4 Does this graph have:
   i a Eulerian path?
   ii a Eulerian circuit?

5


   a What name is given to the type of graph shown at right?
   b Find the minimum spanning tree for this graph?
   c What does this tree represent?

6 a What is a directed graph?
   b What name is given to the type of graph shown at right?

7 What makes plane or solid objects topologically equivalent?

Answers

Chapter 1: Basic Number Skills

Exercise 1:01

1 **1** *metres* **2** *decimetres* **3** *centimetres* **4** *millimetres*
5 *kilometres* **6** *square metres* **7** *square centimetres* **8** *square kilometres*
9 *hectares* **10** *cubic metres* **11** *cubic centimetres* **12** *seconds*
13 *minutes* **14** *hours* **15** *metres per second* **16** *kilometres per hour*
17 *grams* **18** *milligrams* **19** *kilograms* **20** *tonnes*
21 *litres* **22** *millilitres* **23** *kilolitres* **24** *degrees Celsius*

2 **1** *is equal to* **2** *is approximately equal to* **3** *is not equal to* **4** *is less than*
5 *is less than or equal to* **6** *is not less than* **7** *is greater than* **8** *is greater than or equal to*
9 *4 squared* **10** *4 cubed* **11** *the square root of 2* **12** *the cube root of 2*
13 *is perpendicular to* **14** *is parallel to* **15** *is congruent to* **16** *is similar to*
17 *per cent* **18** *therefore* **19** *for example* **20** *that is*
21 *pi* **22** *the sum of* **23** *the mean* **24** *probability of event E*

3 **1** $6 - 2 = 4$ **2** $6 + 2 = 8$ **3** $6 \div 2 = 3$ **4** $6 - 2 = 4$
5 $6 \div 2 = 3$ **6** 2 **7** 6 **8** $6 \times 2 = 12$
9 $6 - 2 = 4$ **10** $6 \times 2 = 12$ **11** $2 + 6 = 8$ **12** $6 - 2 = 4$
13 $6^2 = 36$ **14** $\sqrt{36} = 6$ **15** $6 - 2 = 4$ **16** $6 \times 2 = 12$
17 $(6 + 2) \div 2 = 4$ **18** $6 + 2 = 8$ **19** $6^2 = 36$ **20** $6 - 2 = 4$
21 $6 - 2 = 4$ **22** $6 \neq 2 = 8$ **23** $6 \div 2 = 3$ **24** $6 + 2 = 8$

4 **a** true **b** true **c** true **d** false **e** true **f** true **g** false **h** true **i** true **j** true
k false **l** true **m** true **n** false **o** true

Diagnostic Test 1:02A Integers

1 **a** 7 **b** −13 **c** −10
2 **a** 9 **b** 7 **c** 11
3 **a** −6 **b** 30 **c** −63
4 **a** 5 **b** −7 **c** 52
5 **a** −56 **b** 1 **c** 7

Diagnostic Test 1:02B Fractions

1 **a** $1\frac{3}{4}$ **b** $4\frac{1}{3}$ **c** $14\frac{1}{10}$
2 **a** $\frac{5}{2}$ **b** $\frac{53}{10}$ **c** $\frac{22}{7}$
3 **a** $\frac{2}{3}$ **b** $\frac{2}{13}$ **c** $\frac{1}{15}$
4 **a** 21 **b** 85 **c** 375
5 **a** $\frac{5}{8}$ **b** $1\frac{1}{5}$ **c** 1
6 **a** $\frac{1}{5}$ **b** $\frac{2}{7}$ **c** $\frac{13}{50}$
7 **a** $1\frac{11}{20}$ **b** $\frac{7}{10}$ **c** $\frac{29}{200}$
8 **a** $\frac{1}{8}$ **b** $\frac{13}{20}$ **c** $\frac{7}{30}$
9 **a** $8\frac{1}{10}$ **b** $12\frac{9}{20}$ **c** $2\frac{17}{24}$
10 **a** $3\frac{5}{18}$ **b** $5\frac{13}{20}$ **c** $20\frac{7}{40}$
11 **a** $6\frac{5}{8}$ **b** $3\frac{9}{10}$ **c** $1\frac{2}{3}$
12 **a** $\frac{12}{55}$ **b** $\frac{21}{100}$ **c** $\frac{3}{50}$
13 **a** $\frac{3}{8}$ **b** $\frac{3}{8}$ **c** $\frac{7}{12}$
14 **a** $2\frac{1}{2}$ **b** $2\frac{17}{50}$ **c** 14
15 **a** $15\frac{1}{5}$ **b** $6\frac{3}{4}$ **c** $31\frac{7}{8}$
16 **a** 4 **b** 3 **c** 1
17 **a** $1\frac{1}{5}$ **b** $1\frac{5}{27}$ **c** $1\frac{3}{32}$
18 **a** $2\frac{1}{2}$ **b** $1\frac{3}{7}$ **c** $1\frac{1}{4}$
19 **a** $2\frac{1}{2}$ **b** $\frac{7}{10}$ **c** $1\frac{3}{8}$
20 **a** 20 **b** 50 **c** 40

Diagnostic Test 1:02C Decimals

1 **a** 0·5, 0·505, 0·55 **b** 8·4, 8·402, 8·41 **c** 1·01, 1·011, 1·1
2 **a** 5·74 **b** 21·6 **c** 0·265
3 **a** 11·63 **b** 7·176 **c** 4·015
4 **a** 4·2 **b** 0·09 **c** 0·034
5 **a** 314·2 **b** 40 **c** 0·65
6 **a** 21 000 **b** 8 040 000 **c** 125
7 **a** 2·04 **b** 2·42 **c** 0·0475
8 **a** $0{\cdot}41\dot{6}$ **b** $0{\cdot}59\dot{1}$ **c** $9{\cdot}\dot{3}$
9 **a** 2·435 **b** 0·067 **c** 0·0007
10 **a** 32 **b** 10·3 **c** 130
11 **a** $\frac{1}{2}$ **b** $\frac{9}{50}$ **c** $9\frac{21}{200}$
12 **a** 0·8 **b** 0·375 **c** 0·83

Diagnostic Test 1:02D Percentages

1 **a** $\frac{9}{50}$ **b** $\frac{7}{100}$ **c** $2\frac{6}{25}$ **2** **a** $\frac{19}{200}$ **b** $\frac{1}{16}$ **c** $\frac{49}{400}$

3 **a** 55% **b** $83\frac{1}{3}$% **c** 125% **4** **a** 0·09 **b** 0·16 **c** 1·1

5 **a** 0·238 **b** 0·125 **c** $0{\cdot}04\dot{6}$ **6** **a** 51% **b** $8\frac{1}{2}$% **c** 180%

7 **a** 210 m **b** \$12·96 **8** **a** 5·901 m **b** 2·75 t

9 **a** \$16 **b** 60 kg **10** **a** 25% **b** 40%

11 **a** \$69 **b** \$5.20

Exercise 1:04

1 **a** 250%, 2·5 **b** 1·3, $1\frac{3}{10}$ **c** 280%, $2\frac{4}{5}$ **d** 125%, 1·25

2 **a** 0·805, 0·85, 0·9, 1 **b** $12\frac{1}{4}$%, 87·5%, 100%, 104% **c** $\frac{6}{100}$, $\frac{4}{7}$, $\frac{5}{8}$, $\frac{2}{3}$ **d** 150%, 1·65, $1\frac{3}{4}$, 2

e 140%, 1·41, $\sqrt{2}$, 1·42 **f** 3·1, π, $3\frac{1}{4}$, $\sqrt{12}$

3 **a** 6·85 **b** 16% **c** $\frac{3}{5}$ or 0·6 **d** 6·35 **4** $\frac{1}{2}$ and $\frac{1}{8}$

5 **a** 3·2, 6·4, 12·8 **b** 0·15, 0·075, 0·0375 **6** **a** 29 **b** 9

7 Each year, \$8 is paid in interest for every \$100 invested. **8** The building can house three tenants.

9 7 years **10** −33° **11** 54 BC **12** **a** 370 **b** −120 **c** −440

13 **a** $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{1}{9}$ **d** $\frac{5}{9}$

14 **a** 0·888 89 **b** 0·285 71 **c** 0·538 46 **d** 0·952 38 **e** 0·363 64 **f** 0·277 78

15 \$64 500 **16** **a** What the maximum possible score was.

b The number of marks lost for those 3 mistakes. **c** The number who have cancer and the number over 65.

Prep Quiz 1:05

1 0·25 **2** 0·4 **3** $0{\cdot}3\dot{}$ **4** $0{\cdot}8\dot{3}$ **5** $0{\cdot}\dot{4}$ **6** $0{\cdot}\dot{6}3\dot{1}$ **7** $0{\cdot}1\dot{6}$ **8** $0{\cdot}72\dot{6}\dot{9}$ **9** $\frac{3}{4}$ **10** $\frac{7}{8}$

Exercise 1:05

1 **a** 0·75 **b** 0·8 **c** 0·625 **d** 0·7 **e** 0·07

f 1·75 **g** 0·16 **h** 0·34 **i** 0·475 **j** 0·936

2 **a** $0{\cdot}\dot{6}$ **b** $0{\cdot}\dot{5}$ **c** $0{\cdot}\dot{8}$ **d** $1{\cdot}\dot{1}\dot{8}$ **e** $0{\cdot}\dot{1}42857\dot{}$

f $0{\cdot}1\dot{6}$ **g** $0{\cdot}0\dot{6}$ **h** $0{\cdot}4\dot{6}$ **i** $0{\cdot}041\dot{6}$ **j** $0{\cdot}5\dot{6}$

3 **a** $\frac{47}{100}$ **b** $\frac{4}{25}$ **c** $\frac{1}{8}$ **d** $\frac{17}{20}$ **e** $\frac{7}{200}$

4 **a** $\frac{4}{9}$ **b** $\frac{19}{33}$ **c** $\frac{173}{999}$ **d** $\frac{7}{9}$ **e** $\frac{4}{11}$ **f** $\frac{1234}{9999}$

5 $\frac{1}{3}$

6 **a** $\frac{5}{6}$ **b** $\frac{629}{990}$ **c** $\frac{89}{450}$ **d** $\frac{29}{45}$ **e** $\frac{221}{300}$ **f** $\frac{8167}{9900}$ **g** $\frac{853}{1665}$ **h** $\frac{4751}{9000}$ **i** $\frac{6467}{9990}$

Challenge 1:05 Try this with repeating decimals

1 $\frac{7}{9}$ **2** $\frac{67}{99}$ **3** $\frac{104}{333}$ **4** $\frac{1}{6}$ **5** $\frac{637}{1980}$

Prep Quiz 1:06

1 $\frac{5}{6}$ **2** $\frac{4}{5}$ **3** $\frac{6}{7}$ **4** $\frac{1}{5}$ **5** $\frac{1}{2}$ **6** $\frac{1}{4}$ **7** $\frac{4}{5}$ **8** $\frac{2}{3}$ **9** $\frac{2}{3}$ **10** $\frac{4}{5}$

Exercise 1:06

1 **a** $\frac{7}{15}$ **b** $\frac{10}{21}$ **c** $\frac{3}{4}$ **d** $\frac{1}{10}$ **e** $\frac{4}{5}$ **f** $\frac{1}{10}$ **g** $\frac{15}{16}$ **h** $\frac{1}{5}$ **i** $\frac{5}{24}$

2 **a** 3 : 2 **b** 2 : 1 **c** 13 : 3 **d** 2 : 5 **e** 4 : 9 **f** 7 : 10 **g** 4 : 1 **h** 9 : 10 **i** 200 : 1

j 11 : 8 **k** 11 : 4 **l** 4 : 7 **m** 2 : 21 **n** 36 : 1 **o** 1 : 26 **p** 20 : 1 **q** 1 : 5 **r** 5 : 4

s 11 : 4 **t** 8 : 13 **u** 9 : 4 **v** 3 : 2 **w** 11 : 8 **x** 4 : 3

3 **a** 7 : 9 **b** 13 : 15 **c** 7 : 8 **d** 2 : 3 **e** 4 : 5 **f** 3 : 17 **g** 9 : 10 **h** 1 : 2 **i** 1 : 8

j 32 : 25 **k** 5 : 3 **l** 5 : 2

4 **a** 2 : 1 **b** 2 : 1 **c** 1 : 5 **d** 5 : 3 **e** 1 : 8 **f** 1 : 10 **g** 3 : 5 **h** 5 : 3 **i** 4 : 1

j 3 : 1 **k** 5 : 6 **l** 1 : 2

5 **a** $\frac{2}{5}$: 1 **b** $\frac{3}{4}$: 1 **c** $\frac{7}{10}$: 1 **d** $1\frac{2}{3}$: 1 **e** $\frac{20}{9}$: 1

6 **a** 1 : $\frac{5}{2}$ **b** 1 : $\frac{4}{3}$ **c** 1 : $\frac{10}{7}$ **d** 1 : $\frac{3}{5}$ **e** 1 : $\frac{9}{20}$

7 **a** For every 2 cups of sugar you should use 1 cup of flour and 1 cup of custard powder.
A mixture of these three would be 50% sugar, 25% flour and 25% custard powder.

b The mixture would be $\frac{2}{13}$ cement, $\frac{10}{13}$ sand and $\frac{1}{13}$ lime. **c** 1 : 40 **d** 1 : 20

Prep Quiz 1:07

1 $32 **2** $48 **3** $80 **4** $8 **5** 1000 g **6** 1000 kg **7** 60 min **8** 10 mm **9** 10 000 cm^2 **10** 1500

Exercise 1:07

1 **a** 3 km/h **b** 2 kg/$ **c** 50c/kg **d** 1 mL/cm^3 **e** 40 L/h
f $20/h **g** $45/day **h** 7 km/L **i** 4 deg/min **j** 70 g/cm
k 25 t/block **l** 120 km/h **m** 11 runs/wicket **n** 3 children/mother **o** 7·5 g/cm^3

2 **a** 15 km **b** $49 **c** 16 **d** $101.40 **e** 225
f 2010 kJ **g** $16 **h** 7·5 kg **i** 18 000 000 km **j** $800

3 **a** 60 **b** 40 **c** 5000 **d** 5 **e** 40
f 16·6 **g** 7 **h** $42 **i** 0·108 **j** 1·5
k 19·2 **l** 50 **m** 600 **n** $500 **o** 1050

Exercise 1:08

1 **a** 2 **b** 3 **c** 3 **d** 2 **e** 4 **f** 3 **g** 2 **h** 3 **i** 1
j 2 **k** 1 **l** 2 **m** 3 **n** 4 **o** 1 **p** 3 **q** 4 **r** 3
s 3 **t** 3 **u** 1

2 **a** 1 **b** 2 **c** 2 **d** 3 **e** 4 **f** 3 **g** 3 **h** 6

3 2 significant figures (ie nearest thousand), or 3 significant figures (ie nearest hundred)

Prep Quiz 1:09

1 4 **2** 1 **3** 4 **4** 3·08 **5** 0·80 **6** 2·410 **7** 4 **8** 2·33 **9** 3·55 **10** 0·065

Exercise 1:09

1 **a** 7900 **b** 1100 **c** 67 300 **d** 900 **e** 600 **f** 400 **g** 74 900 **h** 7900

2 **a** 9 **b** 80 **c** 45 **d** 3 **e** 2 **f** 18 **g** 237 **h** 100

3 **a** 243·13 **b** 79·66 **c** 91·35 **d** 9·81 **e** 0·30 **f** 0·09 **g** 0·10 **h** 1·99

4 **a** 6·7 **b** 8·5 **c** 2·1 **d** 6·1 **e** 0·1 **f** 0·0 **g** 29·9 **h** 10·0

5 **a** 8200 **b** 3500 **c** 660 **d** 850 **e** 15 000 **f** 76 000 **g** 50 000 **h** 77 000

6 **a** 8000 **b** 4000 **c** 700 **d** 800 **e** 10 000 **f** 80 000 **g** 50 000 **h** 80 000

7 **a** 695 **b** 35·1 **c** 321 **d** 0·0815 **e** 0·667 **f** 9·33 **g** 10·1 **h** 9·10

8 **a** 1·8 **b** 1·78 **c** 1·778 **d** 2 **e** 1·8 **f** 1·78

9 **a** 5 cm **b** 55 mm **c** 5·5 cm **d** 5·45 cm **e** 5·455 cm **f** 5 cm **g** 5·5 cm **h** 5·45 cm

10 **a** $141 **b** $140.60 **c** 14059.7 cents **d** 14059.71 cents **e** 14059.705 cents
f $100 **g** $140 **h** $141

11 **a** nearest thousand, 500 **b** nearest billion (ie $1 000 000 000), 500 000 000

12 **a** nearest tenth, 0·05 cm **b** nearest thousandth, 0·0005 mg

13 Any number between 2·145 and 2·55 or equal to 2·145, 2·145, No because the number can have any number of decimal places and must be less than 2·155.

14 Any number between 2 500 000 and 3 500 000 or equal to 2 500 000

15 $26.80, Answers will vary but the 15c over could be given as a tip.

16 There will be insufficient tiles. Because parts of tiles are required in some places, you must round up, so that the entire floor can be covered using parts of tiles and you must also allow for breakages as the tiles are cut.

17 **a** $0{\cdot}000\,000\,000\,\dot{3}$ **b** $0{\cdot}000\,000\,000\,\acute{6}$ **c** $0{\cdot}000\,000\,000\,\bar{5}$

18 **a** 90 000 **b** 119 370·25 **c** 122 500 **19** 29 335·7025 **20** 2859·2 m^3

Reading Maths 1:09

1 4 mg three times a day **2** three times a day, 6 **3** 15 years **4** 5 mL **5** 24 mg

Prep Quiz 1:10

1 200 **2** 20 **3** 156·1 **4** 1561 **5** <1 **6** < **7** true **8** *b* **9** true **10** false

Exercise 1:10

For all these exercises it is possible to have more than one answer.

1 **a** 22 **b** 36 **c** 78 **d** 140 **e** 5 **f** 0·02 **g** 0·6 **h** 100 **i** 50 **j** 1·7

2 **a** 1 **b** 1000 **c** 2 **d** 3 **e** 2 **f** 2 **g** 25 **h** 32 **i** 60

3 **a** iii or 3·1 kg **b** i or 27 m **c** iii or $276 **d** ii or $9400

4 **a** 3 **b** 5 **c** 2 **d** 4 **e** 11 **f** 1 **g** 8 **h** 11

5 **a** 0·01 cm or 0·1 mm **b** 3200
c Yes, because in 8×80 both numbers have been rounded down and in 9×90 both have been rounded up.

6 73·6575 and 75·3875 (ie 73·6575 $\leqslant$ the measurement < 75·3875)

Diagnostic Test 1 Basic number skills

1 **a** $0.\dot{6}$ **b** $0.1\dot{6}$ **c** $0.\dot{7}$ **d** $0.2\dot{6}$ **2** **a** $\frac{5}{9}$ **b** $\frac{37}{99}$ **c** $\frac{26}{45}$ **d** $\frac{683}{990}$
3 **a** 6 **b** 5 **c** 5 **d** can't tell, may be 2, 3 or 4
4 **a** 57 000 000 **b** 0·6662 **c** 8·0 **d** 4·67 m
5 **a** 0·006 **b** 510 000 000 **c** 0·061 **d** 93 800 000
6 answers may vary slightly **a** 36 **b** 5 **c** 20 **d** 200

1A Revision Assignment

1 **a** 9 **b** 16 **c** −3
2 **a** $\frac{4}{5}$ **b** $\frac{1}{6}$ **c** $\frac{1}{10}$ **d** $\frac{5}{6}$ **e** $5\frac{19}{20}$ **f** $1\frac{11}{15}$ **g** $9\frac{1}{3}$ **h** 6
3 **a** 4·47 **b** 4·34 **c** 40·5 **d** 0·09 **4** **a** $\frac{7}{20}$ **b** $\frac{7}{8}$ **c** $\frac{23}{99}$ **d** $\frac{172}{495}$
5 **a** \$26.25 **b** 20% **6** **a** 5 : 9 **b** 1 : 3 **c** 1 : 4 **d** 1 : 6
7 **a** 3 **b** 3 **c** 3 **d** 2
8 **a** 6·4 **b** 6·92 **c** 47 644 000 **d** 648 **e** 6·4 **f** 0·005 82 **g** 47 600 000 **h** 0·70
9 **a** 3·650 **b** 3·749
10 answers may vary slightly **a** 16 **b** 5 **c** 2 **d** 200

1B Working Mathematically

1 **a** *hectares* **b** *cubic metres* **c** *metres per second* **d** *kilometres per hour*
e *milligrams* **f** *kilograms* **g** *tonnes* **h** *millilitres*
i *kilolitres* **j** *degrees Celsius*
2 **a** *is approximately equal to* **b** *is less than or equal to* **c** *4 cubed* **d** *the square root of 2*
e *is parallel to* **f** *is congruent to* **g** *is similar to* **h** *therefore*
i *for example* **j** *the mean*
3 11 **4** 39
5 **a** **i** 50 **ii** 67 **iii** 20 **iv** 27 **b** **i** 48 **ii** 54 **iii** 34 **iv** 23
c 17 **d** 12 **e** No. You cannot score more than 60 marks for a test out of 60.
6 **a** 14 litres **b** 2 litres **c** 10:00 am and 1:00 pm **d** No petrol; the car was stationary **e** 30 litres **f** 2 litres

Chapter 2: Working Mathematically

Exercise 2:01A

1 **a** **i** \$93.80 **ii** 6·4 **b** **i** \$336 **ii** 250 **c** **i** 4c/folder **ii** €28
d \$1204 **e** **i** £340 **ii** €735 **f** 101·25 **g** **i** 540 km **ii** 12·25 h
2 **a** 2·8 L/100 km **b** about 2·98 km
3 S\$920.65 **4** 7 heartbeats/breath **5** 13 m

Exercise 2:01B

1 **a** 63 kg **b** 35 hours **c** 72 **d** 15 **2** **a** 36 **b** 5000 **c** \$4976 **d** 36
3 **a** 560 m **b** 9 cm **4** **a** \$180 **b** \$19 600 **c** 1·5 km **d** 480 mL
5 **a** 2 : 3 **b** 1 : 1 **c** 72 cm

Exercise 2:01C

1 **a** *A*, \$3000; *B*, \$7000 **b** *G*, 42 : *D*, 63 **c** 1·5 L **d** *M*, \$330 : *J*, \$440
e *M*, 15 : *S*, 25 **f** €35, €21, 7 hours **g** Army, \$124 200 000 : Navy, \$55 200 000
2 **a** *R*, \$60; *A*, \$100; *N*, \$40 **b** 9 **c** 270, 300, 330 **d** \$540 **e** 36 cm, 45 cm, 27 cm

Challenge 2:01 Mixing drinks

No, orange is only $\frac{1}{3}$ part of *A* and only $\frac{1}{5}$ part of *B*. Adding the two mixtures would give a mixture where the fraction of orange is between $\frac{1}{3}$ and $\frac{1}{5}$. Container *C* has $\frac{3}{7}$ orange.

1 **a** 333 mL **b** 200 mL **c** 429 mL **2** **a** 667 mL **b** 800 mL **c** 571 mL
3 **a** 533 mL **b** 629 mL **4** **a** 1467 mL **b** 1371 mL **5** **a** 27% **b** 31% **6** **a** 27 : 73 **b** 31 : 69

Exercise 2:01D

1 **a** 960 **b** \$232.38 **c** 805 **d** \$13 904 **e** \$49 115
2 **a** 5·5% **b** 85% (to nearest whole %) **c** **i** \$45 000 **ii** 21·4% (correct to 1 dec pl)
d 12% **e** 2%
3 **a** \$1299 **b** 16 880 **c** \$1 450 000 **d** \$78 000 **e** \$7200

Exercise 2:01E

1 **a** $1{\cdot}152\,m^3$ **b** $1{\cdot}68\,m^3$ (correct to 2 dec pl) **c** 8143 L (to the nearest litre)
d 6·5 cm (correct to 1 dec. pl.) **e** Volume = $140\,cm^3$ Mass = 1092 g
2 **a** Area = $11{\cdot}34\,m^2$; Cost = $340.20 **b** Area = $50{\cdot}08\,m^2$; Cost = $2253.60
c $34.20 **d** $3022\,cm^2$ (to the nearest cm^2) **e** $78{\cdot}4\,m^2$
3 **a** 384 m **b** 35 m **c** **i** 4·005 km **ii** 5 **d** 318 mm **e** 796 revolutions

Fun Spot 2:01

a 80 m **b** 160 m **c** 200 m **d** 220 m **e** 230 m **f** 238 m

Exercise 2:02

1 **a** 32 **b** $592.90 **c** 54 **d** 3
e The first number must be $\frac{1}{2}$. If so, the other can be any number.
f 1296 **g** 1·2 kg **h** 12
2 **a** $42{\cdot}25\,m^2$. One quarter of the perimeter of the outside of the square path gives the length of one side (ie 8·5 m). The length of the inside of the path is therefore 2 m less (ie 6·5 m). The area of the garden is the area of the square bounded by the inside of the path. The area of the garden is $6{\cdot}5 \times 6{\cdot}5\,m^2$.
b length = 24 cm, breadth = 6 cm. Let the breadth be x cm. The length is therefore $4x$ cm. The perimeter, $60\text{ cm} = 2(x) + 2(4x)$. Solve this equation to find the breadth, x (ie $x = 6$). The length is four times the breadth (ie 24 cm).
c 15. This could have been done by *guess and check*. However, you could let the number of 5-cent coins be x. The number of 10-cent coins is then $37 - x$. The total, 295 cents = $x \times 5$ cents + $(37 - x) \times 10$ cents. Solving the equation $295 = 5x + 10(37 - x)$, we get $x = 15$. There are 15 five-cent coins. Check your answer by working out the value of 15×5 cents + $(37 - 15) \times 10$ cents.
3 **a** 1 **b** 4 **c** 9 **d** 16 **e** 25 **f** 36 **g** 49 **h** 64
Total = $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$
4 **a** **i** 2 **ii** 3 **iii** 4 **iv** 5 **v** 16 **vi** 187.
The number of pieces is always one more than the number of cuts.
b **i** 4 **ii** 8 **iii** 16 **iv** 32 **v** 64 **vi** 16 384.
The number of additional folds can be found by raising 2 to the power of one less than the fold number.
5 **a** 56, 23 **b** 67, 78 **c** 634, 566 **6** 30

7

Number	0	1	2	3	4	5	6	7	8	9
Number needed	16	15	15	15	15	57	5	5	5	5

8 20 **9** 12 **10** 30

Investigation 2:02 Line marking

146·304 m

Challenge 2:03

1 **a** 3 **b** 18 **c** 3 **2** **a** 33 **b** 42 **c** 35
3 **a** 5 **b** 8 **c** 10 **4** **a** 2 **b** 8 **c** 5 **d** 2 **e** 14
5 **a** 10 **b** 0 **c** 5 **d** 8 **e** yes
6 **a** 6 **b** 5 **c** 3 **d** 7 **e** 20 **f** 9 **g** 28 **h** 32 **i** 23

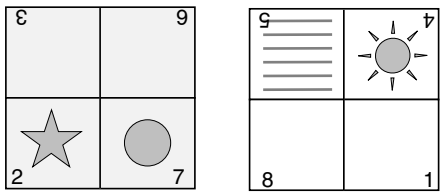
2A Revision Assignment

1 **a** $70 **b** 5·3 runs/over; 79 or 80 runs **c** 8 min 20 s **d** 15 or 16 drops/min
2 **a** 161° **b** 52 mm, 195 mm **c** $231\,cm^2$ **d** $26{\cdot}95\,m^3$
3 **a** 120 g **b** $0{\cdot}53\,m^3$ **c** 400 g in the smaller plot and 600 g in the larger plot **d** 60%, 40%
4 1250 g of packaged puff pastry, $2\frac{1}{2}$ cups sugar, $1\frac{7}{8}$ cups cornflower, $1\frac{1}{4}$ cups custard powder, 3·125 litres milk, 150 g butter, 5 egg-yolks, 5 teaspoons vanilla
5 **a** $2612.50 **b** 6·3% **c** **i** $2800 **ii** $2632 **6** **a** 12 **b** $1{\cdot}92\,m^3$
7 €47 308.80 **8** 15 games **9** **a** 2·4 m **b** 1·5 m **10**

6	3
6	9

2B Working Mathematically

1 **a** 15 to 24 **b** 15, 16, 17 **c** 18, 19, 20 **d** 21, 22, 23, 24 **e** 17
2 **a** 21, 22, 23 **b** 5, 16, 19, 23 **c** 12, 13, 14 **d** 9
3 4 hectares **4** **a** 14 **b** 23 **c** 46 **d** 42
5 **a** June **b** July, 24°C **c** 4 **d** 148 mm
6



Front Back

Chapter 3: Ratio and Proportion

Exercise 3:01

1a i 3:1 **ii** 1:3 **b** No

2 a i 9:11 **ii** 9:20 **iii** 11:20 **b i** $\frac{9}{20}$ **ii** $\frac{11}{20}$

3 a i 5:4 **ii** 5:3 **iii** 4:3 **iv** 5:4:3 **b i** $\frac{5}{12}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{4}$

4 a i 4:3 **ii** 3:7 **b i** $\frac{3}{7}$ **ii** $\frac{4}{7}$

5 a 50 **b** 80

6 a i $\frac{1}{10}$ **ii** $\frac{2}{5}$ **iii** $\frac{5}{1}$ **iv** $\frac{2}{5}$ **b** $\frac{2}{5}$

7 a $\frac{4}{7}$ **b** 3:4

8 a 12 **b** $\frac{1}{3}$ **c** twice

9 a 2:3:4 **b** 400 g : 600 g : 800 g

10 a 8 cm long and 6 cm wide **b** 48 cm^2

11 €3000 : €9000 : €12 000

12 a 32.2 cm **b** 1:12

13 a 7:6:4 **b** 84 sedans, 72 station sedans and 48 motorcycles

14 a 90 dollar coins, 108 fifty cent pieces and 90 twenty cent pieces **b** $162

15 22

Exercise 3:02

1 a €64 **b** €49 **c** 64 m **d** 36 m **e** $480

2 15 cm

3 a 720 **b** $\frac{4}{5}$

4 a 36 000 **b** 20% **c** 120%

5 a $660 **b** 660:550 = 6:5

6 a 270 **b** increase in the ratio 27:25

7 3 852 000 **8 a** $\frac{1}{4}$ **b** 7.5 cm **c** 3:4

9 a 6300 million **b** 7:8 **c** 7:8

10 a 3060 **b** 17:20 **c** $\frac{17}{20}$ **11** 4 **12 a** £25 **b** £48

Exercise 3:03

1 675 mL **2** 15 metres

3 a 2:5:8 **b** 120 m^3 of cement, 300 m^3 of sand and 480 m^3 of aggregate

4 175 mL of baking powder, 475 mL of sugar and 1050 mL of flour

5 $16\frac{2}{3}$ m **6 a** 2.25 litres **b** 2.7 litres **7** 320 pixels **8** 12

9 a Jeremy's **b** 60 m **c** 20 cm **d** 3:2

10 a $\frac{5}{14}$ **b** $\frac{1}{14}$ **c** 280g **d** 320g

Diagnostic Test 3

1 a $200:$280 **b** $160:$320 **c** $330:$150

2 a 16 **b** 25 **c** 20 men and 25 women

3 12 cm **4 a** $\frac{1}{3}$ **b** 42

5 a 82% **b** ₩4 100 000 **c** 50:41

6 a 200g **b** 270g **c** 330g

7 a 3 **b** 23 **c** 115%

8 a 134 m **b** $\frac{1}{200}$ **c** $\frac{1}{2}$%

9 a 1:25 **b** 75cm **c** $\frac{1}{25}$

10 35 pieces

3 Revision Assignment

1 a 5:12 **b** 7:5 **c** 7:12

2 a i $\frac{1}{4}$ **ii** $\frac{1}{3}$ **iii** $\frac{5}{12}$ **b** 3:4:5

3 a i $\frac{2}{5}$ **ii** 3:2 **b** 45

4 a i $\frac{5}{8}$ **ii** $\frac{3}{8}$ **b** 40

5 a $980 **b** 280 **c** $\frac{2}{5}$ **6** 57 kg

7 a $550 **b** $\frac{3}{10}$ **8** 294 mm **9** 16 days **10** 30 km

Chapter 4: Algebraic Expressions

Prep Quiz 4:01

1 19 **2** -4 **3** 1 **4** -11 **5** 15 **6** $5+2$ **7** $9-4$ **8** a **9** 12 **10** $2a$

Exercise 4:01

1 **a** $5x$ **b** $13a$ **c** $31p$ **d** $3a$ **e** $6b$ **f** $10q$ **g** $2p$ **h** $7x$
i $4x$ **j** $12x^2$ **k** $3a^2$ **l** $2y^2$ **m** $-2y$ **n** $-2x$ **o** $3n$ **p** $16ab$
q pq **r** $-2x^2y$

2 **a** $5a+b$ **b** $13x-9$ **c** $17+3a$ **d** $4q-p$ **e** $3y+x$ **f** $m+n$ **g** $q+3p$ **h** $k-2h$
i $7m+4n$ **j** $3a+5b$ **k** $9t+u$ **l** $5p-q$ **m** $a+4p$ **n** $2m$ **o** $1-3x$ **p** -2
q $3x^2+x$ **r** $4p^2+5p$ **s** $2q^2+4q$ **t** $2y^2$ **u** $2-p^2+p$ **v** $3a+a^2+7$
w $x-7-3x^2$ **x** $8ab-16$

Prep Quiz 4:02

1 -24 **2** 6 **3** -7 **4** 9 **5** 4 **6** $7a$ **7** xy **8** a^2 **9** $6x$ **10** $6x$

Exercise 4:02

1 **a** $10x$ **b** $21p$ **c** $24y$ **d** $9a$ **e** $5b$ **f** $108p$ **g** xy **h** $6xb$
i $24xy$ **j** $30mn$ **k** $17xy$ **l** $50ax$ **m** $2x$ **n** $5xy$ **o** $3a$

2 **a** x^2 **b** $2y^2$ **c** $3a^2$ **d** $8m^2$ **e** $14p^2$ **f** $80b^2$ **g** $121t^2$ **h** $30x^2$
i $2a^2b$ **j** xy^2z **k** a^2bc **l** $6p^2q$ **m** $20mn^2p$ **n** $8a^2b^2$ **o** $3xy^2z$ **p** $2m^2n$
q $-12xy$ **r** $6x^2$ **s** $-35a^2$ **t** $9a^2b$ **u** $28am$ **v** $-10p^2g$ **w** ab^2c **x** $-7a^2b^2$

3 **a** $42a$ **b** $30xyz$ **c** $10x^2$ **d** $a^2b^2c^2$ **e** $6xyz$ **f** $60a^2$ **g** $7mn$ **h** $6k^2l$
i $70x$ **j** $-120a^2$ **k** $180xy$ **l** $-mnp$

Prep Quiz 4:03

1 9 **2** -2 **3** -4 **4** 4 **5** 3 **6** $2\frac{1}{2}$ **7** $2\frac{2}{5}$ **8** $2\frac{6}{7}$ **9** 1 **10** 1

Exercise 4:03

1 **a** $\frac{x}{2}$ **b** $\frac{2b}{5}$ **c** a **d** $\frac{1}{2}$ **e** $\frac{1}{5}$ **f** 3 **g** $2a$ **h** $\frac{4a}{b}$ **i** $\frac{1}{t}$

2 **a** $3x$ **b** $2y$ **c** p **d** $3m$ **e** ab **f** $4xy$ **g** $3x^2$ **h** $2x$ **i** 4
j 2 **k** 1 **l** 4 **m** $\frac{3m}{2n}$ **n** $\frac{3}{2}$ or $1\frac{1}{2}$ **o** $\frac{8a}{3b}$ **p** $\frac{1}{4x}$ **q** $4y$
r $3b$ **s** $\frac{9}{7y}$ **t** $\frac{6}{5b}$ **u** $\frac{1}{9k}$ **v** $\frac{1}{3}$ **w** $\frac{m}{2}$ **x** $\frac{9a}{4c}$

3 **a** $-4p$ **b** $3x$ **c** $-5p$ **d** 2 **e** $\frac{-5}{3a}$ **f** $\frac{-x}{z}$ **g** $\frac{ab}{16}$ **h** 6 **i** -14
j $\frac{5}{3}$ **k** $\frac{-3}{y}$ **l** $4n$

Exercise 4:04A

1 **a** $\frac{2x}{3}$ **b** $\frac{5m}{7}$ **c** $\frac{7n}{6}$ **d** $\frac{x}{5}$ **e** $\frac{5t}{8}$ **f** $\frac{2m}{7}$

2 **a** $\frac{4p}{5}$ **b** $\frac{t}{2}$ **c** $\frac{2y}{3}$ **d** $\frac{3x}{5}$ **e** $\frac{7x}{6}$ **f** $\frac{7m}{8}$

3 **a** $\frac{5x}{6}$ **b** $\frac{11a}{12}$ **c** $\frac{9m}{10}$ **d** $\frac{7t}{6}$ **e** $\frac{16x}{15}$ **f** $\frac{a}{6}$ **g** $\frac{4w}{15}$ **h** $\frac{11n}{10}$ **i** $\frac{m}{6}$ **j** $\frac{3a}{20}$

4 **a** $\frac{5a}{6}$ **b** $\frac{7y}{3}$ **c** $\frac{m}{2}$ **d** $\frac{m}{2}$ **e** $\frac{3t}{10}$ **f** $\frac{5m}{18}$ **g** $\frac{11a}{12}$ **h** $\frac{19x}{20}$ **i** $\frac{29n}{24}$ **j** $\frac{21y}{8}$

5 **a** $\frac{7}{12}m$ **b** $\frac{5}{24}y$ **c** $\frac{9}{20}x$ **d** $\frac{25}{4}y$ **e** $\frac{6}{5}m$

Exercise 4:04B

1 **a** $\frac{xy}{6}$ **b** $\frac{ab}{30}$ **c** $\frac{mn}{12}$ **d** $\frac{2a}{3x}$ **e** $\frac{5n}{3m}$ **f** $\frac{10p}{3q}$

2 **a** $\frac{15xy}{8}$ **b** $\frac{8mn}{9}$ **c** $\frac{7xy}{6}$ **d** $\frac{6ab}{25}$ **e** $\frac{15x^2}{8}$

3 **a** $\frac{m^2}{6}$ **b** $\frac{xy}{4}$ **c** $\frac{a}{12}$ **d** $2xy$ **e** $\frac{15t^2}{4}$

4 **a** 6 **b** $\frac{1}{4}$ **c** 1 **d** 6 **e** $\frac{a^2}{5}$

5 **a** $2y$ **b** $\frac{2n}{3}$ **c** $15a$ **d** $4y$ **e** $\frac{3a}{2}$

6 **a** $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** $\frac{5}{6}$ **d** $\frac{5}{2}$ **e** 5 **f** $\frac{a}{8}$ **g** $\frac{2x}{5}$ **h** $\frac{5m}{2t}$ **i** $\frac{10m}{n}$ **j** $\frac{2x}{25y}$

7 **a** $\frac{10}{3}$ **b** $\frac{3a}{2b}$ **c** $\frac{3}{2}$ **d** $\frac{2a}{x}$ **e** $\frac{5}{9}$

8 **a** $\frac{10}{x^2}$ **b** $\frac{a^2}{6}$ **c** $\frac{5m}{6}$ **d** $\frac{2b}{3c}$ **e** $\frac{2c}{3b}$

Prep Quiz 4:05

1 $10a$ **2** $2a^2$ **3** $10x^2$ **4** 20 **5** $-6x$
6 $-x^2$ **7** $-14p^2$ **8** 8 **9** 3 **10** 8 and 5

Exercise 4:05

1 **a** $2x + 6$ **b** $3a + 15$ **c** $5x - 5$ **d** $10a + 10$ **e** $3x - 6$
f $8y + 32$ **g** $10a + 2$ **h** $6x - 15$ **i** $36x + 63$ **j** $5 + 5x$
k $12 - 4a$ **l** $10 - 6m$ **m** $6a + 4b$ **n** $5x + 5y$ **o** $21x - 35y$

2 **a** $x^2 + 7x$ **b** $a^2 - a$ **c** $m^2 + 10m$ **d** $n^2 - 3n$ **e** $2a^2 + a$
f $3b^2 - 5b$ **g** $5x + x^2$ **h** $3y - y^2$ **i** $2x^2 + 6x$ **j** $3y^2 - 3y$
k $10z + 5z^2$ **l** $10m - 10m^2$ **m** $6a^2 - 3a$ **n** $18x^2 + 63x$ **o** $16p - 40p^2$
p $21q + 14q^2$ **q** $ax + bx$ **r** $2y^2 + xy$ **s** $2m^2 + 2mn$ **t** $10a^2 + 15ab$

3 **a** $-2x - 6$ **b** $-3a - 15$ **c** $-2y + 2$ **d** $-5p + 15$ **e** $-21a - 14$
f $-10x + 5$ **g** $-21 - 14m$ **h** $-28 + 4x$ **i** $-a - 1$ **j** $-3x - 7$
k $-8 + 2p$ **l** $-3a - 2b$ **m** $-x^2 - 10x$ **n** $-5y + y^2$ **o** $-6x^2 - 21x$
p $-80mn + 10n^2$

4 **a** $2x + 12$ **b** $4x - 6$ **c** x **d** $-y$

Prep Quiz 4:06

1 $10x$ **2** $3a^2$ **3** $6x + 8$ **4** $x + 2$ **5** $5y^2 + 4y$ **6** $13 + 2a$ **7** $3x - 21$ **8** $18 - 45y$ **9** $2a^2 + 6a$ **10** $-5x - 35$

Exercise 4:06

1 **a** $6x + 15$ **b** $4a + 31$ **c** $2y + 8$ **d** $3p + 14$ **e** $9a + 12$
f $10m + 9$ **g** $7a + 8$ **h** $14x + 25$ **i** $12a + 16$ **j** $12m + 14$

2 **a** $5x + 8$ **b** $8y + 22$ **c** $5m + 23$ **d** $x^2 + 6x + 3$ **e** $a^2 + 10a + 25$
f $2n^2 + 7n + 3$

3 **a** $2a - 2$ **b** $7a + 4$ **c** $4m + 14$ **d** $3x - 2$ **e** $4m - 15$
f $6p - 7$ **g** $8y - 17$ **h** $10a - 9$ **i** $12x + 2$

4 **a** $-2a$ **b** $2 - 3x$ **c** -3 **d** $m + 8$ **e** $5a - 24$
f $6y$ **g** $3m$ **h** $2x + 23$ **i** $4a^2 + 3ab - 2b^2$

5 **a** $17k - 6$ **b** $13m - 5$ **c** $2a + 15$ **d** $14 + 2y$ **e** $28 - 6x$
f $14x - 13$ **g** $2a + 66$ **h** $t^2 - 9t + 20$ **i** $4x^2 - 11x + 6$ **j** $2m^2 - 11m + 15$
k $3y - 17$ **l** $6a^2 - 4a + 9$ **m** $a^2 + ab$ **n** $x^2 - xy - 6x + 3y$ **o** $6x^2 - 5xy + 2y$

Prep Quiz 4:07

1 $2x + 10$ **2** $4x^2 - x$ **3** $6a^2 + 21a$ **4** $2m^2 + 6mn$ **5** $\{1, 2, 3, 4, 6, 12\}$ **6** $\{1, 2, 4, 5, 10, 20\}$
7 $\{1, 2, 3, 5, 6, 10, 15, 30\}$ **8** 4 **9** 6 **10** 10

Exercise 4:07

1 **a** $x + 2$ **b** $a + 3$ **c** $2a - 3$ **d** $x + 2y$ **e** $3x - 2y$
f $4pq - 5x$ **g** $m - p$ **h** $b - c$ **i** $x + 2y$ **j** $y - 5$
k $5 - 3a$ **l** $2m - 1$

2 **a** $2(x+5)$ **b** $2(3a+2)$ **c** $7(y+3)$ **d** $4(7+x)$ **e** $3(9-y)$
f $6(4x+1)$ **g** $9(x-5)$ **h** $4(4-3a)$ **i** $3(3x+y)$ **j** $5(a+2b)$
k $5(3m-4n)$ **l** $2(2b-3a)$ **m** $m(p+n)$ **n** $a(x+y)$ **o** $x(x+y)$
p $p(p-q)$ **q** $a(p+3)$ **r** $x(5+a)$ **s** $m(4-n)$ **t** $t(x-1)$

3 **a** $3a(x+2y)$ **b** $5m(n-2p)$ **c** $2b(2a-3c)$ **d** $3q(3p-2r)$ **e** $5x(x-2y)$
f $3a(b+2a)$ **g** $2m(5m-2n)$ **h** $4x(3x+y)$ **i** $bc(a+d)$ **j** $pq(a-b)$
k $xy(z+1)$ **l** $mn(1-p)$ **m** $xa(x-y)$ **n** $5x(a-2y)$ **o** $ap(a-5)$
p $xy(y+z)$ **q** $5b(2a-3c)$ **r** $xy(5x-3y)$ **s** $ap(p-a)$ **t** $ab(5-ab)$

4 **a** $-2(a+3)$ **b** $-5(x+3)$ **c** $-4(2m+3)$ **d** $-5(2x+1)$ **e** $-4(2x-1)$
f $-3(n-3)$ **g** $-7(y-5)$ **h** $-2(3a-2)$ **i** $-x(x+3)$ **j** $-m(m+1)$
k $-x(3x-2)$ **l** $-5y(y-2)$ **m** $-p(4+p)$ **n** $-x(3+2x)$ **o** $-m(1-7m)$
p $-2a(2-9a)$

5 **a** $a(b+c+d)$ **b** $x(3+y+z)$ **c** $m(m-3+n)$ **d** $a(7-b+a)$ **e** $p(p+q-5)$
f $2(x+2y-3z)$ **g** $5(2a-b+3c)$ **h** $3(3x^2+2x-4)$ **i** $2(4-2x+3x^2)$ **j** $5(5+3y-4y^2)$
k $x(xy-3y+1)$ **l** $2a(b-2c+5)$ **m** $3x(x+2y-3)$ **n** $xy(x+1+y)$ **o** $ab(ab+3a+2b)$
p $mn(1+4m-8n)$

6 **a** $(a+2)(a+3)$ **b** $(m+2)(m+4)$ **c** $(x-1)(x+5)$ **d** $(b+1)(b-5)$ **e** $(y-2)(7-y)$
f $(t-7)(t-9)$ **g** $(2m-3)(4+3m)$ **h** $(7x+1)(2x-5)$ **i** $(a+3)(x-1)$ **j** $(2y-1)(y-1)$
k $(p-3)(p-3)$ **l** $(5x+3)(1-x)$

Diagnostic Test 4 Algebraic Expressions

1 **a** $2a+3b$ **b** $2p^2+2p$ **c** $2ab$ **d** $4a-x-2$

2 **a** $56m$ **b** $30ab$ **c** $10y^2$ **d** $-8ny$

3 **a** $3a$ **b** $5y$ **c** $\frac{3c}{2b}$ **d** $\frac{-1}{3y}$

4 **a** $\frac{4a}{5}$ **b** $\frac{2m}{7}$ **c** $\frac{x}{2}$ **d** $\frac{2p}{5}$

5 **a** $\frac{7a}{12}$ **b** $\frac{m}{15}$ **c** $\frac{5x}{4}$ **d** $\frac{n}{4}$

6 **a** $\frac{xy}{12}$ **b** $\frac{a^2}{10}$ **c** 4 **d** $\frac{a}{2}$

7 **a** $\frac{4}{3}$ **b** $\frac{a^2}{10}$ **c** $\frac{3}{2}$ **d** $\frac{n^2}{3}$

8 **a** $9x+63$ **b** $30a-12$ **c** p^2+3p **d** $15a-6a^2$

9 **a** $-3x-6$ **b** $-2m+16$ **c** $-15x-20$ **d** $-7+2m$

10 **a** $2a+11$ **b** $5x^2-7x$ **c** $11m-2$ **d** $6x-18$

11 **a** $5(m+2)$ **b** $x(x-3)$ **c** $3a(2b+5)$ **d** $-4(2y+3)$

4A Revision Assignment

1 **a** $5x$ **b** $6x^2$ **c** $6x$ **d** $2x^2$ **e** $2x$ **f** 2 **g** $2x+3y$ **h** $6xy$

2 **a** $6ab^2$ **b** $2ab$ **c** $5ab$ **d** $3a^2-a$ **e** $6x+3y$
f $15+6x$ **g** $7a+1$ **h** $4x^2-x$

3 **a** $3x+15$ **b** $2x-14$ **c** $-3x-6$ **d** $-4+12x$ **e** a^2+3a
f $xy+xz$ **g** $3p^2-3pq$ **h** $10x^2-35x$

4 **a** $3(a+5)$ **b** $3(2m+3)$ **c** $5(3-y)$ **d** $x(a-3)$ **e** $2x(1+3y)$
f $2x(2x-1)$ **g** $3b(3a-2c)$ **h** $3x(2x-3+y)$

5 **a** $4x+13$ **b** $10y+25$ **c** $3a+6$ **d** $2a-10$ **e** $5m-14$
f $3x^2-8x+4$

6 **a** $\frac{2a}{3}$ **b** $\frac{9a}{20}$ **c** $\frac{4m}{5}$ **d** $\frac{17y}{12}$ **e** $\frac{d}{2}$ **f** $\frac{3x}{10}$ **g** $\frac{a}{4}$ **h** $\frac{9m}{20}$

7 **a** $\frac{a^2}{12}$ **b** $\frac{3ab}{10}$ **c** $3m$ **d** $\frac{a^2}{12}$

8 **a** 3 **b** $\frac{3}{10}$ **c** 4 **d** $4n$

9 **a** Incorrect **b** Incorrect **c** Correct **d** Incorrect

10 **a** $4(3a-4)=12a-16$ **b** $-3(x-2)=-3x+6$

c
$$\begin{aligned}&2(a+3)-3(a+4)\\&=2a+6-3a-12\\&=-a-6\end{aligned}$$

d
$$\begin{aligned}&4(3a-2)-3(2a+1)\\&=12a-8-6a-3\\&=6a-11\end{aligned}$$

4B Working Mathematically

1 **a** *square* **b** *rectangle* **c** *parallelogram* **d** *rhombus*
e *trapezium* **f** *pentagon* **g** *hexagon* **h** *octagon*
i *kite* **j** *isosceles triangle*

2 **a** An octagonal prism **b** 200 mL

3 17 **4** **a** 4 **b** 10 **5** **a** 40% **b** 5%

6 **a** Tasmania; 60% **b** Victoria; over 90% **c** Queensland, *just under 3000 per 10 000*
d About 50%; less than 40%. Perhaps some could read but not write, or perhaps some children were not tested.

Chapter 5: Pythagoras' Theorem

Exercise 5:01

1

	a	b	c	a^2	b^2	c^2	$a^2 + b^2$
a	2	3	4	4	9	16	13
b	3	4	5	9	16	25	25
c	2·4	3·2	4	5·76	10·24	16	16
d	2·5	3	3	6·25	9	9	15·25
e	2	4·8	5·2	4	23·04	27·04	27·04

2 **b**, **d**, **f**; right-angled.

3 **a**

a	b	c
3	4	5

A_1	A_2	A_3
9	16	25

b

a	b	c
6	8	10

A_1	A_2	A_3
36	64	100

c

a	b	c
5	12	13

A_1	A_2	A_3
25	144	169

b ΔABE **c** 90°

4 **a**

Triangle	AB^2	*(right side)*^{2}	*(left side)*^{2}
ΔABC	16	9	$(2{\cdot}5)^2 = 6{\cdot}25$
ΔABD	16	9	16
ΔABE	16	9	25
ΔABF	16	9	36

Investigation 5:01B

a yes **b** yes **c** yes **d** $a^2 + b^2$ **e** c^2 **f** yes
In any right-angled triangle the square on the longest side is equal to the sum of the squares on the other two sides.

Prep Quiz 5:02

1 A **2** B **3** B **4** $c^2 = n^2 + m^2$ **5** 4 **6** 25 **7** 25 **8** 25 **9** 6 **10** No

Exercise 5:02A

1 **a** $c^2 = 8^2 + 15^2$ **b** $a^2 = 9^2 + 12^2$ **c** $h^2 = 10^2 + 15^2$ **d** $p^2 = 8^2 + 10^2$

2

Number	3	4	5	6	7	8	9	10	12	13	15	16	17	18	20	24	25	26	30
Square	9	16	25	36	49	64	81	100	144	169	225	256	289	324	400	576	625	676	900

3 **a** 5 **b** 10 **c** 13 **d** 15 **e** 30 **f** 25 **g** 17
4 **a** 5 m **b** 25 cm **c** 10 cm **d** 26 m
5 **a** 5·1 cm **b** 2·8 m **c** 4·5 cm **d** 4·1 m **e** 4·2 cm **f** 3·6 m **g** 2·2 m
6 **a** 5·7 cm **b** 4·2 m **c** 17 cm **7** **a** 5 cm **b** 17 m **c** $a = 17$; $b = 10$
8 **a** 5 units **b** 5 units **c** 13 units **9** **a** 5·1 m **b** 10 km **c** 5·4 m

Exercise 5:02B

1 **a** 256 **b** 6·25 **c** 46·24 **d** 17 **e** 41 **f** 5·8 **g** 9·8 **h** 12·17
2 **a** 6·1 m **b** 2 m **c** 4·5 m **d** 20 cm
3 **a** 4·2 m **b** 21·6 cm **c** 12·2 cm **d** 5·7 m **e** 10·4 m **f** 94·3 cm **g** 5·7 m
4 **a** 8·5 cm **b** 17·5 cm **c** 14·4 cm **d** 21·5 m **e** 47·7 cm (or 47·6 cm)

Investigation 5:02

1 $(3x)^2 + (4x)^2 = 9x^2 + 16x^2$
$= 25x^2$
$(5x)^2 = 25x^2$
$\therefore (3x)^2 + (4x)^2 = (5x)^2$.
$\therefore \{3x\ 4x, 5x\}$ is a Pythagorean triad.

a	b	c
3	4	5
6	8	10
9	12	15
12	16	20
27	36	45
3·6	4·8	6

2 a 11 60 61
13 84 85
15 112 113
b i 27, 364, 365 **ii** 51, 1300, 1301
iii 101, 5100, 5101

3 {3, 4, 5}, {5, 12, 13}, {7, 24, 25}, {9, 40, 41}, {11, 60, 61}, {13, 84, 85}

4 {8, 15, 17}, {12, 35, 37}, {16, 63, 65}, {20, 21, 29}, {27, 45 53}

Exercise 5:03A

1 a $5^2 = a^2 + 4^2$ **b** $15^2 = b^2 + 12^2$ **c** $13^2 = 5^2 + x^2$ **d** $25^2 = y^2 + 15^2$

2 a 3 **b** 12 **c** 20 **d** 18 **e** 8 **f** 9

3 a $13^2 = 12^2 + x^2$
$169 = 144 + x^2$
$x^2 = 25$
$x = \sqrt{25}$
$x = 5$

b $30^2 = 24^2 + a^2$
$900 = 576 + a^2$
$a^2 = 324$
$a = \sqrt{324}$
$a = 18$

c $26^2 = y^2 + 10^2$
$676 = y^2 + 100$
$y^2 = 576$
$y = 24$

d $10^2 = p^2 + 8^2$
$100 = p^2 + 64$
$p^2 = 36$
$p = 6$

4 a 3 **b** 9 **c** 12 **d** 20 **5 a** 16 cm **b** 15 cm **c** 10 cm **d** 12 cm

6 a 3 **b** 12 **c** 8 **d** 8 **e** 34 cm

Exercise 5:03B

1 a 1·6 **b** 1·5 **c** 40 **d** 21 **2 a** 7·4 **b** 2·1 **c** 4·1 **d** 2·0

3 a 1·5 **b** 6 **c** 96

4 a 12·65 m (corr. to 2 dec. pl.) **b** 19·36 m **c** 14·14 nm (corr. to 2 dec. pl.)

5 a 4·8 m **b** $x = 12$; $y = 9$ **c** $h = 3{\cdot}2$; $l = 8{\cdot}32$ **d** $h = 1{\cdot}4$

6 a 5·7 cm **b** 33·5 cm

Exercise 5:04

1 a 15 **b** 15 **c** 16 **d** 7·5 **e** 20 **f** 6 **g** 14

2 a 34 cm **b** 11·83 m **c** 5·4 km **d** 8·42 m **e** 3 m **f** 1200 m **g** yes **h** 2·42
i Bill, 30 km; Bev, 40 km; 50 km **j** 9140 mm **k** 4081 mm **l** 10·20 m

3 a $x = 5{\cdot}6$; $y = 11{\cdot}9$ **b** $x = 60$; $y = 27$ **c** $x = 5$; $y = 3$

4 a 21·7 cm **b** 59·4 cm **c** 19·0 cm **d** 2·3 m **5 a** 19·85 cm **b** 11·18 cm

Investigation 5:04

iii Wendy will take 24·4 min walking (missing train) or 12·2 minutes running (early but flustered). Wendy should run part of the way and walk the rest.

Diagnostic Test 5: Investigating Pythagoras' Theorem

1 a no **b** yes **c** yes **2 a** l, m, n **b** c, b, a **c** n, p, m

3 a $c^2 = 6^2 + 8^2$ **b** $l^2 = 3^2 + 6^2$ **c** $x^2 = 5{\cdot}6^2 + 4{\cdot}2^2$

4 a 10 m **b** 6·7 m (to 1 dec. pl.) **c** 7 m

5 a $17^2 = a^2 + 8^2$ **b** $4^2 = b^2 + 3{\cdot}2^2$ **c** $25^2 = a^2 + 20^2$

6 a 15 **b** 2·4 **c** 15 **7 a** 16 **b** 39 **c** 7

5A Revision Assignment

1 a triangular prism **b** trapezoidal prism **c** 6 faces, 12 edges, 8 vertices **d** pentagon

2 a rectangular prism **b** cylinder **c** triangular prism

3 a acute **b** obtuse **c** obtuse **d** reflex **e** reflex

4 a obtuse **b** acute **c** $\angle BCD$ **d** ΔABC

5 a 48 **b** 109 **c** 1565 **d** 1993

6 a 5000 **b** 250 **c** 5200 **d** 0·56 **e** 3·3

7 a \$2217.60 **8 a** 11:20 am **b** 7:45 pm

9 a 20 m^2 **b** 1428 mm^2 **c** 31·25 m^2 **10 a** 16·2 cm^3 **b** 125 cm^3 **c** 12 cm^3

5B Working Mathematically

1 28 **2** 28 **3** (Kate 1, Sarah 10), (Kate 3, Sarah 5), (Kate 5, Sarah 0) **4** 57 **5** 9 **6** 7 drinks

Chapter 6: Indices

Exercise 6:01

1 **a** 2^4 **b** 3^2 **c** 5^3 **d** 7^5 **e** 10^3 **f** 9^4 **g** x^2 **h** a^4
i n^3 **j** m^5 **k** p^6 **l** y^2 **m** 4^3 **n** t^4 **o** x^5

2 **a** $2 \times 2 \times 2$ **b** 4×4 **c** $6 \times 6 \times 6 \times 6 \times 6$ **d** $10 \times 10 \times 10 \times 10$ **e** $7 \times 7 \times 7$ **f** 3
g $a \times a \times a$ **h** $x \times x \times x \times x \times x$ **i** $y \times y$ **j** $m \times m \times m \times m \times m \times m$
k $n \times n \times n \times n \times n \times n \times n$ **l** $p \times p \times p$

3 **a** 32 **b** 81 **c** 49 **d** 121 **e** 10 000 **f** 216 **g** 256 **h** 78 125
i 32 768 **j** 59 049 **k** 16 777 216 **l** 14 348 907 **m** 8000 **n** 27 783 **o** 455 625 **p** 592 704

4 **a** 10^3 **b** 10^5 **c** 10^6 **d** 10^9 **5** **a** 2^6 **b** 2^8 **c** 2^{14} **d** 2^{20}

6 **a** 3^3 **b** 3^5 **c** 3^{10} **d** 3^{14} **7** **a** $2^2 \times 3^2$ **b** $2^3 \times 5^2$ **c** $2^2 \times 7^2$ **d** $2^4 \times 5$

8 **a** $x = 2$ **b** $x = 3$ **c** $x = 5$ **d** $x = 4$ **9** 2 147 483 648 **10** less

Challenge 6:01 Family trees

1 8 **2** 20. No, the names of some people would be repeated.
3 The estimate should be less than 2 000 000 and probably less than 1 000 000.
4 Answers will vary but at least 12 generations would be needed and perhaps many more.

Prep Quiz 6:02

1 5^4 **2** a^3 **3** $2 \times 2 \times 2 \times 2$ **4** $m \times m \times m \times m \times m$ **5** 4 **6** 8 **7** 32 **8** 27 **9** 3 **10** 1

Exercise 6:02

1

Power	11^6	6^8	8^{10}	5^6	8^7	7^8	5^4	3^5
Answer	1 771 561	1 679 616	1 073 741 824	15 625	2 097 152	5 764 801	625	243

Expression	$7^3 \times 7^5$	$5^8 \div 5^2$	$(11^3)^2$	56^0	$3^{10} \div 3^5$	$8^2 \times 8^5$	$(13)^0$	$(6^2)^4$
Answer	5 764 801	15 625	1 771 561	1	243	2 097 152	1	1 679 616

2 **a** true **b** false **c** true **d** true **e** true **f** true **g** true **h** false **i** true
3 **a** 10^5 **b** 10^3 **c** 10^6 **d** 5^6 **e** 2^5 **f** 7^6 **g** 3^9 **h** 2^8 **i** 10^{10}
4 **a** 10^1 **b** 10^3 **c** 10^1 **d** 5^4 **e** 7^3 **f** 3^9 **g** 2^1 **h** 5^0 or 1 **i** 2^6
5 **a** 10^6 **b** 10^9 **c** 10^{12} **d** 2^{12} **e** 2^4 **f** 2^{35} **g** 3^8 **h** 5^{15} **i** 7^8
j $2^6 \times 3^4$ **k** $7^6 \times 11^8$ **l** $3^4 \times 2^8$
6 **a** x^5 **b** y^6 **c** m^6 **d** m^5 **e** p^{10} **f** a^2 **g** y^7 **h** x^3 **i** m^7
j $3y^5$ **k** $3m^6$ **l** $15x^3$
7 **a** x^3 **b** x^4 **c** x^5 **d** m^2 **e** y^3 **f** m^2 **g** 1 **h** 1 **i** y^6
j $2m^6$ **k** $2y^8$ **l** $4x^5$
8 **a** x^6 **b** y^8 **c** a^{15} **d** 1 **e** 1 **f** 1 **g** y^6 **h** a^{18} **i** x^9
j $8x^3$ **k** $9x^4$ **l** $625m^8$

Exercise 6:03

1 **a** $\frac{1}{3}$ **b** $\frac{1}{5}$ **c** $\frac{1}{2}$ **d** $\frac{1}{36}$ **e** $\frac{1}{16}$ **f** $\frac{1}{1000}$ **g** $\frac{1}{16}$ **h** $\frac{1}{10\,000}$ **i** $\frac{1}{25}$

2 **a** 11^{-1} **b** 3^{-1} **c** 5^{-1} **d** 7^{-1} **e** 3^{-3} **f** 5^{-4} **g** 2^{-8} **h** 7^{-2} **i** 10^{-2}
j 10^{-3} **k** 10^{-6} **l** 10^{-5}

3 **a** true **b** false **c** true **d** false **e** false **f** false **g** true **h** false

4 **a** 10^{-3} **b** 10^1 **c** 10^{-5} **d** 10^{-3} **e** 10^{-1} **f** 10^{-6} **g** 10^6 **h** 10^{-4}

5 **a** $\frac{1}{a}$ **b** $\frac{1}{x}$ **c** $\frac{1}{m}$ **d** $\frac{1}{y}$ **e** $\frac{1}{x^3}$ **f** $\frac{1}{y^2}$ **g** $\frac{1}{x^4}$ **h** $\frac{1}{m^6}$ **i** $\frac{2}{x}$
j $\frac{5}{a^3}$ **k** $\frac{10}{y^2}$ **l** $\frac{36}{q^4}$

6 **a** x^{-1} **b** x^{-2} **c** x^{-3} **d** x^{-4} **e** $5y^{-2}$ **f** $3a^{-1}$ **g** $10m^{-4}$ **h** $75x^{-3}$ **i** xy^{-2}
j ma^{-3} **k** $3ab^{-2}$ **l** $4xy^{-1}$

7 **a** 2 **b** 3 **c** $1\frac{1}{2}$ **d** 10 **e** 4 **f** 9 **g** $2\frac{1}{4}$ **h** $11\frac{1}{9}$

8 **a** $\frac{1}{x^2}$ **b** $\frac{1}{a^5}$ **c** $\frac{3}{x}$ **d** $\frac{5}{m^2}$ **e** $\frac{1}{(x+1)^2}$ **f** $\frac{1}{(3+a)^1}$ **g** $\frac{1}{(6x)^2}$ or $\frac{1}{36x^2}$
h $\frac{4}{(x+2)^1}$ or $\frac{4}{x+2}$

9 **a** 0·125 **b** 0·0625 **c** 0·04 **d** 0·015 625 **e** 0·015 625 **f** 0·015 625 **g** 8
h 25 **i** 400 **j** 0·064 **k** 100 000 **l** 2·56

Investigation 6:03 Zero and negative indices

1

Power of 10	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Answer	10 000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$

• Yes

Power of 5	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}	5^{-4}
Answer	625	125	25	5	1	$\frac{1}{5}$	$\frac{1}{25}$	$\frac{1}{125}$	$\frac{1}{625}$

• Yes

2 **a** true **b** true **c** true **d** true **e** true **f** true **g** true **h** true **i** true

Diagnostic Test 6 Indices

1 **a** 3^4 **b** 5^2 **c** m^3

2 **a** 9 **b** 16 **c** 1000

3 **a** 3^7 **b** x^5 **c** m^3n^5 **4** **a** x^5 **b** a^3 **c** a^2b

5 **a** a^8 **b** x^{12} **c** $8a^{12}$ **6** **a** 1 **b** 5 **c** 3

7 **a** $\frac{1}{9}$ **b** $\frac{1}{5}$ **c** $\frac{27}{8}$ or $3\frac{3}{8}$ **8** **a** x^4 **b** $\frac{1}{x^2}$ **c** $\frac{9}{x^2}$

6A Revision Assignment

1 **a** a^5 **b** a^3b^2 **c** $12a^3b^3$ **d** 3^5 **e** a^3 **f** 4^4 **g** 3^8 **h** x^6
i a^{11} **j** m

2 **a** 1 **b** 6 **c** $125x^9$ **d** $1000a^6$ **e** $8x$

3 **a** 1024 **b** 531 441 **c** 145 800 000 **d** 351 232

4 **a** 7 **b** 5 **c** 8 **5** **a** $3^7 = 2187$ **b** $10^3 = 1000$ **c** $2^8 = 256$

6 **a** $\frac{1}{5}$ **b** $\frac{1}{16}$ **c** $\frac{1}{9}$ **d** $\frac{1}{1000}$

6B Working Mathematically

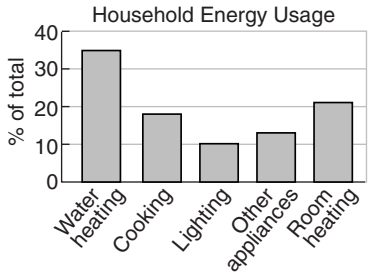
1 **a** *hectare* **b** *cross-section*
c *coordinates* **d** *tally*
e *picture* graph **f** *column* graph
g *line* graph **h** *sector* (or *pie*) graph
i *bar* graph **j** *scatter diagram*

2 (1) *parallel* lines (2) *perpendicular* lines
(3) *vertical*, *horizontal* (4) *concurrent* lines
(5) *angle ABC* or *CBA* (6) *acute* angle
(7) *right* angle (8) *obtuse* angle
(9) *straight* angle (10) *reflex* angle
(11) *revolution* (12) *adjacent* angles

3 **a** 3, 6 **b** **i** 20 **ii** 35 **iii** 405

4 **a** 35% **b** cooking 66°, lighting 39°, other appliances 50°, room heating 79°.

c



5 $31.63 **6** **a** 1900, ten shillings **b** 15·7

c Inflation has caused the value of the dollar to drop. **d** Australia became a nation in 1901. Before this the prices may have been kept high by British taxes or the high cost of imports. There could have been many reasons.

e **i** 52c (or 52%) **ii** 18c (or 18%) approx. **iii** 48c (or 48%)

Chapter 7: Equations and Inequations

Prep Quiz 7:01

1 2 **2** 3 **3** 12 **4** 5 **5** $\frac{x}{6}$ **6** $5y$ **7** $\frac{y}{2}$ or $y \div 2$ **8** $p \div 4$ or $\frac{1}{4}p$ **9** 12 **10** 4

Exercise 7:01

1 **a** 5 **b** 13 **c** 4 **d** 6 **e** 6 **f** 6 **g** 2 **h** 30
i 6 **j** 10 **k** 4 **l** 20 **m** 18 **n** 20 **o** 2 **p** 3
2 **a** 11 **b** 27 **c** 14 **d** 7 **e** 1 **f** 0 **g** 15 **h** $1\frac{1}{2}$
i 8 **j** 12 **k** 4 **l** 8 **m** 24 **n** 14 **o** 15 **p** 7
3 **a** correct **b** incorrect **c** incorrect **d** correct **e** correct **f** incorrect **g** incorrect **h** incorrect
i correct **j** incorrect **k** incorrect **l** correct **m** correct **n** incorrect **o** incorrect **p** incorrect
4 **a** 26 **b** −2 **c** 16 **d** 23 **e** 32 **f** 2 **g** 69 **h** 14
5 **a** A **b** F **c** D **d** C **e** E **f** G **g** H **h** B

Prep Quiz 7:02

1 $x + 7$ **2** $4c$ **3** $\frac{y}{3}$ **4** $a - 9$ **5** $4m + 8$ **6** $4(x - 5)$ **7** subtracting 6 **8** adding 11
9 dividing by 3 **10** multiplying by 7

Exercise 7:02

1 **a** ×5 **b** +7 **c** −4 **d** ÷3 **e** ×4; +1 **f** −8; ×3 **g** ×−7; +2 **h** +3; ÷10
i ÷7, +6 **j** +9; ×−6
2 **a** ÷6 **b** ×2 **c** −9 **d** +4 **e** +7; ÷8 **f** −2; ÷9 **g** −4; ÷−5 **h** +1; ×4
i ×4; −2 **j** −5; ×4

3 **a** $m \xrightarrow{\times 7} 7m$ **b** $y \xrightarrow{\times 3} 3y \xrightarrow{-5} 3y-5$ **c** $a \xrightarrow{\times 8} 8a \xrightarrow{+1} 8a+1$
d $a \xrightarrow{\times(-3)} -3a \xrightarrow{+2} -3a+2$ **e** $p \xrightarrow{+4} p+4 \xrightarrow{\times 5} 5(p+4)$ **f** $m \xrightarrow{\times 4} 4m \xrightarrow{+9} 9+4m$
g $x \xrightarrow{-7} x-7 \xrightarrow{\times 10} 10(x-7)$ **h** $c \xrightarrow{\times(-5)} -5c \xrightarrow{+7} 7-5c$ **i** $x \xrightarrow{\div 4} \frac{x}{4} \xrightarrow{+1} \frac{x}{4}+1$
j $m \xrightarrow{+6} m+6 \xrightarrow{\div 2} \frac{m+6}{2}$ **k** $a \xrightarrow{-8} a-8 \xrightarrow{\div 7} \frac{a-8}{7}$ **l** $c \xrightarrow{\div 3} \frac{c}{3} \xrightarrow{+6} 6+\frac{c}{3}$

4 **a** $x + 4$ **b** $5m$ **c** $\frac{t}{2}$ **d** $y - 4$ **e** $2a$; $2a + 1$ **f** $x + 2$; $9(x + 2)$ **g** $-6n$; $-6n - 7$
h y **i** a **j** n **k** x **l** $7x$, x **m** $a + 3$; a **n** $-3n$; n
5 **a** ×2; +7 **b** ×(−6), −2 **c** ×9; −1 **d** −6; ×4 **e** +5; ×7 **f** +4; ×3
6 **a** −10; ÷3 **b** −15; ÷[−11] **c** +200; ÷20 **d** ×3 **e** −6; ×5 **f** ×4; −3
7 **a** ×4; +1; ÷6 **b** ÷10; −2; ÷7 **c** ×3; −7; ÷[−2] **d** ÷8; −4; ÷[−5] **e** +4; ×3; ÷5
f −6; ×11; ÷10 **g** ×10; ÷7; −4; ÷[−3] **h** ×5; ÷6; +3; ÷2

Prep Quiz 7:03

1 x **2** x **3** a **4** m **5** dividing by 7 **6** subtracting 8 **7** adding 1 **8** multiplying by 4
9 −10; ÷7 **10** +2; ÷3

Exercise 7:03

1 **a** 66 **b** 31 **c** 73 **d** 252 **e** 301 **f** 151 **g** 841 **h** 505 **i** 192 **j** 280
k 644 **l** 650 **m** $9\frac{3}{5}$ **n** $8\frac{2}{7}$ **o** $20\frac{2}{9}$
2 **a** correct **b** incorrect **c** incorrect **d** correct **e** correct **f** correct **g** correct **h** correct
i correct
3 **a** 5 **b** 10 **c** 4 **d** 9 **e** 9 **f** 100 **g** 3 **h** 5 **i** 1 **j** 0
k 0 **l** 4 **m** 8 **n** 7 **o** 2 **p** 13 **q** −1 **r** 1
4 **a** $\frac{3}{4}$ **b** $\frac{5}{8}$ **c** $\frac{1}{2}$ **d** $\frac{6}{7}$ **e** $1\frac{2}{3}$ **f** $1\frac{1}{2}$ **g** $1\frac{4}{5}$ **h** $\frac{2}{3}$ **i** $2\frac{1}{3}$ **j** $2\frac{2}{3}$
k $\frac{1}{3}$ **l** 0 **m** $-\frac{3}{4}$ **n** $-\frac{3}{5}$ **o** $-2\frac{1}{2}$ **p** $2\frac{1}{3}$
5 **a** 5 **b** 15 **c** 8 **d** 12 **e** 12 **f** −1 **g** −2 **h** 2 **i** −2 **j** $1\frac{2}{3}$
k $\frac{2}{3}$ **l** −3 **m** −3 **n** −3 **o** 200 **p** $\frac{-3}{2}$

Prep Quiz 7:04

1 x **2** $10a$ **3** 0 **4** 0 **5** -9 **6** $+2$ **7** $\times 10$ **8** 14 **9** 4 **10** 7

Exercise 7:04

1 **a** 7 **b** 7 **c** 1 **d** 10 **e** 8 **f** 6 **g** 6 **h** 1 **i** 2 **j** 12
k 6 **l** -4 **m** 4 **n** 5 **o** 1 **p** 2 **q** 2 **r** 2 **s** 9 **t** 2
u no solution **v** 1 **w** 2 **x** 5

2 **a** correct **b** incorrect **c** correct **d** correct **e** correct
f incorrect **g** correct **h** correct **i** correct **j** correct
k correct **l** correct

3 **a** -2 **b** $1\frac{1}{2}$ **c** $1\frac{1}{3}$ **d** $\frac{1}{4}$ **e** $-\frac{1}{3}$ **f** 0 **g** $1\frac{1}{6}$ **h** -5 **i** $4\frac{1}{2}$ **j** $2\frac{1}{2}$
k $\frac{5}{6}$ **l** -1 **m** -4 **n** $2\frac{1}{6}$ **o** $2\frac{2}{3}$ **p** $\frac{3}{4}$ **q** $1\frac{4}{7}$ **r** $-\frac{4}{3}$ **s** -20 **t** $\frac{2}{3}$ **u** $4\frac{1}{3}$

Prep Quiz 7:05

1 $7x + 28$ **2** $2a - 6$ **3** $20a + 45$ **4** $12p - 42$ **5** $-3x + 12$ **6** -5 **7** 6 **8** -9 **9** $\frac{1}{2}$ **10** -7

Exercise 7:05

1 **a** 4 **b** 1 **c** 3 **d** 5 **e** 6 **f** 13 **g** 2 **h** 5 **i** -1 **j** 4
k 2 **l** 3 **m** 3 **n** 8 **o** 6 **p** 8 **q** 1 **r** 4

2 **a** $\frac{3}{5}$ **b** $1\frac{1}{2}$ **c** $7\frac{3}{4}$ **d** -1 **e** 4 **f** 4 **g** $\frac{1}{2}$ **h** $1\frac{7}{10}$ **i** $1\frac{1}{3}$ **j** 2
k -2 **l** 25 **m** $\frac{5}{7}$ **n** $5\frac{1}{2}$ **o** 3 **p** $1\frac{1}{5}$ **q** 3 **r** -12

3 **a** -4 **b** 13 **c** 18 **d** 4 **e** 4 **f** 6 **g** 2 **h** 2 **i** 7 **j** $4\frac{1}{2}$
k 3 **l** $2\frac{2}{7}$ **m** 2 **n** -5

4 **a** 1 **b** 1 **c** 1 **d** 2 **e** 1 **f** 2 **g** 2 **h** 2 **i** 2 **j** -46

5 **a** 2 **b** 3 **c** 2 **d** 2 **e** 1 **f** 2 **g** $-1\frac{3}{8}$ **h** $3\frac{1}{3}$

Prep Quiz 7:06

1 0 **2** 3 **3** -3 **4** $3\frac{3}{4}$ **5** $-3\frac{1}{5}$ **6** p **7** 12 **8** $x + 7$ **9** $5m$ **10** $12 - p$

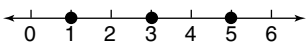
Exercise 7:06

1 **a** 24 **b** 20 **c** 36 **d** 15 **e** 12 **f** 28 **g** 50 **h** 18 **i** $1\frac{3}{5}$ **j** 8
k 9 **l** $11\frac{2}{3}$ **m** 29 **n** 6 **o** 1 **p** 3 **q** 2 **r** 6 **s** $2\frac{3}{4}$ **t** $1\frac{1}{3}$
u -2 **v** -22 **w** $17\frac{2}{3}$

2 **a** correct **b** incorrect **c** correct **d** correct **e** incorrect **f** incorrect **g** correct
h correct **i** correct **j** incorrect **k** correct **l** incorrect

3 **a** -12 **b** $\frac{15}{9}$ **c** 10 **d** $\frac{8}{3}$ **e** $\frac{12}{5}$ **f** 10 **g** 3 **h** $-\frac{1}{7}$ **i** $\frac{3}{10}$ **j** $\frac{1}{9}$
k -1 **l** $\frac{9}{7}$

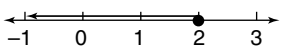
Prep Quiz 7:07

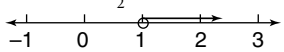
1 -6 **2** 12 **3** -2 **4** 2 **5** false **6** false **7** 'greater than' **8** 

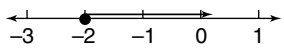
9 $-2 > -4$ **10** $-4 < 6$

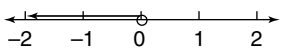
Exercise 7:07

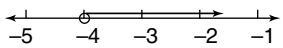
1 **a** $x \leqslant 0$ **b** $x \geqslant -3$ **c** $x \leqslant -2$ **d** $x > -2$ **e** $x > 3$ **f** $x < 1$ **g** $x > 5$ **h** $x = -1$
i $x < -1$ **j** $x \leqslant 0$ **k** $x \geqslant \frac{1}{2}$ **l** $x < 4\frac{1}{2}$

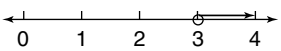
2 **a** 

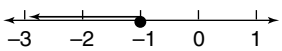
b 

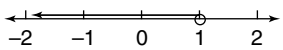
c 

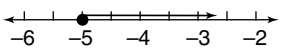
d 

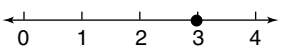
e 

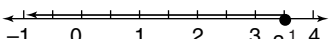
f 

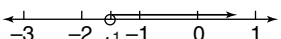
g 

h 

i 

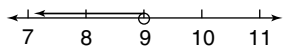
j 

k 

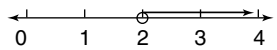
l 

Exercise 7:08

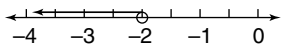
1 **a** $x < 9$



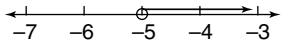
b $y > 2$



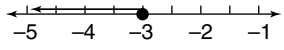
c $m < -2$



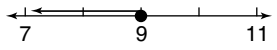
d $m > -5$



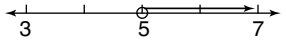
e $p \leqslant -3$



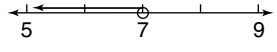
f $m \leqslant 9$



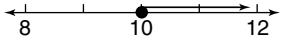
g $y > 5$



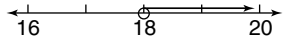
h $m < 7$



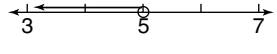
i $x \geqslant 10$



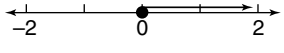
j $m > 18$



k $p < 5$



l $x \geqslant 0$



2 **a** $m > 7$ **b** $p \leqslant 7$ **c** $m < 2$ **d** $y \geqslant -8\frac{2}{5}$ **e** $x < 3\frac{3}{4}$ **f** $x > -2$ **g** $x < 15$ **h** $y > 12$
i $m \geqslant 4$ **j** $x < -50$ **k** $x < 6$ **l** $x > -12$

3 **a** $m > -8$ **b** $x < -4$ **c** $p \geqslant -4$ **d** $x > 2$ **e** $x < -2\frac{1}{2}$ **f** $x \geqslant -3\frac{1}{3}$ **g** $x < -4$ **h** $x > -3$
i $x \geqslant -5$ **j** $x < -6$ **k** $x < -5$ **l** $x \leqslant -8$ **m** $x > -2$ **n** $x < -6$ **o** $x < -12$

4 **a** $x > 3$ **b** $m < 4$ **c** $p \geqslant 3$ **d** $p \leqslant -1$ **e** $p \geqslant 1\frac{1}{5}$ **f** $x < -3\frac{1}{2}$ **g** $x \leqslant 4$ **h** $p \geqslant 2$
i $y < 1\frac{1}{4}$ **j** $x < 3$ **k** $x > 2$ **l** $x \leqslant 1\frac{1}{2}$ **m** $x < 4$ **n** $m > 3$ **o** $x \leqslant 3\frac{1}{2}$ **p** $x \geqslant 4\frac{1}{2}$
q $m > 2\frac{1}{2}$ **r** $p > 3$

5 **a** $m < -2$ **b** $y > -2$ **c** $x \leqslant -1\frac{1}{2}$ **d** $x \geqslant -2$ **e** $m \leqslant 6$ **f** $m \geqslant 8$ **g** $p > \frac{1}{2}$ **h** $x > -2$
i $y < -1$ **j** $x \leqslant -1$ **k** $y \geqslant 1$ **l** $p < -\frac{1}{2}$ **m** $x \geqslant -\frac{1}{6}$ **n** $a \geqslant 3\frac{3}{4}$ **o** $x < \frac{1}{6}$

6 **a** $x > 3$ **b** $x < 5\frac{1}{2}$ **c** $a \leqslant 2$ **d** $b \leqslant -3$ **e** $x < 10$ **f** $p > 15$ **g** $y \geqslant -1\frac{1}{2}$ **h** $p < 9$
i $x \leqslant 1$

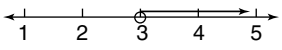
Fun Spot 7:08

a Steve **b** Alan

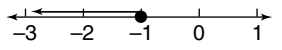
Diagnostic Test 7 Equations and inequalities

1 **a** $x = 6$ **b** $y = 9$ **c** $p = 14$ **d** $m = 10$
2 **a** $x = 6$ **b** $x = 8$ **c** $y = 27$ **d** $y = 20$
3 **a** $p = 18$ **b** $x = 20$ **c** $x = 6$ **d** $t = 12$
4 **a** incorrect **b** incorrect **c** incorrect **d** correct
5 **a** $p = 7$ **b** $m = 12$ **c** $p = 8\frac{4}{9}$ **d** $m = -10$
6 **a** $x = -4$ **b** $a = 3$ **c** $b = -5$
7 **a** $x = 2$ **b** $a = 9$ **c** $x = 2{\cdot}6$
8 **a** $y = 12$ **b** $m = 12$ **c** $p = 18$ **d** $m = 10$
9 **a** $m = 5$ **b** $m = 9$ **c** $p = 7$ **d** $x = 6\frac{2}{3}$
10 **a** $m = \frac{5}{2}$ **b** $x = -2$ **c** $n = 1$ **d** $a = \frac{11}{8}$

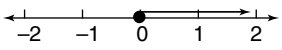
11 **a**



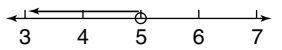
b



c



d

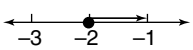


12 **a** $x \geqslant \frac{1}{2}$ **b** $x < 9\frac{1}{2}$ **c** $x > 5$
13 **a** $x < -2$ **b** $x < -63$ **c** $a < \frac{1}{3}$

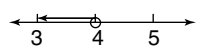
7A Revision Assignment

1 **a** $x = \frac{13}{4}$ **b** $a = 12$ **c** $m = 44$ **d** $n = 88$
2 **a** $m = 3$ **b** $y = -1$ **c** $m = 3$ **d** $n = -2$ **e** $p = -\frac{1}{3}$ **f** $k = \frac{17}{2}$ **g** $y = \frac{9}{5}$ **h** $q = \frac{1}{3}$
3 **a** $x = 4$ **b** $x = 4$ **c** $x = -3$ **d** $x = \frac{9}{2}$ **e** $x = \frac{11}{2}$ **f** $x = \frac{-8}{5}$ **g** $x = -11$ **h** $x = \frac{1}{5}$
i $x = 1$
4 **a** $x = -1$ **b** $a = 6$ **c** $m = 1\frac{1}{2}$ **d** $x = -6$ **e** $n = 11$ **f** $x = 0$ **g** $a = \frac{2}{11}$ **h** $n = \frac{-23}{11}$
i $m = -3$ **j** $x = -9$
5 **a** $x = 8$ **b** $m = 15$ **c** $x = 9$ **d** $a = 17$ **e** $y = 8$ **f** $p = 3$ **g** $m = -3$ **h** $m = -\frac{1}{3}$
i $n = \frac{-16}{5}$

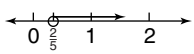
6 **a** $m \geqslant -2$,



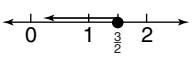
b $x < 4$,



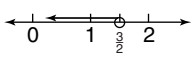
c $n > \frac{2}{5}$,



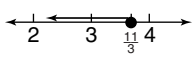
d $x \leqslant \frac{3}{2}$,



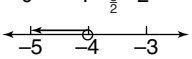
e $y < \frac{3}{2}$,



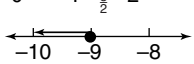
f $n \leqslant \frac{11}{3}$,



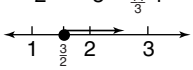
g $x < -4$,



h $x \leqslant -9$,



i $a \geqslant \frac{3}{2}$,



7B Working Mathematically

1 **a** reflection **b** translation **c** rotation **d** tessellation **e** picture **f** column **g** line **h** pie **i** bar **j** scatter diagram

2 **a** (1) $15x^2$ (2) $15x^3$ (3) $-80x^2$ (4) $60x^5$ (5) $14x^4$
(6) $9x^3$ (7) $-30x^3$ (8) $-48x^4$ (9) $-15x^6$ (10) $21x^6$
(11) $4x^4$ (12) $-7x^{11}$

b (1) $6x + 3$ (2) $-4x - 2$ (3) $30x + 10$ (4) $-6x - 6$ (5) $-24x - 64$
(6) $30x + 10$ (7) $-6x - 48$ (8) $60x + 24$ (9) $14x + 28$ (10) $-25x - 20$
(11) $-22x - 77$ (12) $8x + 12$

c (1) $25m^4$ (2) $4m^6$ (3) $64m^{10}$ (4) $36m^4$ (5) m^4
(6) $81m^6$ (7) $4m^{12}$ (8) $9m^6$ (9) m^{14} (10) $64m^8$
(11) $16m^2$ (12) $49m^4$

d (1) $-15x + 40x$ (2) $3x^2 - 3x$ (3) $10x^2 - 50x$ (4) $-30x^2 - 60x$ (5) $21x^2 + 7x$
(6) $9x^2 + 63x$ (7) $-12x + 30x$ (8) $-12x^2 + 120x$ (9) $10x^2 - 20x$ (10) $-3x^2 - 21x$
(11) $-4x^2 + 24x$ (12) $-14x^2 - 21x$

3 **a** 10 minutes **b** 55 minutes **4** $\frac{9}{10}$

5 **a** **i** Argentina **ii** Bolivia **iii** Chile **iv** Brazil **b** Bolivia, Chile, Argentina **c** 56% **d** 36% **e** no

6 **a** $2\frac{1}{2}$ minutes **b** 46° **c** 27° **d** 73°

Chapter 8 Coordinate Geometry: Graphing Straight Lines

Prep Quiz 8:01

1 x-axis **2** y-axis **3** origin (0, 0) **4** (3, 5) **5** (3, −1) **6** (−4, −2) **7** (0, 6) **8** (−3, 0) **9** 7 units **10** 3 units

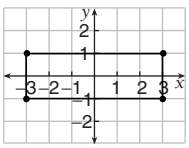
Exercise 8:01

1 **a** V **b** B **c** J **d** T **e** F **f** X **g** O **h** G **i** D

2 **a** (1, 2) **b** (−2, 4) **c** (−3, −3) **d** (3, −2) **e** (2, 0) **f** (0, 3) **g** (−3, 0) **h** (0, −1)
i (2, −3) **j** (3, 4) **k** (−4, 4) **l** (−2, −4)

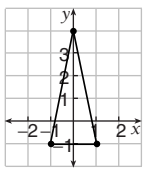
3 **a** R, U, A, V **b** C, S, G, R

4 **a**



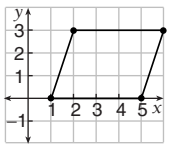
Rectangle

b



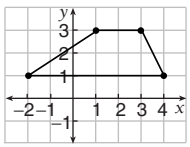
Isosceles triangle

c



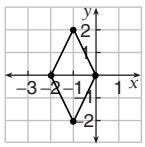
Parallelogram

d



Trapezium

e



Rhombus

5 **a** 3 **b** 3 **c** 2 **d** 4 **e** 4 **f** 2 **g** 4 **h** 4 **i** 7 **j** 3
k 3 **l** 6 **m** 3 **n** 4 **o** 8 **p** 3 **q** 4 **r** 2

6 **a** 1st **b** 2nd **c** 3rd **d** 4th **e** 3rd **f** 1st **g** 3rd **h** 2nd **i** 4th **j** 2nd
k 3rd **l** 4th **m** 1st **n** 4th **o** 2nd **p** 1st **q** 3rd **r** 1st

7 **a** **i** (1·0, 0·8) **ii** (−2·5, 2·8) **iii** (3·1, 2·4) **iv** (−2·3, −1·8)

b **i** F **ii** I **iii** E **iv** H

8 **a** **i** H **ii** B **iii** E **iv** G

b **i** (1·4, 0·8) **ii** (−1·4, 1·6) **iii** (1·4, −2·6) **iv** (−0·8, −3·4)

9 **a** $A(-1, 4)$; $B(1, 2)$; $C(2, 1)$; $D(3, 0)$ **b** $A(-1, -3)$; $B(0, -2)$; $C(1, -1)$; $D(3, 1)$
c $A(-3, 4)$; $B(-2, 2)$; $C(0, -2)$ $D(1, -4)$

10 **a** Line A

x	−3	−2	0	1
y	4	2	−2	−4

Line B

x	−4	−2	0	2
y	−1	0	1	2

Line C

x	−2	−1	2	3
y	−4	−3	0	1

Line D

x	−1	0	2	4
y	5	4	2	0

b $a = 3$, $b = -6$

11 **a** (1, 1), $(1\frac{1}{2}, 1\frac{1}{2})$, (2, 2) are three of an infinite number of answers.

b $(\frac{3}{4}, -1\frac{1}{2})$, (1, −2), $(1\frac{1}{4}, -2\frac{1}{2})$ **c** $a = -4$, $b = 4$, $c = 5$

12 **a** A **b** C **c** D **d** B **e** E **f** F

Prep Quiz 8:02

1 1 **2** 3 **3** −3 **4** 2 **5** 5 **6** −2 **7** −5 **8** C **9** A **10** B

Exercise 8:02

1 a $y = 3x$

x	0	1	2
y	0	3	6

b $y = x + 1$

x	0	1	2
y	1	2	3

c $y = 5 - x$

x	0	1	2
y	5	4	3

d $y = 6x - 7$

y	2	3	4
y	5	11	17

e $y = 18 - 5x$

x	2	3	4
y	8	3	−2

f $y = 7x + 2$

x	2	3	4
y	16	23	30

g $y = x - 7$

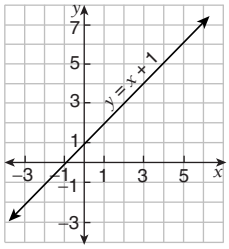
x	0	1	2
y	−7	−6	−5

h $y = 1 - x$

x	0	1	2
y	1	0	−1

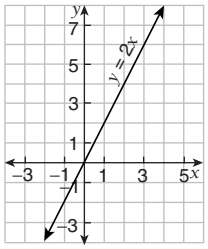
2 a $y = x + 1$

x	0	1	2
y	1	2	3



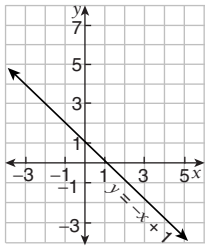
b $y = 2x$

x	0	1	2
y	0	2	4



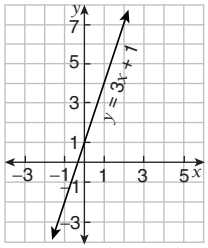
c $y = -x + 1$

x	0	1	2
y	1	0	−1



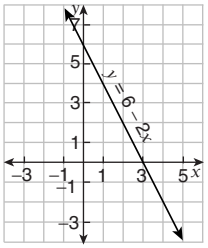
d $y = 3x + 1$

x	0	1	2
y	1	4	7



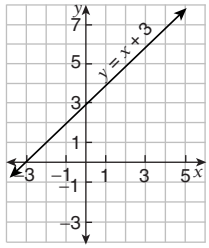
e $y = 6 - 2x$

x	0	1	2
y	6	4	2

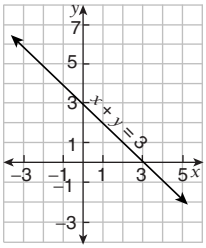


f $y = x + 3$

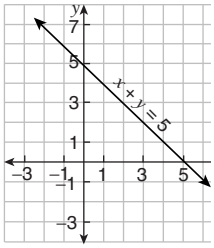
x	0	1	2
y	3	4	5



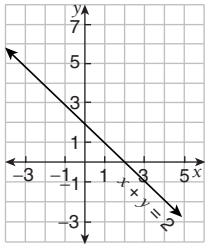
3 a



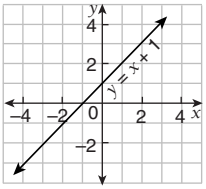
b



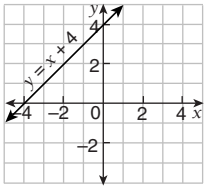
c



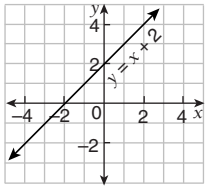
d



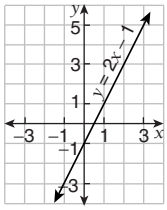
e



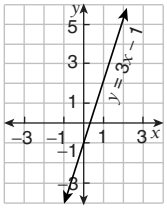
f



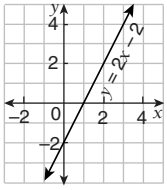
g



h



i



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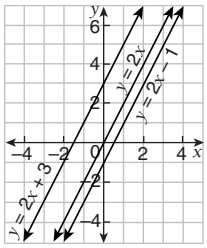
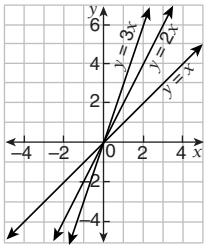


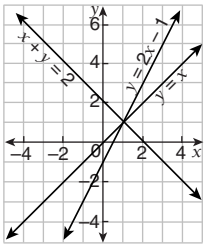
The lines are parallel.

5



The lines all pass through the origin.

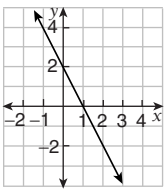
6



The lines are concurrent at (1, 1).

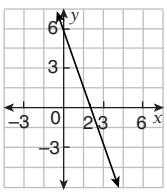
7 a, b, d **8** **a** yes **b** yes **c** (7, −2) **d** (5, 0)

9 **a**



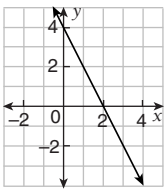
x-intercept is 1
y-intercept is 2

b



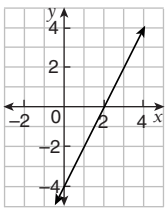
x-intercept is 2
y-intercept is 6

c



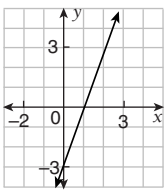
x-intercept is 2
y-intercept is 4

d



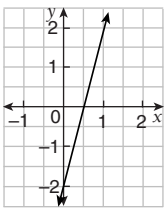
x-intercept is 2
y-intercept is −4

e



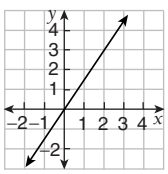
x-intercept is 1
y-intercept is −3

f



x-intercept is $\frac{1}{2}$
y-intercept is −2

10 **a**



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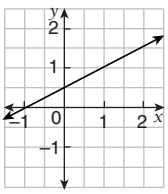

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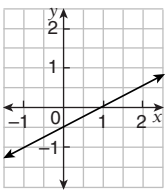

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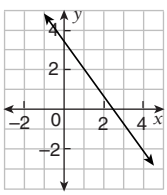

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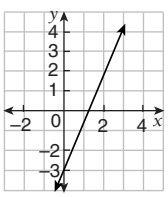
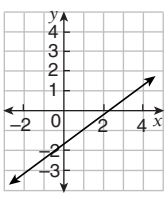


f



Investigation 8:02

1 **a** D **b** B **c** E **2** **a** A **b** B **c** C
3 **a** (4, 0) **b** (0, 0) **c** (−2, 0); the *y*-coordinate is zero for each point.
4 (0, 2), (0, 0), (0, −4); the *x*-coordinate is zero for each point. **5** The *y*-coordinate is always zero.
6 The *x*-coordinate is always zero. **7** Substitute $y = 0$ into the equation and solve for x.
8 Substitute $x = 0$ into the equation and solve for y.

9 Lines in question 3

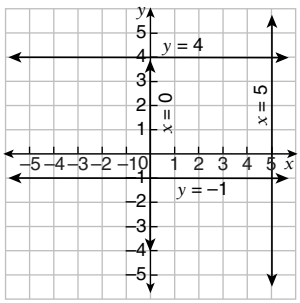
	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
x-intercept	3	5	2	−1	−4	−2	$\frac{1}{2}$	$\frac{1}{3}$	1
y-intercept	3	5	2	1	4	2	−1	−1	−2

Lines in question 10

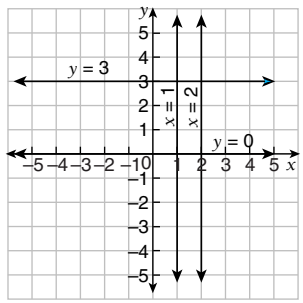
	a	*b*	*c*	*d*	*e*	*f*
x-intercept	0	−1	1	$2\frac{1}{3}$	$1\frac{1}{5}$	$2\frac{1}{2}$
y-intercept	0	$\frac{1}{2}$	$-\frac{1}{2}$	$3\frac{1}{2}$	−3	$-1\frac{2}{3}$

Exercise 8:03

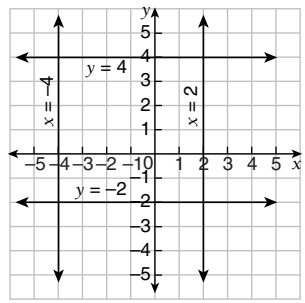
1 **a** $x = 2$ **b** $y = -2$
2 **a** *A:* $x = -4$, *B:* $x = -1$, *C:* $x = 3$, *D:* $y = 3$, *E:* $y = 1$, *F:* $y = -4$
b *A:* $x = -3$, *B:* $x = 2$, *C:* $x = 4$, *D:* $y = 4$, *E:* $y = 2$, *F:* $y = -2$
3 **a** $x = 0$ **b** $y = 0$
4 **a**



b



c



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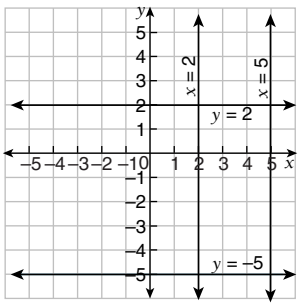
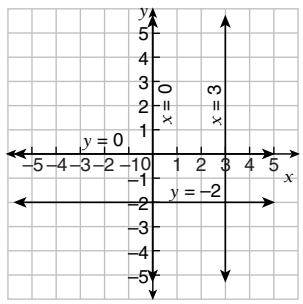


e



The lines in **a** and **c** enclose square regions.

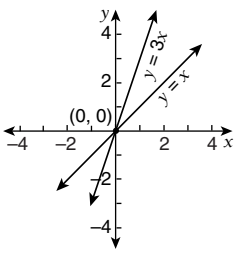
5 **a** (3, 1) **b** (−4, 3) **c** (−1, −4) **d** (2, −2) **e** (−3, 2) **f** (4, 4) **g** (0, 3) **h** (2, 0) **i** (0, 0)

Prep Quiz 8:04

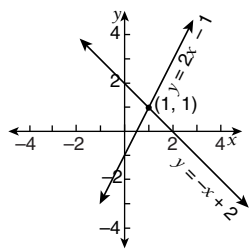
1 (0, 0) **2** $y = 0$ **3** $x = 0$ **4** (0, 0) **5** {5} **6** yes **7** yes **8** yes **9** yes **10** (1, 1)

Exercise 8:04

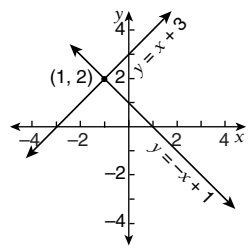
1 **a** (1, 2) **b** (1, −2) **c** (−1, −1) **d** (0, 2)
2 **a** (1, 1) **b** (2, 0) **c** (0, −1) **d** (−2, 1)
3 **a** $(\frac{1}{2}, 1\frac{1}{2})$ **b** (−0·7, 0·3) **c** (−0·2, −0·8) **d** (1, −0·5)
4 **a**



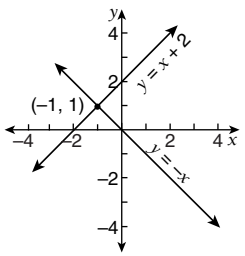
b



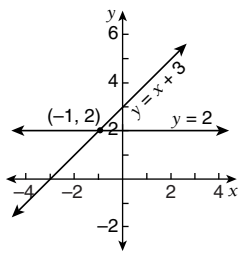
c



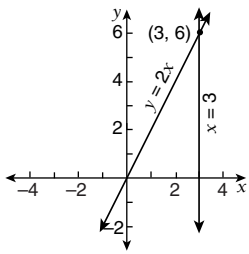
d



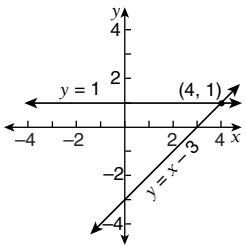
e



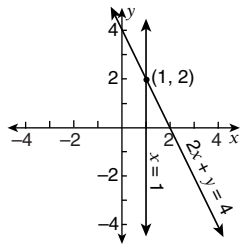
f



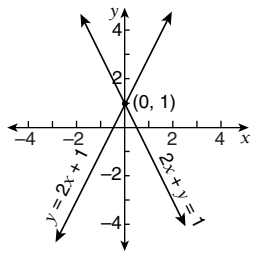
g



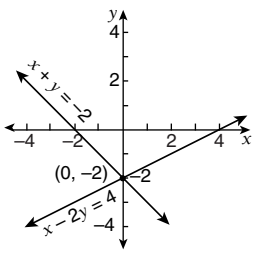
h



i



j



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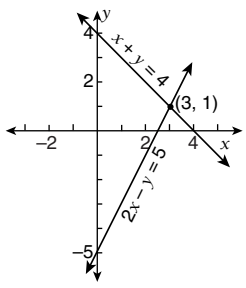
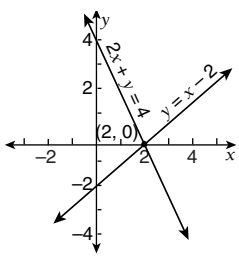


l



5 **a** $(-1, -1)$ **b** $(2, 4)$ **c** $(0, 2)$ **d** $(4, 0)$ **e** $(-1, -4)$
f Substituting $(1, 3)$ into both equations shows that $(1, 3)$ lies on both lines and hence is the point of intersection.
g $(-2, -3)$

6 **a** **i** $(2, 5)$ **ii** $(0, 3)$ **iii** $(4, 2)$ **(iv)** $(6, 4)$ **b** Parallelogram **c** No. They are parallel.

Prep Quiz 8:05

1 *EF* **2, 3** and **4** *A* and *B*; *C* and *D*; *E* and *F* **5, 6** and **7** *B* and *C*; *D* and *E*; *F* and *G* **8** up **9** down **10** not at all

Exercise 8:05

1 **a** positive **b** positive **c** negative **d** negative **e** positive
f negative **f** positive

2 **a** 1 **b** 5 **c** $\frac{1}{5}$ **d** $-\frac{1}{2}$ **e** -2 **f** -5

3 **a** $\frac{1}{2}$ **b** $\frac{3}{2}$ **c** $\frac{1}{2}$ **d** $-\frac{1}{2}$ **e** -2 **f** $-\frac{2}{3}$

4 **a** $-\frac{1}{2}$ **b** $-\frac{4}{3}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$ **e** $-\frac{1}{3}$ **f** 1

5 **a** 2 **b** 1 **c** $\frac{1}{2}$ **d** -1 **e** $-\frac{3}{2}$ **f** -3

6 **a** $\frac{2}{5}$ **b** $\frac{2}{5}$ **c** Yes

Investigation 8:05

1 **a** *C* **b** *B* **c** *A* **2** **a** 8 m **b** 5·6 m **c** 12·8 m

3 **a** $1\frac{1}{2}$ m **b** $\frac{1}{2}$ m **4** No, he would need a run of 11·2 m. He could curve the rampway across the slope.

Prep Quiz 8:06

1 0 **2** *b* **3** 2 **4** -1 **5** 2 **6** $\frac{1}{2}$ **7** $(0, 2)$ **8** $(0, -1)$ **9** Yes **10** Yes

Investigation 8:06

1

Line	*Equation*	*Gradient*	*y-intercept*
A	$y = x + 2$	1	2
B	$y = x - 2$	1	-2
C	$y = \frac{1}{2}x + 1$	$\frac{1}{2}$	1
D	$y = \frac{1}{2}x - 1$	$\frac{1}{2}$	-1
E	$y = 2x + 2$	2	2
F	$y = 2x - 3$	2	-3

2

Line	*Equation*	*Coefficient of x*	*Constant*
A	$y = x + 2$	1	2
B	$y = x - 2$	1	-2
C	$y = \frac{1}{2}x + 1$	$\frac{1}{2}$	1
D	$y = \frac{1}{2}x - 1$	$\frac{1}{2}$	-1
E	$y = 2x + 2$	2	2
F	$y = 2x - 3$	2	-3

3 The gradient of the line is the same as the coefficient of x. The y-intercept of the line is the same as the constant.

4 $y = -2x + 3$

5 It tells us that m is the gradient and c is the y-intercept.

Exercise 8:06

1 The gradient is stated first.

a 2; 3 **b** 5; 1 **c** 3; 2 **d** 1; 6 **e** 4; 0 **f** 1; 0 **g** 1; -2 **h** 5; -1 **i** 6; -4 **j** -2; 3
k -1; -2 **l** -3; 1 **m** $\frac{1}{2}$; 4 **n** $\frac{3}{4}$; -2 **o** $\frac{1}{3}$; 5 **p** -3; 4 **q** 4; -2 **r** $-\frac{1}{2}$; 3

2 **a** $y = 4x + 9$ **b** $y = -2x + 3$ **c** $y = 7x - 1$ **d** $y = -5x - 2$ **e** $y = \frac{1}{2}x + 5$
f $y = \frac{2}{3}x - 4$ **g** $y = 3x + 1$ **h** $y = 2x - 3$ **i** $y = 5x + \frac{1}{2}$ **j** $y = x + 1\frac{1}{2}$

3 **a** 2; 1; $y = 2x + 1$ **b** $\frac{1}{4}$; 3; $y = \frac{x}{4} + 3$ **c** $\frac{1}{2}$; 2; $y = \frac{1}{2}x + 2$ **d** $\frac{1}{3}$; 4; $y = \frac{1}{3}x + 4$ **e** 3; -1; $y = 3x - 1$
f -1; -2; $y = -x - 2$ **g** $-\frac{1}{6}$; $-2\frac{1}{2}$; $y = -\frac{x}{6} - 2\frac{1}{2}$ **h** $\frac{3}{2}$; 0; $y = \frac{3x}{2}$

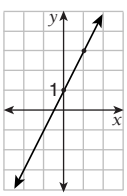
4 **a** *E* **b** *B* **c** *D* **d** *G* **e** *F* **f** A

5 (In each part the gradient is stated first.) **a** 2; 5 **b** 3; -3 **c** 2; 4 **d** 2; -1 **e** 5; 8 **f** 6; -12
g 3; -2 **h** -2; -4 **i** 2; 7 **j** -1; 4 **k** $\frac{1}{2}$; $1\frac{1}{2}$ **l** -3; 5 **m** -3; 2 **n** $-\frac{3}{2}$; 3 **o** $\frac{2}{5}$; $\frac{1}{5}$

6 **a** yes **b** no **c** yes **d** yes

7 **a** $2x - y + 6 = 0$; $y = 2x - 3$ **b** $x + y = 3$ and $y = -x + 8$

8 **a**



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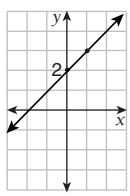

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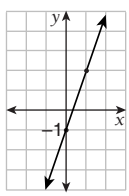

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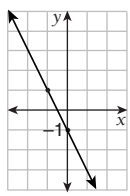

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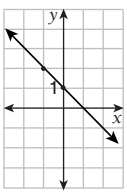

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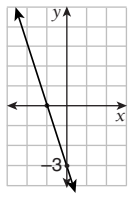

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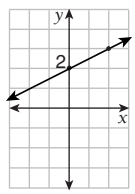

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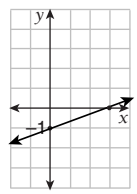
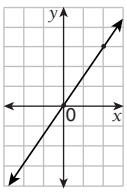


i



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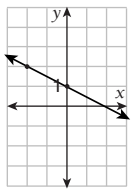

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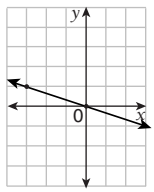

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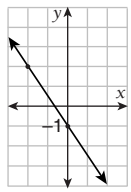
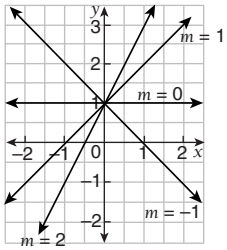


9 a



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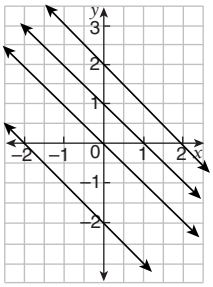


10 a -2
b 2
c $-\frac{1}{2}$
d $\frac{4}{3}$
e 4
f $-\frac{1}{3}$

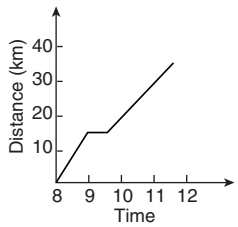
Prep Quiz 8:07

1 200 km **2** 450 km **3** 25 km **4** 4 h **5** 10 h **6** 70 km/h **7** 500 km/h **8** *A* **9** *B* **10** *B*

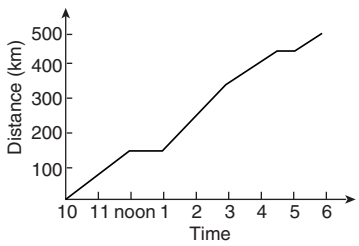
Exercise 8:07

1 a 30 km **b** 1:30 pm **c** 50 km **d** 20 km **e** Yes at 11 am
2 a 8 am **b** 2 pm **c** $1\frac{1}{2}$ h **d** 30 km **e** 180 km
3 a 5 km **b** 11:30 am **c** 60 km **d** 11–11:30 am **e** 5 km/h
4 a 20 km **b** 20 km **c** 1 pm
d Increases. The slope of the line becomes steeper. **e** 40 km
5 a 5 km **b** 20 km **c** 35 km **d** increases **e** 4–5 pm
6 a Bartley **b** Twice **c** 1 pm **d** 150 km
e Bartley travels 200 km, Conway travels 350 km.
7 a Brown starts from *A* at 3 pm. Jones starts 20 km from *A* at 3 pm.
b 4:30 pm; 7:30 pm **c** 20 km **d** 6:30 pm **e** Brown travels 160 km. Jones travels 100 km.
8 a 75 km **b** 10:45 am and 12:15 pm **c** noon **d** 75 km **e** Doyle; $\frac{1}{2}$ h
9 a Jackson **b** 75 km/h **c** increase **d** 10–11 am **e** 125 km
10 a 11:30 am **b** 35 km **11 a** 5:45 pm **b** 510 km **c** noon **d** 4:30 pm

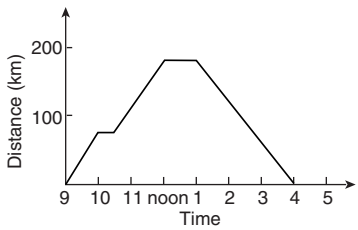
c



e

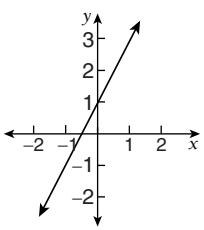


12

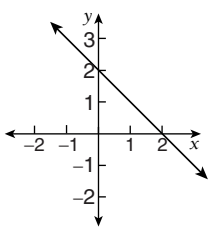


Diagnostic Test 8

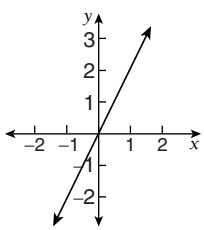
1 a J **b** P **c** Q **d** L **2 a** $(-3, 3)$ **b** $(-3, -2)$ **c** $(3, -2)$ **d** $(0, 3)$
3 a 5 **b** 5 **c** 7 **d** 8 **4 a** F **b** D **c** G **d** J
5 a $(-1{\cdot}6, -0{\cdot}8)$ **b** $(2{\cdot}6, -1{\cdot}8)$ **c** $(0{\cdot}8, -2{\cdot}4)$ **d** $(-0{\cdot}8, 2{\cdot}4)$ **6 a** *B* **b** *A* **c** *C*
7 a



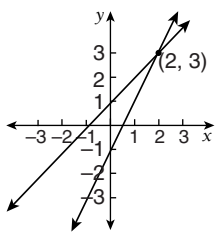
b



c

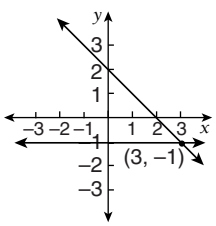


8 a Yes **b** No **c** Yes **9 a** 3 **b** 4 **c** 4
10 a 4 **b** -1 **c** -5
11 a $x = 2$ **b** $x = -3$ **c** $y = 1$ **d** $y = -2$
12 a



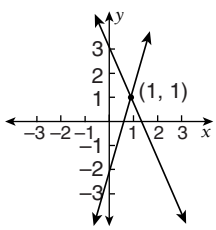
Point of intersection is (2, 3).

b



Point of intersection is (3, −1).

c



Point of intersection is (1, 1).

13 **a** 1 **b** -2 **c** $\frac{1}{2}$

14 **a** $y = 3x + 2$ **b** $y = \frac{1}{2}x - 3$ **c** $y = -x + 3$ **d** $y = \frac{1}{3}x - \frac{1}{2}$

15 **a** $m = 2, b = 3$ **b** $m = \frac{1}{2}, b = -3$ **c** $m = -2, b = 3$ **d** $m = -1, b = 4$

16 **a** $y = x + 1$ **b** $y = -2x + 2$ **c** $y = \frac{1}{2}x - 1$

8A Revision Assignment

1 **a** (1, 0) **b** (0, 4) **c** $(-2, 2)$ **2** **a** *F* **b** *H* **c** *N*

3 **a** *AB* **b** *EF* **c** *EF* **4** **a** (2, 2) **b** $(-0{\cdot}7, 4{\cdot}7)$

5 **a** **i** -3 **ii** 4 **b** **i** -2 **ii** 4 **6** **a** **i** (3, 1) **ii** (3, 4) **b** **i** $(5, -1)$ **ii** $(-3\frac{1}{2}, -1)$

7 **a** (1, 3) **b** $(-4, -2)$ **c** (4, 6) **8** **a** $y = 2x + 6$ **b** $x + y = 4$

9 Intersects *AB* at $(-1{\cdot}3, 3{\cdot}3)$; Does not intersect *EF*. **10** $(0{\cdot}7, -0{\cdot}7)$

8B Working Mathematically

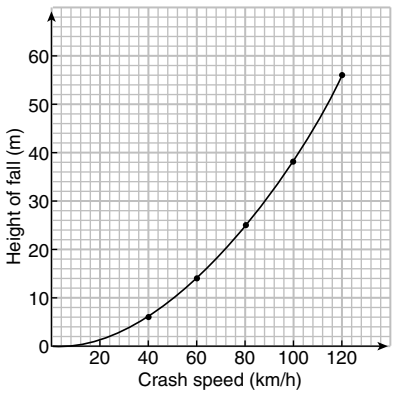
1 **a** *parallelogram* **b** *rhombus* **c** *trapezium* **d** *kite*
e *scalene triangle* **f** *isosceles triangle* **g** *equilateral triangle* **h** *triangular prism*
i *triangular pyramid* **j** *hemisphere*

2 **a** *diagonals* **b** *acute-angled triangle* **c** *right-angled triangle* **d** *obtuse-angled triangle*
e *vertices* **f** ΔABC **g** *hypotenuse* **h** *180°*
i *[b]* $\alpha = \beta$ **j** *AB* is a *tangent*, *CD* is an *arc*, *EF* is a *chord*.

3 **a** 5 **b** 42 **c** 45

4 **a** Four times the damage

b



c **i** 6 m **ii** 25 m **iii** 56 m
iv 46 m **v** 2 m

d **i** 114 km/h **ii** 88 km/h **iii** 50 km/h
iv 104 km/h

5 **a** 6

b 168 (120 five-digit numbers and 48 four-digit numbers)

6 **a**

1	2	3	4
3	4	1	2
4	3	2	1
2	1	4	3

b

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

Another solution is possible.

Chapter 9: Formulae and Problem Solving

Prep Quiz 9:01

1 $7 + 5 = 12$ **2** $9 - 2 = 7$ **3** $25 - 8 = 17$ **4** $48 \div 6 = 8$ **5** $7 \times 3 = 21$

6 $8 + 12 = 20$ **7** $(41 + 47) \div 2 = 44$ **8** $13 + 21 = 34$ **9** $138 \div 23 = 6$ **10** $4 \times 5 - 8 = 12$

Exercise 9:01

1 **a** C **b** F **c** A **d** D **e** B **f** E

2 **a** $2n$ **b** $n + 3$ **c** $8 - n$ **d** $2n + 3$ **e** $\frac{n}{2}$ or $\frac{1}{2}n$ **f** $2n - 5$

3 **a** $5 + 7$ **b** $5 + y$ **c** $x + y$ **4** **a** 3×7 **b** $7a$ **c** ab

5 **a** $8 - 3$ **b** $8 - p$ **c** $q - p$ **6** **a** $\frac{8 + 12}{2} = 10$ **b** $\frac{8 + x}{2}$ **c** $\frac{w + x}{2}$

7 **a** 5×75 cents **b** $75a$ cents **c** ab cents **8** **a** $\frac{30}{5}$ cm **b** $\frac{30}{t}$ cm **c** $\frac{A}{t}$ cm

9 **a** $15 + 6$ **b** $15 + y$ **10** **a** (60×3) km **b** $60h$ km

11 **a** $(10 - 3 \times 2)$ m **b** $(X - 2x)$ m **12** **a** $(100 - x)$ cents

13 **a** 8 **b** $y + 2$ **c** $y - 1$ **14** **a** hm km **b** $\frac{k}{h}$ km/h **c** $\frac{k}{m}$ hours

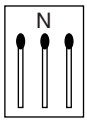
15 **a** $\$(R - P)$ **b** $\$(P + G)$

16 **a** $(a + b + 4)$ cm **b** $(2x + 2y)$ cm **c** $4s$ cm **d** $(4x + 10)$ cm

17 **a** x^2 cm^2 **b** $x(x + 2)$ cm^2 **c** $\frac{xy}{2}$ cm^2 **d** $\frac{bh}{2}$ cm^2

18 **a** $N+3$ **b** $3N+1$ **c** $2(N+1)$ **d** $(3N+2)+4$ **e** $3(N+3)+1$ **f** $2(N+4)+6$

You could possibly use 'red' to indicate a negative number of matches, like:
where the three matches are drawn in red.



$= N - 3$,

19 **a** Add four to the product of three and m. **b** Subtract five from the product of two and y.
c Three subtract the product of four and x. **d** Add one to half of a. **e** Half of the sum of a and one.
f Subtract half of a from four. **g** Twice the sum of m, and one.
h Subtract a from three and multiply the result by four. **i** Multiply the sum of a and b by nine.
j Multiply the sum of a and three by five and then add four.
k Take away one from x, multiply the result by two and then subtract three.
l Subtract the product of three and the sum of a and five, from ten.

Fun Spot 9:01

1 E, the first letters of the words One, Two, Three, etc.
2 The beggar was the man's daughter.
3 Ask either guard "Which sack will the other guard tell me has the gold?"

Prep Quiz 9:02

1 $2L+2B$ **2** LB **3** πr^2 **4** $2r+2w$ **5** 14 **6** 22 **7** 100 **8** -12 **9** -1 **10** 3

Exercise 9:02

1 **a** 18 **b** 40 **c** 17 **d** 24 **e** 32 **f** 6 **g** 3 **h** 2 **i** 5 **j** 2
k 7 **l** 12·5 **m** 4 **n** 1 **o** 8 **p** 48 **q** 36 **r** 100 **s** 144 **t** 3000
u 12·5 **v** 4 **w** 3 **x** 80

2 **a** 7 **b** 17 **c** 2 **d** 36 **e** 49 **f** 25 **g** 56 **h** 14 **i** -12 **j** 3·5
k 2·5 **l** 56 **m** 84 **n** -8 **o** -20 **p** -432 **q** -9 **r** -48 **s** -4 **t** $2\frac{1}{12}$
u $\frac{7}{12}$ **v** -7 **w** 384 **x** 6

3 **a** **i** 31 **ii** 17 **iii** -18 **b** **i** 24 **ii** 35 **iii** 21·93 **c** **i** 198·4 **ii** 19·625 **iii** 31 420

Prep Quiz 9:03

1 $2ab$ **2** $3ab^2$ **3** $3a+2b$ **4** $4\times x\times y\times y$ **5** $(x+y)\times(x+y)$
6 $x\times x+y\times y$ **7** True **8** 300 **9** 8 **10** -12

Exercise 9:03

1 **a** I **b** J **c** N **d** F **e** E **f** B **g** M **h** H **i** O **j** D
k K **l** C **m** G **n** L **o** A

2 **a** $A = 8{\cdot}64$ **b** $A = 54$ **c** $A = 27$ **d** $A = 12{\cdot}25$ **e** $A = 44$
f $A = 36{\cdot}3$ **g** $A = 80$ **h** $C = 21{\cdot}4$ **i** $V = 262{\cdot}44$ **j** $V = 2100{\cdot}84$
k $P = 12{\cdot}4$ **l** $C = 13$ **m** $\alpha = 105°$ **n** $\alpha = 140°$ **o** $E = 15$

3 **a** **i** $C = 100$ **ii** $C = 0$ **iii** $C = 40$ **b** $E = 18{\cdot}75$ **c** $S = 44{\cdot}7$ **d** $V = 2011\text{ cm}^3$

4 **a** 0·9 **b** 828 **c** 36 **d** 729·6 **e** -4 **f** -48 **g** 189 **h** 12·48 **i** 1·61

5 **a** 7·1 **b** 19·9 **c** 6·9 **d** 25·0

Prep Quiz 9:04

1 $I = 163$ **2** $y = 222$ **3** $x = 49$ **4** $x = 370$ **5** $x = 96$ **6** $y = 183$ **7** $p = 28$ **8** $m = 54$
9 $a = 5$ **10** $t = 2$

Exercise 9:04

1 **a** 10 **b** 15 **c** 2·4 **d** 13·2 **e** 40 **f** 6 **g** 6·38 **h** $26\frac{2}{3}$

2 **a** 5 **b** 4 **c** 4·8 **d** 15 **e** 3·6 **f** 3·75 **g** 30 **h** 27

3 **a** 30 **b** $3\frac{1}{3}$ **c** 0·4 **d** 56 **e** ± 5 **f** ± 3 **g** ± 12 **h** $\pm\frac{1}{2}$

4 **a** 2 **b** 3 **c** 0 **d** 12 **e** 0·6 **f** 2 **g** 6 **h** 0·8

5 **a** 4 **b** 25 **c** 3·1 **d** 6·79 **e** 1·6

6 **a** 3·64 **b** 0·4 **c** 13·5 **d** 13·2 **e** 100

7 **a** 3·13 **b** 1·78 **c** 1·45 **d** 7·72 **e** 37·78

8 **a** 113·097 cm (to 3 dec. pl.) **b** 113·097, yes

Prep Quiz 9:05

1 7 **2** $x+3$ **3** $x+y$ **4** 12 **5** $3x$ **6** xy **7** $\$(50-x)$ **8** $\$5x$
9 xy **10** $x+5$ years

Exercise 9:05

1 **a** $8x = 32$; $x = 4$ **b** $4x = 100$; $x = 25$ **c** $7x = 42$; $x = 6$ **d** $12x = 78$; $x = 6\frac{1}{2}$ **e** $6x = 15{\cdot}6$; $x = 2{\cdot}6$

2 **a** $x + 6 = 10$; $x = 4$ **b** $x + 5 = 22$; $x = 17$ **c** $x + 22 = 50$; $x = 28$ **d** $x + 17 = 12$; $x = -5$ **e** $12 + x = 38$; $x = 26$

3 **a** $x - 3 = 10$; $x = 13$ **b** $x - 5 = 22$; $x = 27$ **c** $x - 25 = 10$; $x = 35$ **d** $x - 17 = 18$; $x = 35$ **e** $6 - x = 4$; $x = 2$

4 **a** $\frac{x}{5} = 4$; $x = 20$ **b** $\frac{x}{8} = 7$; $x = 56$ **c** $\frac{1}{10}x = 8$; $x = 80$ **d** $\frac{x}{6} = 7$; $x = 42$ **e** $\frac{10}{x} = 2$; $x = 5$

5 **a** $2x + 6 = 14$; $x = 4$ **b** $3x + 5 = 20$; $x = 5$ **c** $5x - 8 = 22$; $x = 6$ **d** $4(x + 5) = 56$; $x = 9$ **e** $\frac{1}{2}x - 5 = 3$; $x = 16$

6 **a** $2x + 3 = 33$; $x = 15$ **b** $4x - 3 = 25$; $x = 7$ **c** $2(x + 3) = 22$; $x = 8$ **d** $\frac{x}{4} - 7 = 1$; $x = 32$ **e** $\frac{x + 4}{3} = 8$; $x = 20$

7 **a** 11 **b** 7 **c** 17 **d** 8

Reading Maths 9:05

1 Both weigh the same (3 kg) **2** 742 **3** 18 **4** 8:15 am

Exercise 9:06

1 **a** 13 yrs, 39 yrs **b** \$480, \$80 **c** Anne \$25, Joan \$18 **d** Jim \$165, Alan \$335 **e** 40 years

2 **a** 13 **b** 19 cm by 13 cm **c** 4 kg **d** 24 km

3 **a** \$2 **b** They meet at 2:48 pm when *X* has travelled 24 km and *Y* has travelled 16 km
c The son is 28 yrs old and his father is 56 yrs old **d** 3600 litres

4 **a** Bill is 20 yrs old and Fred is 45 yrs old **b** 16 five-cent coins, 34 ten-cent coins **c** 12
d small tank 900 L; large tank 1200 L **e** 50 km

Diagnostic Test 9 Formulae and Problem Solving

1 **a** $y = 4x$ **b** $y = 2x + 2$ **c** $y = x^2$

2 **a** $x + y$ **b** $\frac{5 + m}{2}$ or $(5 + m) \div 2$ **c** \$$pb$ **d** $200 - x$

3 **a** 13 **b** 45 **c** 310 **4** **a** 18 **b** 36·2 **c** 7·76

5 **a** 157 **b** 16·1 **c** 3·54 **6** **a** 4·5 **b** 8 **c** 8·875

7 **a** 22·5 **b** 11·1 **c** 20 **8** **a** $2a + 7 = 10$ **b** $\frac{a}{3} - 4 = 4$ **c** $3(a + 6) = 32$

9 **a** $x + (x + 5) = 57$ **b** $3x + x + 3x + x = 48$ **c** $3x + 10 = 2(x + 10)$

9A Revision Assignment

1 **a** $12ab$ **b** $7x + 9y$ **c** $\frac{d}{s}$ **d** $mn + \frac{1}{2}mn = \frac{3mn}{2}$

2 **a** **i** 11 **ii** 20 **b** **i** 11·3 **ii** 0·96 **c** **i** 20·25 **ii** 3·375

3 **a** **i** 22 **ii** $12\frac{1}{2}$ **iii** 26·26 **iv** −66·25 **b** **i** 3 **ii** 2 **iii** 24·5 **iv** 20 **v** 14·15
c **i** 4 **ii** 13·3 **iii** 10 **iv** 4·032 **v** 3 **vi** 0·3 **4** **a** 66 **b** 8 **c** 4·1

5 **a** $3x + 7 = 15$, $x = \frac{8}{3}$ **b** $5(x - 9) = 30$, $x = 15$ **c** $8x + 10 = 12x - 7$, $x = \frac{17}{4}$

9B Working Mathematically

1 **a** *collinear* points **b** *vertices* **c** *hypotenuse* **d** $\alpha + \beta$ **e** $360°$
f $3 \times 180° = 540°$ **g** *AB* is a *diameter*, *OC* is a *radius* **h** *circumference* **i** *semicircle*
j *AB* is a *tangent*. *CD* is an *arc*. *EF* is a *chord*.

2 **a** *concurrent* lines **b** *adjacent* angles **c** *complementary* angles **d** *supplementary* angles
e *vertically opposite* angles **f** *transversal* **g** *bisecting* an interval **h** *bisecting* an angle
i $\angle CAB = 60°$ **j** *CD* is *perpendicular* to *AB*

3 84 : 60 : 36 or 7 : 5 : 3 **4** 17 **5** 120 g

6 **a** 37% **b** 9% **c** No, because there are many more car drivers than motor cycles.
It is probable that the rate of fatalities is much higher for motor cyclists.

Chapter 10: Geometry

Prep Quiz 10:01

1 *B* **2** *BA*, *BC* **3** $\angle ABC$ or $\angle CBA$ **4** $\angle DBC$ **5** $\angle ABD$ **6** 90° **7** 180° **8** 100° **9** 20° **10** 80°

Exercise 10:01

1 **a** 60 **b** 30 **c** 130 **d** 65

2 **a** $\alpha = 20°$ (adj. comp. angles) **b** $\beta = 30°$ (adj. supp. angles) **c** $m = 60°$ (adj. comp. angles)
d $y = 49$ (adj. supp. angles) **e** $x = 60$ (adj. supp. angles) **f** $\theta = 47°$ (adj. supp. angles)
g $b = 61$ (adj. comp. angles) **h** $m = 42$ (adj. comp. angles) **i** $a = 135$ (adj. supp. angles)

3 **a** 40° **b** 85° **c** 115° **4** (a), (d), (f)

5 **a** $x = 60$ (adj. supp. angles) **b** $x = 36$ (adj. supp. angles) **c** $a = 50$ (adj. supp. angles)
d $m = 25$ (adj. comp. angles) **e** $x = 16{\cdot}5$ (adj. supp. angles) **f** $\alpha = 17°$ (adj. supp. angles)
6 **a** $\theta = 20°$ (adj. supp. angles) **b** $a = 60$ (adj. supp. angles); $b = 30$ (adj. comp. angles); $c = 150$ (adj. supp. angles)
c $a = 50$ (adj. comp. angles); $b = 20$ (adj. supp. angles); $c = 40$ (adj. supp. angles)

Prep Quiz 10:02

1 360° **2** Because the angles at a point must sum to 360° **3** 60° **4** 140° **5** 260° **6** 360° **7** 130°
8 50° **9** 130° **10** 50°

Exercise 10:02

1 **a** 155° (angles at a point) **b** 60° (angles at a point) **c** 50° (angles at a point)
d 145° (angles at a point) **e** 99° (angles at a point)
2 **a** 114 (vert. opp. ∠s) **b** 147° (vert. opp. ∠s) **c** 65 (vert. opp. ∠s)
d 62 (vert. opp. ∠s) **e** 110° (vert. opp. ∠s) **f** 64° (vert. opp. ∠s)
3 **a** $5x = 80$ (vert. opp. ∠s) **b** $4a = 80$ (vert. opp. ∠s) **c** $3\alpha = 18°$ (vert. opp. ∠s)
$x = 16$ $a = 20$ $\alpha = 6°$
d $3a = 204$ (angles at a point) **e** $2x = 160$ (angles at a point) **f** $5m = 360$ (angles at a point)
$a = 68$ $x = 80$ $m = 72$
g $a = 117$ (vert. opp. ∠s) **h** $\alpha = 98°$ (adj. supp. angles) **i** $m = 125$ (vert. opp. ∠s)
$b = 115$ (vert. opp. ∠s) $\beta = 98°$ (vert. opp. ∠s) $n = 102$ (adj. supp. angles)
4 $a = 60$, $b = 50$, $c = 60$, $d = 30$, $e = 50$, $f = 40$ There are several values that could be found first. Angles $d°$ and $f°$ could have been found using vertically opposite angles while $a°$ could be found using complementary angles.

Exercise 10:03

1 **a** 2 and 8 **b** b and h **c** $\angle ABE$ and $\angle BEF$
3 and 5 c and e $\angle CBE$ and $\angle BED$
2 **a** 1 and 5, 2 and 6 **b** a and e, b and f **c** $\angle GBA$ and $\angle BED$, $\angle GBC$ and $\angle BEF$
4 and 8, 3 and 7 d and h, c and g $\angle ABE$ and $\angle DEH$, $\angle CBE$ and $\angle FEH$
3 **a** 2 and 5 **b** b and e **c** $\angle ABE$ and $\angle BED$
3 and 8 c and h $\angle CBE$ and $\angle BEF$
4 **a** alternate **b** co-interior **c** corresponding **d** co-interior **e** corresponding **f** alternate
5 **a** $a = 120$, $b = 60$, $c = 120$, $e = 40$, $f = 140$, $g = 40$ **b** $a = 160$, $b = 20$, $c = 160$, $e = 92$, $f = 88$, $g = 92$
6 **a** b, n **b** m **c** e, m

Exercise 10:04

1 **a** No **b** No. On the right side. **c** No **d** No. On the left side. **e** Yes
f The corresponding angles are only equal when AB is parallel to CD.

2 **a**

Angle number	1	2	3	4	5	6	7	8
Angle size	120°	60°	120°	60°	120°	60°	120°	60°

b

Angle number	1	2	3	4	5	6	7	8
Angle size	105°	75°	105°	75°	105°	75°	105°	75°

3 **a** The angles which make each pair are equal to each other.
b The angles which make each pair are equal to each other.
c The angles which make each pair sum to 180°.

Exercise 10:05

1 **a** $a = 112$ (corresp. ∠s and || lines) **b** $b = 80$ (co-int. ∠s and || lines) **c** $c = 81$ (alt. ∠s and || lines)
d $e = 121$ (alt. ∠s and || lines) **e** $f = 130$ (corresp. ∠s and || lines) **f** $g = 96$ (alt. ∠s and || lines)
g $h = 115$ (co-int. ∠s and || lines)
2 **a** $m = 93$ (co-int. ∠s, $AB \parallel CD$) **b** $n = 105$ (corresp. ∠s, $RS \parallel TU$) **c** $\theta = 76°$ (alt. ∠s $PQ \parallel RS$)
3 **a** $3x + 120 = 180$ (co-int. ∠s and || lines) **b** $6x = 114$ (corresp. ∠s and || lines)
$x = 20$ $x = 19$
c $3m = 99$ (corresp. ∠s and || lines) **d** $22x = 44$ (alt. ∠s and || lines)
$m = 33$ $x = 2$
e $10a + 70 = 180$ (co-int. ∠s and || lines) **f** $2x = 138$ (alt. ∠s and || lines)
$a = 11$ $x = 69$
g $11x = 77$ (corresp. ∠s and || lines) **h** $2x + 110 = 180$ (co-int. ∠s and || lines)
$x = 7$ $x = 35$

i $4a = 96$ (corresp. ∠s and || lines)
$a = 24$

j $7a = 56$ (corresp. ∠s and || lines)
$a = 8$

k $10p = 70$ (alt. ∠s and || lines)
$p = 7$

l $4x + 88 = 180$ (co-int. ∠s and || lines)
$x = 23$

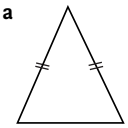
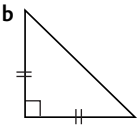
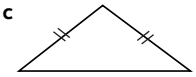
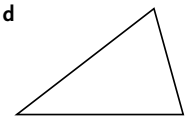
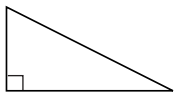
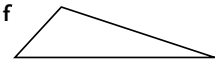
4 **a** corresp. ∠s are equal **b** alt. ∠s are equal **c** co-int. ∠s are supp.
d co-int. ∠s are supp. **e** corresp. ∠s are equal **f** alt. ∠s are equal

5 **a** $a = 27$, $b = 63$, $c = 27$ **b** $a = 60$, $b = 25$ **c** $a = 125$, $b = 55$, $c = 55$, $d = 125$, $e = 55$, $f = 55$, $g = 35$

Prep Quiz 10:06

1 10 **2** 9 **3** 11 **4** scalene **5** isosceles **6** equilateral **7** 100° **8** BC **9** 30° **10** AB

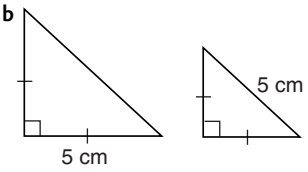
Exercise 10:06

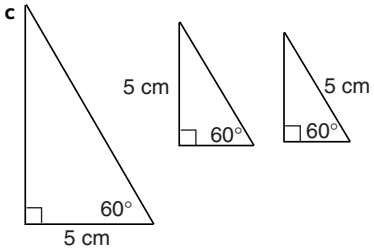
1 **a**  **b**  **c**  **d**  **e**  **f** 

2 In an equilateral triangle all angles are equal to 60°. As 60° is an acute angle, equilateral triangles can only be acute-angled triangles.

3 **a** 

(not to scale)

b 

c 

4 **a** $\angle ACB$ **b** AB **c** Measure the length of each side. The largest angle will be opposite the largest side.

5 **a** $a = 40$, $b = 90$, $c = 90$, $d = 50$, $x = 6$ **b** $a = 20$, $b = 70$, $c = 70$, $x = 3$ **c** $a = 30$, $b = 30$, $x = 3$, $m = 3$, $y = 3$

6 **a** $a = 35$ (angle sum of a Δ) **b** $b = 87$ (angle sum of a Δ) **c** $c = 127$ (angle sum of a Δ)
d $x = 115$ (ext. ∠ of a Δ) **e** $m = 116$ (ext. ∠ of a Δ)
f $105 = 71 + y$ (ext. ∠ of Δ)
$y = 34$
g $b = 42$ (angle sum of a Δ) **h** $95 = x + 70$ (ext. ∠ of a Δ)
$x = 25$
i $118 = 52 + m$ (ext. ∠ of a Δ)
$m = 66$
j $x = 68$ (adj. angles on a straight line) **k** $116 = 80 + y$ (ext. ∠ of a Δ)
$y = 36$
l $m = 46$ (angle sum of a Δ)

7 **a** $a = 75$ (base ∠s of isos. Δ) **b** $b = 55$ (base ∠s of isos. Δ) **c** $c = 69$ (base ∠s of isos. Δ)
d $a = b = 60$ (∠s of an equil. Δ) **e** $c = d = 60$ (∠s of an equil. Δ) **f** $e = f = 60$ (∠s of an equil. Δ)

8 **a** $\angle BCA = 65$ (base ∠s of isos. Δ)
$x + 65 + 65 = 180$ (angle sum of a Δ)
$x = 50$

b $\angle MNV = 58°$ (base ∠s of an isos. Δ)
$y + 58 + 58 = 180$ (angle sum of a Δ)
$y = 64$

c $\angle ACB = 38°$ (base ∠s of isos. Δ)
$m + 38 + 38 = 180$ (angle sum of a Δ)
$m = 104$

d $\angle XZY = a°$ (base ∠s of isos. Δ)
$a + a + 50 = 180$ (angle sum of a Δ)
$a = 65$

e $\angle BAC = b°$ (base ∠s of isos. Δ)
$b + b + 100 = 180$ (angle sum of a Δ)
$b = 40$

f $\angle DEF = c°$ (base ∠s of isos. Δ)
$c + c + 40 = 180$ (angle sum of a Δ)
$c = 70$

9 **a** $2a = 60$ (∠s of equil. Δ equal 60°)
$a = 30$

b $3b = 66$ (base ∠s of isos. Δ are equal)
$b = 22$

c $5a + 60 + 65 = 180$ (angle sum of a Δ)
$a = 11$

d $3a + 2a + 70 = 180$ (angle sum of a Δ)
$a = 22$

e $a + 40 = 60$ (∠s of equil. Δ equal 60°)
$a = 20$

f $104 = a + 3a$ (ext. ∠ of a Δ)
$a = 26$

g $66 + 66 + 6p = 180$ (base ∠s of isos. Δ)
$p = 8$

h $3x + 2x + 4x = 180$ (angle sum of a Δ)
$x = 20$

i $120 = 2a + 4a$ (ext. ∠ of a Δ)
$a = 20$

10 a $\angle ACB = 45°$ (ΔABC is a right isos. Δ)
$x + 45 = 180$ ($\angle BCD$ is a straight $\angle$)
$x = 135$

b $\angle BDA = 70°$ (Base $\angle$s of isos. Δ)
$x + 70 = 180$ ($\angle ADC$ is a straight $\angle$)
$x = 110$

c $\angle BCD = 45°$ (ΔABC is a right isos. Δ)
$x + 45 = 80$ (ext. $\angle$ of a Δ)
$x = 35$

11 a $\angle PLN = x°$ (base $\angle$s of isos. Δ)
$2x + 120 = 180$ (angle sum of Δ)
$x = 30$
$x + y + 90 = 180$ (angle sum of Δ)
$30 + y + 90 = 180$
$y = 60$

b $\angle x = \angle R = 60°$ (ΔXRY is equil.)
$x = 60$ (corresp. to $\angle X$, $RX \| TY$)
$y = 60$ (alternate to $\angle R$, $RX \| TY$)

c $y = 20$ (alternate $\angle$s, $AB \| DC$)
$\angle BAD = x°$ (base $\angle$s of isos. Δ)
$2x + y = 180$ (angle sum of Δ)
$2x + 20 = 180$
$x = 80$

Exercise 10:07

1 a $a = 90$ (angle sum of a quad.)
b $b = 105$ (angle sum of a quad.)
c $c = 99$ (angle sum of a quad.)
d $d = 52$ (angle sum of a quad.)
e $e = 90$ (angle sum of a quad.)
f $f = 104$ (angle sum of a quad.)
g $g = 100$ (angle sum of a quad.)
h $h = 60$ (angle sum of a quad.)
i $i = 82$ (angle sum of a quad.)

2 a $2x + 90 + 114 + 56 = 360$
(angle sum of a quad.)
$x = 50$

b $3a + 75 + 111 + 93 = 360$
(angle sum of a quad.)
$a = 27$

c $7g + 83 + 119 + 67 = 360$
(angle sum of a quad.)
$g = 13$

d $5m + 150 + 5m + 70 = 360$
(angle sum of a quad.)
$m = 14$

e $2x + 2 + 76 + 76 + 2x + 2 = 360$
(angle sum of a quad.)
$x = 51$

f $a + 112 = 360$ (revol'n)
$a = 248$
$248 + 2y + 32 + 2y = 360$ (angle sum of a quad.)
$y = 20$

3 a $x = 55$ (angle sum of a Δ)
$y + 105 + x + 110 = 360$
(angle sum of a quad.)
$y = 90$

b $x = 120$ (co-int. $\angle$s and $\|$ lines)
$x + 40 + y + 110 = 360$
(angle sum of a quad.)
$y = 90$

c $x + 70 + 140 + 100 = 360$
(angle sum of a quad.)
$x = 50$
$\angle ABF = x$ (base $\angle$s of isosceles Δ)
$x + x + y = 180$ (angle sum of a Δ)
$50 + 50 + y = 180$
$y = 80$

Prep Quiz 10:08

1 5 **2** 1 **3** 4 **4** 3 **5** 2 **6** 9 **7** 2 **8** rectangle, square **9** square, rhombus **10** square

Exercise 10:08

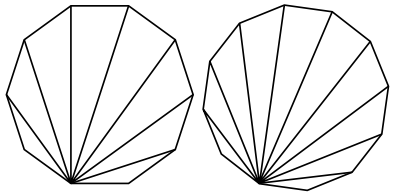
1

Properties	*Parallelogram*	*Rhombus*	*Rectangle*	*Square*
1 Opposite sides parallel	yes	yes	yes	yes
2 Opposite sides equal	yes	yes	yes	yes
3 Opposite angles equal	yes	yes	yes	yes
4 Diagonals bisect one another	yes	yes	yes	yes
5 All sides equal	no	yes	no	yes
6 All angles right angles	no	no	yes	yes
7 Diagonals perpendicular	no	yes	no	yes
8 Diagonals bisect angles through which they pass	no	yes	no	yes
9 Diagonals are equal	no	no	yes	yes

2 **a** True **b** The diagonals intersect at right angles **c** No. Only one diagonal is bisected. **d** Yes.
e It has one pair of opposite angles equal. **3** The numbers refer to the table in Question 1 above.
a 1, 2, 3, 4, 5, 7, 8 **b** 1, 2, 3, 4, 6, 9 **c** 1, 2, 3, 4 **d** 1, 2, 3, 4
4 **a** all sides equal; diagonals bisect angles through which they pass; diagonals are perpendicular
b all angles right angles; diagonals are equal **c** all angles are right angles; diagonals are equal
5 **a** rhombus, square, kite (only one diagonal is an axis of symmetry) **b** 4 **c** 1
6 **a** rectangle **b** parallelogram **c** square
7 **a** It is either a rectangle or a square. **b** square or rectangle **c** square or rhombus
8 **a** $w = 15$, $a = 130$ **b** $b = c = 130$ **c** $y = 7$, $x = 6$
9 **a** Yes **b** Yes **c** No, because it does not have all its sides equal. **d** Yes **e** Yes
f No, because it does not have all its angles equal.
10 (1) quadrilateral (2) trapezium (3) parallelogram (4) rectangle (5) square (6) rhombus
11 **a** both pairs of opposite sides equal **b** diagonals bisect each other
d both pairs of opposite angles are equal **e** both pairs of opposite angles are equal
f one pair of opposite sides equal and parallel **h** both pairs of opposite sides equal
i diagonals bisect each other
12 **b** all sides equal **c** diagonals bisect each other at right angles **e** all sides equal
f diagonals bisect each other at right angles **13** **a** and **d** **14** Yes

Investigation 10:09A The angle sum of a polygon

2



3

No. of sides in polygon	3	4	5	6	7	8	9	10	12
No. of triangles	1	2	3	4	5	6	7	8	10

4 The 'number of triangles' is two less than the 'number of sides'.
5 $n - 2$
6 $(n - 2) \times 180$
7 $\frac{(n-2) \times 180}{n}$

Investigation 10:09B The exterior angle sum of a convex polygon

1 6 **2** $6 \times 180° = 1080°$ **3** 720° **4** $1080° - 720° = 360°$
5 (1)5, 5 (2) $5 \times 180° = 900°$ (3) 540° (4) $900° - 540° = 360°$ **6** The exterior angles sum to 360°.

Exercise 10:09

1 **a** 720 **b** 1440 **c** 2340 **2** **a** **i** 5 **ii** 540° **b** **i** 6 **ii** 720° **c** **i** 7 **ii** 900°
3 **a** 1080° **b** 1440° **c** 900° **4** **a** 108° **b** 120° **c** 135° **d** 144° **e** 140° **f** $158\frac{14}{17}°$
5 **a** 3240° **b** 360° **c** 162° **d** 18° **6** **a** 6 **b** 9 **c** 36 **d** 24
7 **a** 240° **b** 228° **c** 315° **8** **a** 80° **b** 110° **c** 120° **d** 135° **e** 125° **f** 60°

Investigation 10:09C Regular polygons and tessellations

1 120° **2** 3 **3** 360° **4** A regular pentagon will not tessellate because the interior angles (108°) cannot add at a vertex to give 360° **5** Square and equilateral triangle.

Investigation 10:09D Spreadsheet calculations

1 A16 = 24 (number of sides in the polygon)
B16 = 3960 (angle sum of a 24-sided polygon)
C16 = 165 (size of interior angle of a regular 24-gon)
2 15 840°; 176°
3 **a** 30 **b** 36 **c** 46

Diagnostic Test 10 Geometry

1 **a** 25° **b** 44° **c** 40° **2** **a** 67° **b** 212° **c** 120°
3 **a** 48° **b** 93° **c** 91° **4** **a** 40° **b** 49° **c** 113°
5 **a** 54° **b** 120° **c** 17° **6** **a** 60° **b** 50° **c** 110°
7 **a** 93° **b** 86° **c** 42° **8** **a** 720° **b** 135° **c** 360°

10A Revision Assignment

1 **a** $x = 150$ (∠s at a point) **b** $x = 40$ (vert. opp. ∠s); $y = 70$ (∠ sum of Δ)
c $y = 115$ (adj. supp. ∠s); $x = 65$ (alt ∠s, || lines) **d** $m = 38$ (alt ∠s, || lines)
e $x = 60$ (co-int. ∠s, || lines) **f** $y = 78$ (∠ sum of Δ)
g $b = 54$ (∠ sum of quad.)

2 **a** $x = 30$ (co-int. ∠s, || lines) **b** $x = 70$ (corresp. ∠s, || lines) **c** $x = 30$ (base ∠s of isosceles Δ)
$y = 150$ (ext. ∠ of Δ) $y = 130$ (ext. ∠ of Δ) $y = 95$ (co-int. ∠s, || lines)

3 **a** 72 **b** 72°, 144° **c** $a = 108$, $b = 36$, $c = 36$, $d = 72$

10B Working Mathematically

1 The shape of the lamp shade is based on a square pyramid. The top section of the pyramid and its base have been removed.
2 34 **3** 12 L; 33 L
4 **a** 15 m **b** 0·6, 3·4 seconds **c** 20 m **d** 2 seconds **e** 1·4 seconds

Chapter 11: Locus

Practical Activity 11:01

2 **b** They are all the same distance from the circumcentre.
4 parallelogram
5 **a** The bisectors of the chords meet at the centre.
b Draw two chords and construct the perpendicular bisector of each chord. The point where the bisectors meet is the centre of the circle.
6 **c** yes
7 **b** isosceles; because $AO = OB$
d yes
10 **c** yes
d yes
11 **c** rhombus
d rhombus
12 **b** 90°

Prep Quiz 11:02

1 cut in half **2** 30° **3** 90° **4** 45° **5** 60
6 equilateral **7** 60° **8** 90° **9** 45° **10** 30°

Practical Activity 11:02

4 **b** 90° **c** AB = 20 mm; AC = 35 mm
5 **b** 30° **c** AC = 50 mm; BC = 86 mm
6 **b** 60° **c** AD = 54 mm; DC = 29 mm
7 **a** CD should be 36 mm **b** CD should be 20 mm **c** CD should be 35 mm
8 **a** DE should be 3 cm; $\angle CED$ should be 45°
b $\angle MLN$ should be 75°; LM should be 29 mm; LN should be 36 mm

Practical Activity 11:03

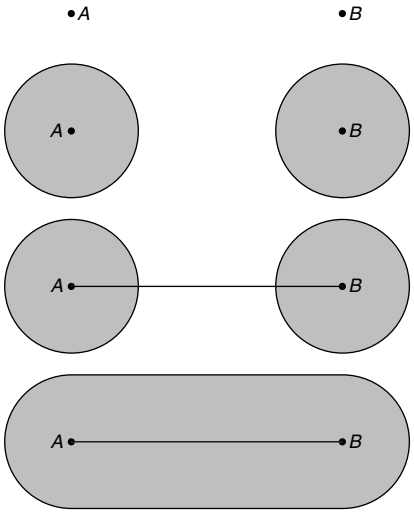
Note: Answers to construction questions can be checked using the answers given.

2 **a** AC = 22 mm; BC = 44 mm **b** AB = 32 mm; BC = 48 mm; CD = 53 mm
c BC = 31 mm; AC = 70 mm; AD = 45 mm; BD = 65 mm
8 **a** $BD = DC$ = 4 cm **b** AD = 3 cm; CD = 5 cm **c** AD = 29 mm

Exercise 11:04

1 **a** They are equidistant from P and Q **c** They make a straight line
d The locus is the perpendicular bisector of PQ **e** …the perpendicular bisector of PQ
2 **a** C and A **b** C and B **c** Yes **g** Yes **i** Yes
j The bisector of the angle ACB **k** …the bisector of the angle ACB
3 **d** Yes **f** the locus of points equidistant from AB and CD

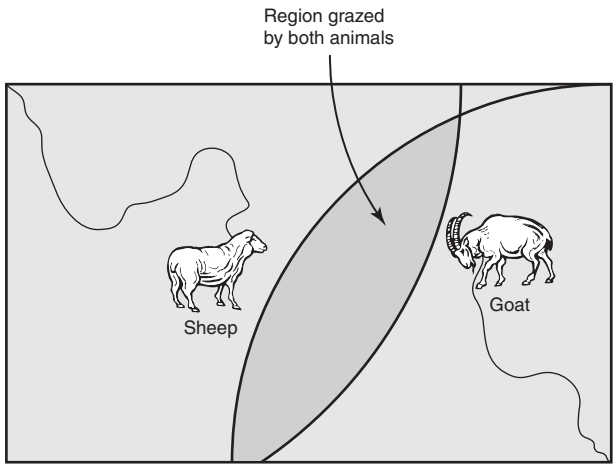
4



5 **a** Locus of points equidistant from *AB* and *CD*
b Locus of points between *AB* and *CD* that are closer to *CD* than *AB*
c Locus of all points closer to *CD* than *AB*
d Locus of points inside the rectangle *ABCD* and 3 cm from *C*
e Locus of points inside the rectangle *ABCD* and less than 3 cm from *C*
f Locus of points inside the rectangle *ABCD* and more than 3 cm from *C*
g Locus of points 4 cm or less from *C* that are not inside the triangle *ABC*

Exercise 11:05

1



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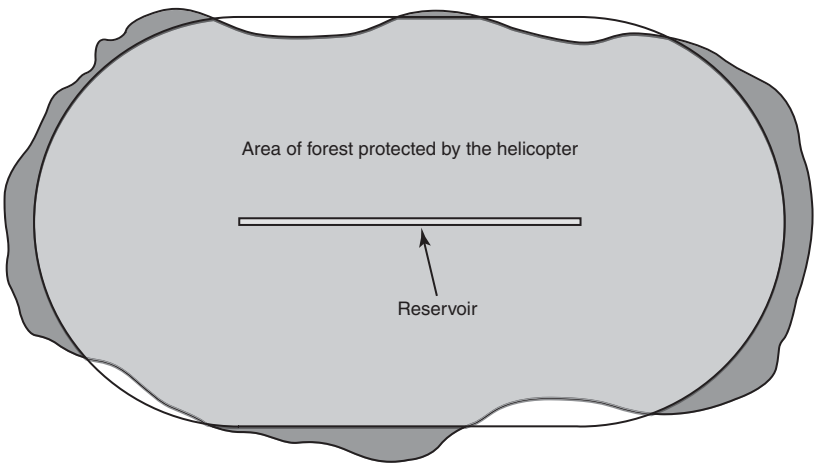

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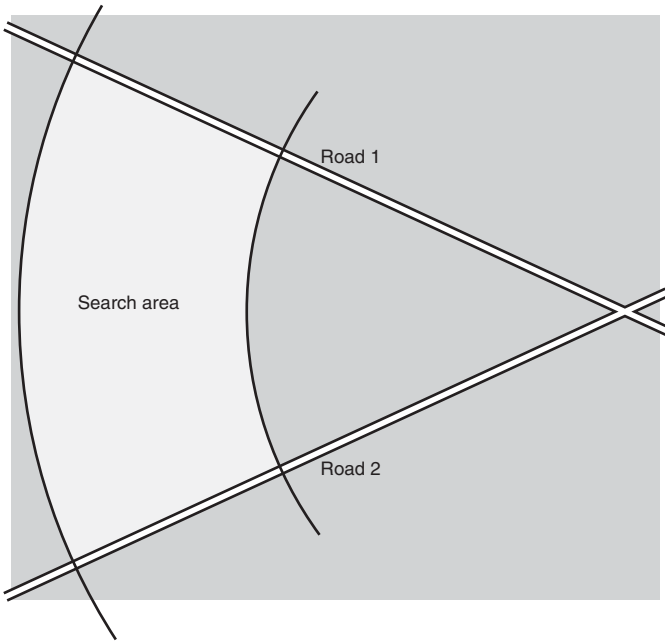

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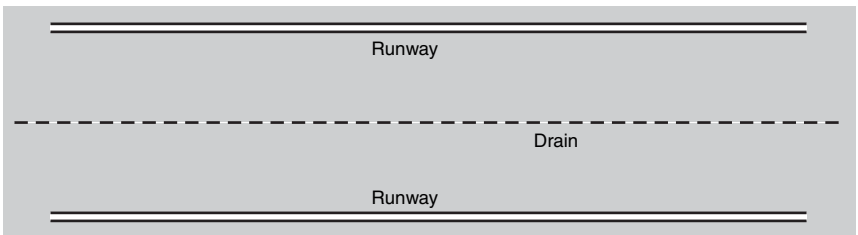
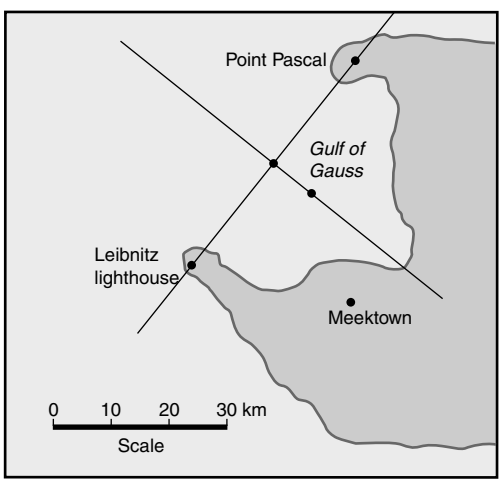


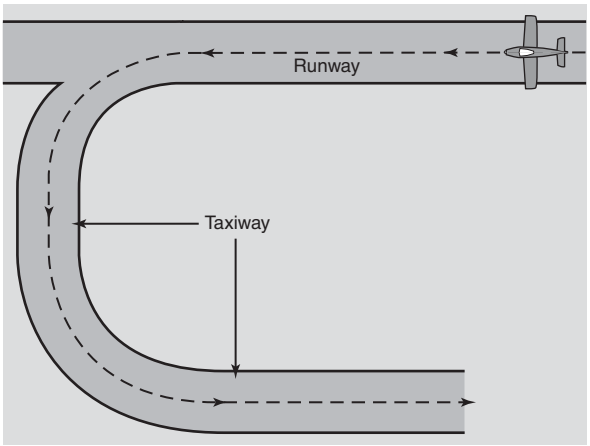
5 To be marked by teacher

6



7 To be marked by teacher

8



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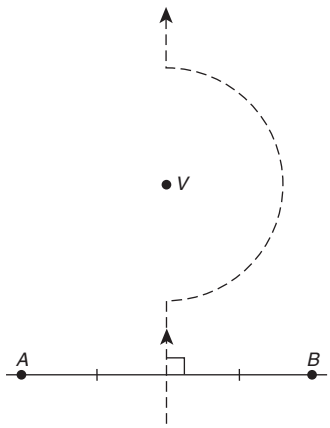

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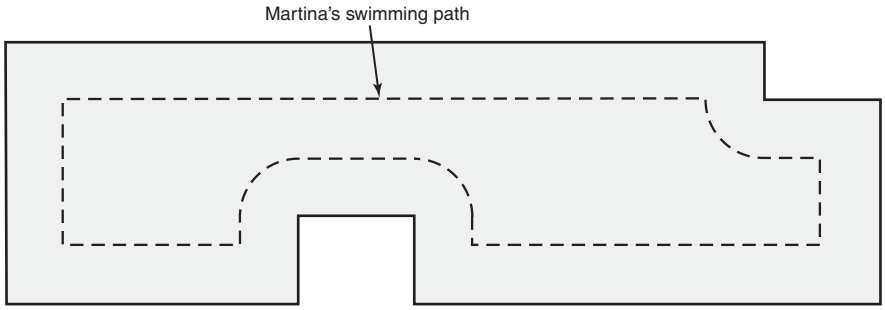


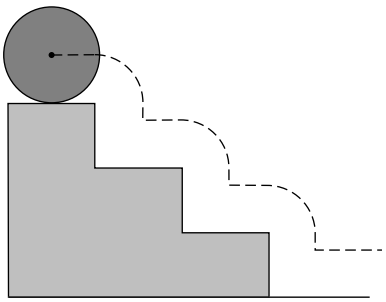
Diagnostic Test 11: Locus

1 To be marked by teacher

2 To be marked by teacher

3 To be marked by teacher

4



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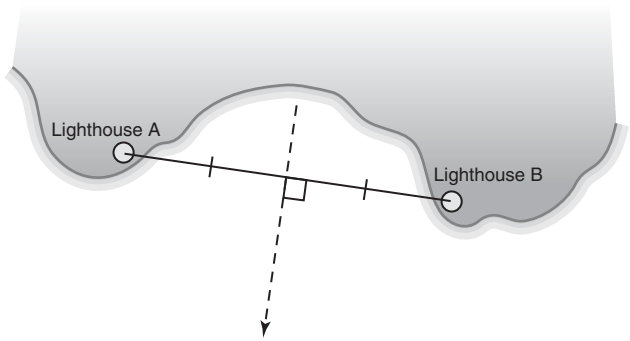

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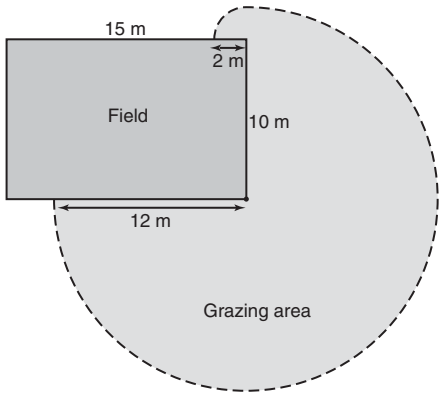

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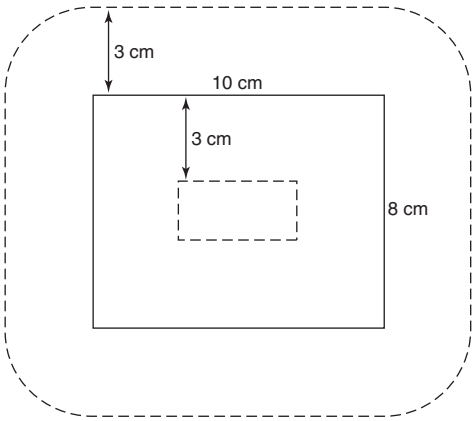

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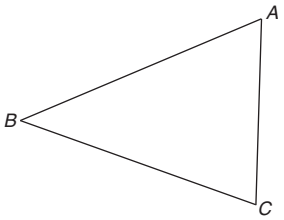
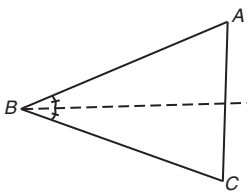


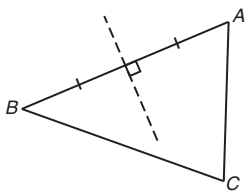
9 **a** To be marked by teacher
b To be marked by teacher

11A Revision Assignment

1



2



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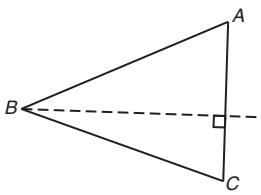
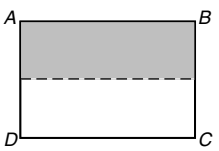


4 **a** Set of points 1 cm from *O*
b Set of points equidistant from *LM* and *PQ*
c Set of points equidistant from *X* and *Y*

5



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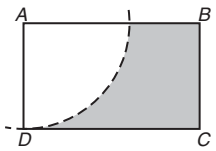

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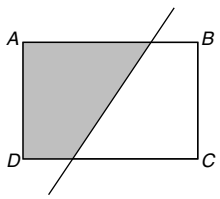

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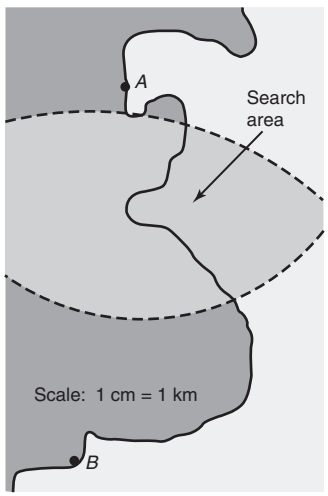


11B Working Mathematically

1 €1449
2 $45
3 840 km
4 31.5 m^2
5 25 and 25, –25 and –26
6 3

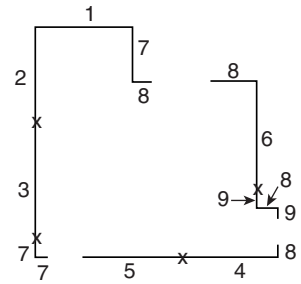
Chapter 12: Perimeter, Area and Surface Area

Exercise 12:01

1 **a** 36·4 m **b** 52·95 m **c** 24·518 m **d** 28·6 m **e** 2500 mm **f** 188·8 mm **g** 52·2 m **h** 56 026 mm
i 70·962 m **j** 40·9 cm **k** 39·51 m
2 **a** 14·04 m **b** 2252 mm **c** 138 cm **d** 22 660 mm **e** 15·544 m
3 **a** 20·1 m **b** 27·0 m **c** 6·28 m **4** **a** 42 cm **b** 56 cm **c** 108·8 cm
5 **a** 41·2 m **b** 42·4 m **c** 46 m **d** 33·318 m **e** 76·4 cm **f** 5824 mm
6 **a** perim. = 1410 m; cost = \$705 **b** 10·3 km **c** 64
7 **a** 7·9 m **b** 910 rev **c** 393 m **d** 23 laps
8 **a** 327 m **b** 36 **c** \$616.65
9 **a** 28·38 m **b** 61 m **c** 112·8 m **d** **i** 3150 m **ii** 3600 m **iii** 3000 m
10 **a** $x = 5$; perimeter = 12 cm **b** $x = 13$; perimeter = 40 m **c** $x = 10$; perimeter = 48 cm
11 **a** $a = 1{\cdot}6$; perimeter = 8 m **b** 66 m **c** 67 cm **12** **a** 12 m **b** 48 cm

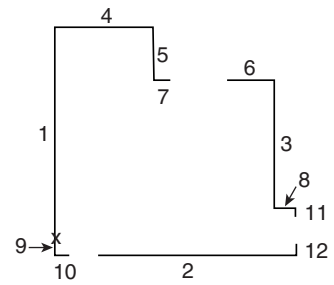
Investigation 12:01

1 No. One wall is 7600 mm long which is longer than the largest length of skirting board available (6·6 m)
2 Total length required = 27 400 (27·4 m)
3



Note: The following is only one possible solution. The numbers show how the 9 skirting boards are used. Joins are indicated by '×'.
Board 1 is 3200 mm long (100 mm waste)
Boards 2, 3, 4, 5, 6 are 3300 mm long.
Board 7 is cut into 3 pieces (1000 mm, 400 mm, 1800 mm)
Board 8 is cut into 4 pieces (600 mm, 1600 mm, 700 mm, 300 mm)
Board 9 is cut into 2 pieces (900 mm, 400 mm)
If joins at corners are not counted the smallest number of joins possible is 4.

4



Boards 1 and 2 (6·6 m); Board 3 (4·2 m); Board 4 (3·3 m);
Boards 5 and 6 (1·8 m); Board 7 (600 mm); Board 8 (900 mm);
Board 9 (1·2 m); Boards 10 and 11 (600 mm);
Board 12 (300 mm)
He would have to order: $2 \times 6{\cdot}6$ m, $1 \times 4{\cdot}2$ m, $1 \times 3{\cdot}3$ m, $1 \times 1{\cdot}8$ m, $1 \times 1{\cdot}2$ m, 1×900 mm, 3×600 mm, 1×300 mm.
This would only require 1 join and give 1·1 m of waste.

Exercise 12:02

1 **a** 130 cm **b** 110 cm **c** 110 cm **2** **a** 28 cm **b** 36 cm **c** 36 cm
3 **a** 11·8 cm **b** 28·3 cm **c** 17·5 cm **d** 39·3 cm **e** 5·2 cm **f** 18·0 cm
4 **a** 21·8 cm **b** 40·3 cm **c** 37·5 cm **d** 64·3 cm **e** 15·2 cm **f** 35·2 cm
5 **a** 70·3 cm **b** 96·0 m **c** 45·1 m **d** 201·1 mm **e** 46·7 m **f** 77·8 cm
6 **a** $x = 17$, $y = 10$ **b** 72 cm **7** 116 cm
8 **a** 120 cm **b** 94 cm **c** 100 cm **9** **a** 76·0 cm **b** 116·1 cm

Prep Quiz 12:03

1 25·44 **2** 2·04 **3** 32·66 **4** *LB* **5** 20 **6** 20 **7** 20 **8** 48 **9** 100 **10** 10

Exercise 12:03

1 **a** 49 cm^2 **b** 161·29 m^2 **c** 0·5625 m^2
2 **a** 60 m^2 **b** 88·4 cm^2 **c** 202·24 m^2 **d** 0·75 m^2 or 7500 cm^2
e 1·188 m^2 or 11 880 cm^2 **f** 1775 mm^2 or 17·75 cm^2
3 **a** 113·10 cm^2 **b** 38·48 cm^2 **c** 21·24 m^2 **d** 38·48 m^2
e 65·04 m^2 **f** 0·64 m^2
4 **a** 1 m^2 **b** 10 000 cm^2 **c** Yes **d** 1 m^2 = 10 000 cm^2
5 **a** 10 000 **b** 21 **c** 31·5 **d** 16 500 m^2 or 1·65 ha
6 **a** 10 **b** 1 cm^2 **c** 100 mm^2 **d** 1 cm^2 = 100 mm^2
7 **a** 1 km^2 = 1 000 000 m^2 **b** 1 km^2 = 100 ha
8 **a** 30·6 m^2 **b** 17·94 m^2 **c** 0·546 m^2

9 **a** $35{\cdot}2\ cm^2$ **b** $63{\cdot}7\ m^2$ **c** $324{\cdot}72\ cm^2$
10 **a** $69{\cdot}6\ cm^2$ **b** $39{\cdot}75\ m^2$ **c** $8{\cdot}172\ m^2$
11 **a** $242\ cm^2$ **b** $81{\cdot}48\ m^2$ **c** $45{\cdot}365\ m^2$

Challenge 12:03

1 250 000 km^2 **2** Columbia 1 100 000 km^2, Venezuela 900 000 km^2
Guyana 200 000 km^2, Surinam 150 000 km^2
French Guiana 100 000 km^2, Brazil 8 500 000 km^2
Uruguay 150 000 km^2, Argentina 2 500 000 km^2
Paraguay 400 000 km^2, Chile 750 000 km^2
Bolivia 1 000 000 km^2, Peru 1 250 000 km^2
Ecuador 250 000 km^2
3 Brazil 4250 km (EW), 4000 km (NS)
Argentina 1000 km (EW), 3500 km (NS)
4 Australia ~ 7 700 000 km^2

Exercise 12:04

1 **a** $19{\cdot}6\ cm^2$ **b** $51{\cdot}3\ cm^2$ **c** $31{\cdot}9\ cm^2$ **d** $154{\cdot}9\ cm^2$ **e** $106{\cdot}2\ cm^2$
2 **a** $60\ cm^2$ **b** $84\ cm^2$ **c** $95\ cm^2$
3 **a** $57{\cdot}1\ m^2$ **b** $137{\cdot}16\ m^2$ **c** $106\ m^2$
4 **a** $31{\cdot}08\ m^2$ **b** $5{\cdot}04\ m^2$ **c** $158{\cdot}8\ m^2$
5 **a** $33{\cdot}77\ cm^2$ **b** $84{\cdot}82\ cm^2$ **c** $103{\cdot}07\ cm^2$
6 **a** $98{\cdot}14\ m^2$ **b** $191{\cdot}42\ m^2$ **c** $78\ m^2$
7 **a** $41{\cdot}56\ cm^2$ **b** $12{\cdot}96\ cm^2$ **c** $168{\cdot}87\ cm^2$ (correct to 2 dec. pl.)
8 **a** $64{\cdot}6\ m^2$ **b** $101{\cdot}4\ m^2$ **c** $44{\cdot}82\ m^2$
9 **a** $311{\cdot}42\ m^2$ **b** $270{\cdot}4\ m^2$ **c** $319{\cdot}975\ m^2$
10 **a** $83\ cm^2$ **b** $164\ cm^2$ **c** $103\ cm^2$
11 **a** $16{\cdot}8\ m^2$ **b** 672 **c** \$168
12 Area = $38{\cdot}43\ m^2$; Cost = \$1537.20
13 **a** $24{\cdot}48\ m^2$ **b** $25{\cdot}52\ m^2$ **c** 1100 bricks would be needed **14** \$1494
15 **a** $48{\cdot}18\ m^2$ **b** $1{\cdot}08\ m^2$ **c** 45 sheets **d** \$216 **e** \$1686.30

Challenge 12:04

2π metres $\div$ 6·3 m

Prep Quiz 12:05

1 $21\ cm^2$ **2** $12\ cm^2$ **3** $48\ cm^2$ **4** 6 **5** squares **6** 6 **7** rectangles **8** opposite faces are congruent
9 5 **10** 13

Exercise 12:05

1 **a** $150\ cm^2$ **b** $952{\cdot}56\ cm^2$ **c** $4{\cdot}2336\ m^2$
2 Area of top and bottom = $140{\cdot}4\ m^2$
Area of sides = $25{\cdot}2\ m^2$
Area of front and back = $87{\cdot}36\ m^2$
Total surface area = $252{\cdot}96\ m^2$
3 **a** $142\ cm^2$ **b** $192\ cm^2$ **c** $229\ cm^2$
4 **a** $136\ cm^2$ **b** $241\ cm^2$ **c** $381{\cdot}72\ cm^2$
5 **a** $648\ m^2$ **b** $312{\cdot}45\ m^2$ **c** $295{\cdot}65\ m^2$
6 **a** **i** $150\ cm^2$ **ii** cube **iii** closed **b** **i** $21\ cm^2$ **ii** rectangular prism **iii** open
c **i** $27{\cdot}46\ cm^2$ **ii** triangular prism **iii** closed **d** **i** $62{\cdot}86\ m^2$ **ii** rectangular prism **iii** closed
7 **a** $x = 5$ cm; S.A. = $96\ cm^2$ **b** $x = 6{\cdot}24$ cm; S.A. = $452{\cdot}4\ cm^2$ **c** $x = 6{\cdot}40$ cm; S.A. = $669{\cdot}43\ cm^2$
d $x = 40{\cdot}02$ m; S.A. = $1470{\cdot}79\ m^2$
8 **a** $3392\ cm^2$ **b** $2800\ cm^2$ **c** $324\ cm^2$ **d** $4704\ cm^2$ **e** $3659\ cm^2$ **f** $79{\cdot}68\ cm^2$

Fun Spot 12:05B

3; 27 blocks (8 with 3 sides painted, 12 with 2 sides painted, 6 with 1 side painted, 1 with no side painted).
64 blocks (8 with 3 sides painted, 24 with 2 sides painted, 24 with 1 side painted, 8 with no sides painted).

Exercise 12:06

1 **a** $18\ cm^2$ **b** $22\ cm^2$ **c** $26\ cm^2$ **d** $24\ cm^2$ **e** $28\ cm^2$
2 **a** $28\ cm^2$ **b** $38\ cm^2$ **c** $36\ cm^2$
3 **a** $840\ cm^2$ **b** $6600\ cm^2$ **c** $1194\ cm^2$
4 **a** $356\ cm^2$ **b** $4980\ cm^2$ **c** $3752\ cm^2$

Investigation 12:07

4 Largest possible area is $25\ cm^2$.

Exercise 12:07

1 $36{\cdot}05\ m^2$ **2** **a** $139{\cdot}9\ m^2$ **b** \$1433.98
3 **a** $57{\cdot}12\ m^2$ **b** $28{\cdot}08\ m^2$ **c** 6 L
4 **a** $5{\cdot}28\ m^2$ **b** \$79.20 **5** **a** $12{\cdot}92\ m^2$ **b** 4·4 m **c** 9·7 m
6 \$45 945 **7** $363\ m^2$ (correct to 2 dec. pl.)
8 $204{\cdot}5\ m^2$ (correct to 1 dec. pl.)

Diagnostic Test 12

1 **a** 44·0 m **b** 23·9 cm **c** 157 mm **d** 9·11 m
2 **a** 34 m **b** 27·8 m **c** 150·2 m
3 **a** 30 cm **b** 36 m **c** 60 cm
4 **a** 64·27 cm **b** 15·35 cm **c** 4·95 m
5 **a** 11·14 m **b** 57·12 m **c** 30 m
6 **a** 60 cm^2 **b** 76·86 cm^2 **c** 25·83 cm^2
7 **a** 58·17 m^2 **b** 21·76 m^2 **c** 17·48 m^2
8 **a** 245·43 cm^2 **b** 14·52 cm^2 **c** 1·34 m^2
9 **a** 7·14 m^2 **b** 196·35 cm^2 **c** 58 m^2
10 **a** 117·14 m^2 **b** 36 cm^2 **c** 666 m^2
11 **a** 400 cm^2 **b** 704 cm^2 **c** 351·97 cm^2

12A Revision Assignment

1 5 **2** 14 m **3** **a** 216 m^2 **b** 16 m^2 **4** $1019.66
5 **a** 7·6 units **b** 139·29 $units^2$ **c** 582·13 $units^2$ **6** **a** 56·9 cm **b** 104·4 cm^2

12B Working Mathematically

1 **a** *complementary angles* **b** *supplementary angles* **c** *vertically opposite angles*
d *angles at a point* **e** *corresponding angles* **f** *alternate angles*
g *co-interior angles* **h** *straight angle* **i** *reflex angle* **j** *revolution*
2 *triangular prism*; the triangle shape is more easily suspended than a square or a rectangle
3 Rachel is 13 years old **4** 34 **5** 4 times, 3 times **6** **a** 10% **b** $33\frac{11}{18}$% **c** 23° **d** 62·5%

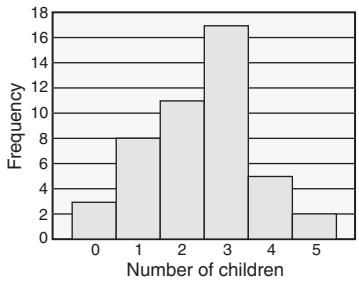
Chapter 13 Statistics

Prep Quiz 13:01

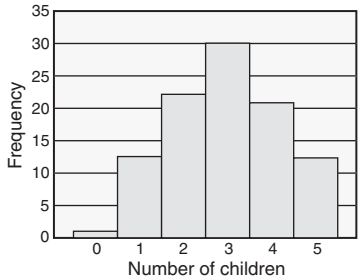
1 20 **2** 1 **3** 1 **4** 7 **5** 4 **6** 9 **7** 13 **8** 23

Exercise 13:01

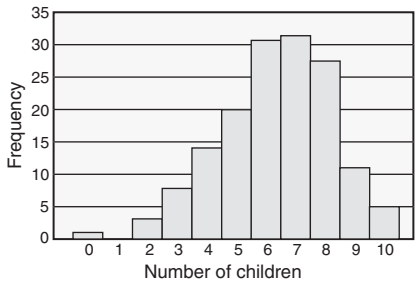
1 **a** 6 **b** 3 heads **c** 1 head **d** 4 **e** 4
2 **a** 4 **b** 2 **c** 0 **d** 21 **e** 32 **f** 70
3 **a**



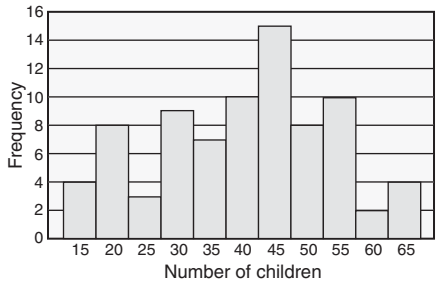
b



c

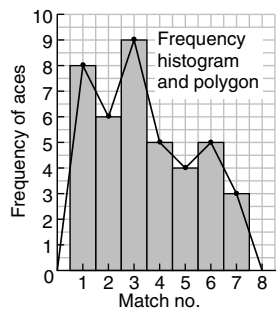


d



4 **a** 9 **b** 34 **c** 123 **d** 22 **e** 14 **f** 38 **g** 103 **h** 20
Amelie actually won the championship although she had more unforced errors throughout the tournament. It would be easy to make the assumption that Justine won, but more information is required to get the whole story of the match.

5 **a**



b In the earlier rounds Jim Courier would have been playing weaker players who would be easier to ace. Other things could also contribute, such as weather changes and physical fitness as the event continues.

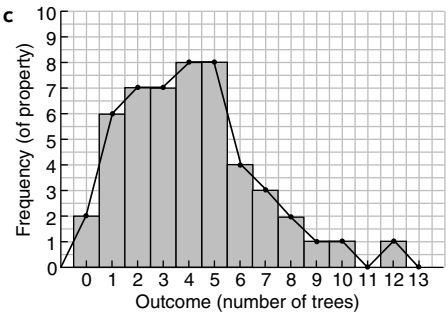
6 a 12, 0

b

x	f
0	2
1	6
2	7
3	7
4	8
5	8
6	4
7	3
8	2
9	1
10	1
11	0
12	1

Total:

c



7 a

Dad
Prog. Total
4
8
14
16
21

b **i** Adam **ii** Dad **iii** Sharon

c 6; 3 times

d Dad

e

	Sharon		*Adam*		*Dad*		*Bron*	
	Score	*Prog. Total*	*Score*	*Prog. Total*	*Score*	*Prog. Total*	*Score*	*Prog. Total*
Round 6	6	28	6	27	7	28	5	24

Exercise 13:02

1 a

Outcome (x)	*f*	*c.f.*
0	3	3
1	8	11
2	11	22
3	17	39
4	9	48
5	2	50

Total: 50

b

Outcome (x)	*f*	*c.f.*
9	1	1
10	13	14
11	22	36
12	30	66
13	21	87
14	13	100

Total: 100

c

x	f	*c.f.*
0	1	1
1	0	1
2	3	4
3	8	12
4	14	26
5	20	46
6	31	77
7	32	109
8	28	137
9	11	148
10	5	153

Total: 153

d

x	f	*c.f.*
15	4	4
20	8	12
25	3	15
30	9	24
35	7	31
40	10	41
45	15	56
50	8	64
55	10	74
60	2	76
65	4	80

Total: 80

2 a 1 **b** 2 **c** 7 **d** 12 **e** 5 **f** 3

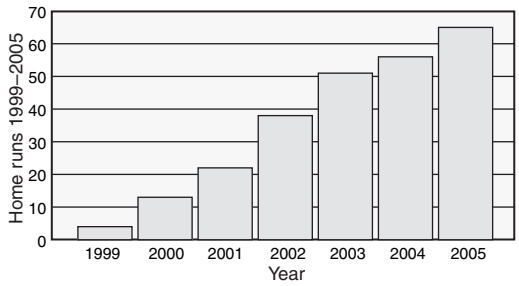
3 a 2 **b** 9 **c** 22 **d** 3 **e** 4 **f** 19 **g** 35 **h** 8

No. Although serving more aces makes the job of winning easier, it does not mean that Federer won. In fact, Nadal won the tournament. More information is required.

4 a

Year	*Home runs*	*c.f.*
1999	4	4
2000	9	13
2001	9	22
2002	16	38
2003	13	51
2004	5	56
2005	9	65

b



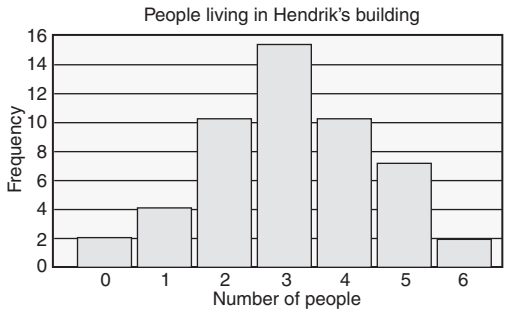
c As this is in the middle it may have been the Peak of his career.

5 a 6 and 0

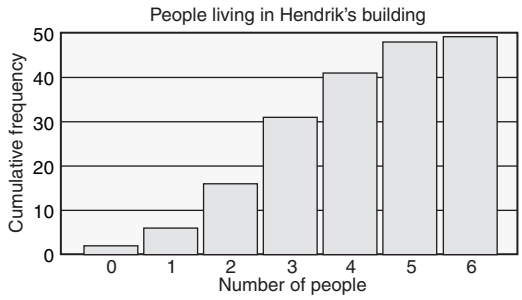
b

No of people	*Frequency*	*c.f.*
0	2	2
1	4	6
2	10	16
3	15	31
4	10	41
5	7	48
6	2	50

c



d



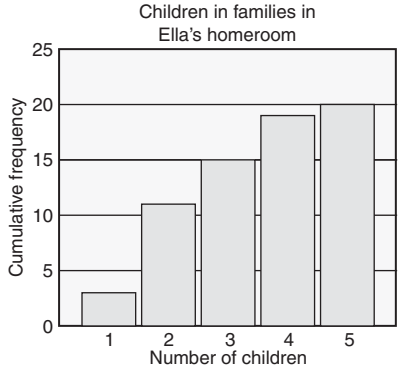
e 50 **f** 156 **g** 2

6 a 5 and 1

b

No of children	*Frequency*	*c.f.*
1	3	3
2	8	11
3	4	15
4	4	19
5	1	20

c



d 20 **e** 52

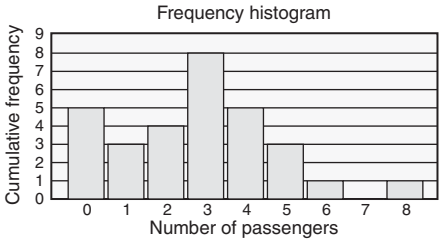
f All families must have at least 1 child in them.

7 a 0 and 8

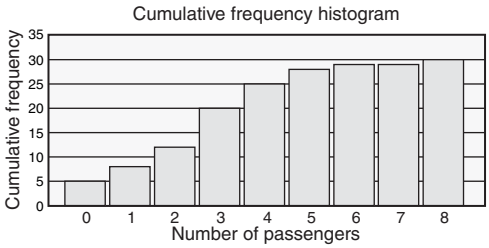
b

No of passengers	*Frequency*	*c.f.*
0	5	5
1	3	8
2	4	12
3	8	20
4	5	25
5	3	28
6	1	29
7	0	29
8	1	30

c



d



e 84 **f** Some passengers got off at stops as well.

8 a

No of calls	*Frequency*
0	1
1	3
2	4
3	8
4	5
5	4
6	3

b 28 **c** 93 **d** 3

Exercise 13:03

1

	Range	*Mode*	*Mean*	*Median*
a	5	5	5·6	5
b	5	2	3·6 (1 dec. pl.)	3
c	4	6, 8	7	7
d	4	5	6·8 (1 dec. pl.)	7
e	8	17	18·9 (1 dec. pl.)	19
f	4	12	11·1 (1 dec. pl.)	12
g	7	29	29·2 (1 dec. pl.)	29
h	6	51, 53	50·6 (1 dec. pl.)	51
i	5	21	22·7 (1 dec. pl.)	22
j	9	88, 93	91·4 (1 dec. pl.)	91·5
k	0·4	5·1	5·1 (1 dec. pl.)	5·1
l	5	99, 102	100·5	100·5

2 a 21·8 **b** 194·3 **c i** 32·68 **ii** 32·66

3 a 181 **b** 394

4 In this case, the mode is also the lowest price paid. The mean is greatly affected by the $3 865 000 price. The median is the middle score, and so is the best measure of central tendency in this case.

5 The mode is most useful to the dress manufacturer as this indicates the size most often purchased.

6 He should use the mean because this takes into account every mark and he is concerned with overall performance.

7 Mode: 79, 82 Mean: 81·1 Median: 79
The median is the best measure of Alan's performance.

8 a i Mean = 74; 4 **ii** Mean = 76; 1
b Not necessarily. **9** 8·22

10 a $\bar{x}$ = 8·17 (corr. to 2 dec. pl.);
Score = 22·88 (corr. to 2 dec. pl.)
b 20·8

Exercise 13:04

1 a 6·75 **b** 12·2 **c** 40·35 **d** 66·05 **e** 287·95 **f** 4·885

2 a 7 **b** 13·3 (1 dec. pl.) **c** 31·0 (1 dec. pl.) **d** 24·9 (1 dec. pl.) **e** 5·5 (1 dec. pl.)

3 a range 4, mode 7, median 7 **b** range 5, mode 13, median 13 **c** range 6, mode 31, median 31
d range 6, mode 24, median 24 **e** range 1, mode 5·6, median 5·6

4 a 100 **b** 7 **c** 6·91

5

No. of goals (x)	*Frequency (f)*	*fx*
0	3	0
1	6	6
2	6	12
3	3	9
4	3	12
5	2	10
6	1	6
Totals	24	55

range = 6
mode = 1, 2
mean = 2·3 (correct to 1 dec. pl.)

6

Outcome (x)	*Frequency (f)*	*fx*
1	2	2
2	4	8
3	3	9
4	6	24
5	8	40
6	7	42
7	5	35
8	1	8
Totals	36	168

a 7 **b** 5 **c** 5
d 4·7 (correct to 1 dec. pl.)

7

Outcome (x)	*Tally*	*Frequency (f)*	*fx*
8	|	1	8
9		0	0
10	||||	4	40
11	𝍸	5	55
12	𝍸 |	6	72
13	𝍸 ||	7	91
14	𝍸 ||	7	98
15	𝍸 |	6	90
16	𝍸 |	6	96
17	𝍸 |||	8	136
18	𝍸 ||	7	126
19	𝍸 |||	8	152
20	𝍸	5	100
Totals		70	1064

range = 12; mode = 17 and 19; mean = 15·2

8

x	*Tally*	*f*	*fx*	*c.f.*
3	||	2	6	2
4	𝍸	5	20	7
5	||||	4	20	11
6	𝍸 ||	7	42	18
7	𝍸 |	6	42	24
8	||||	4	32	28
9	||	2	18	30
Totals		30	180	

a 6 **b** 6 **c** 6 **d** 6

9

Number (x)	*Tally*	*f*	*fx*	*c.f.*
2	|||	3	6	3
3	𝍸	5	15	8
4	𝍸 ||||	9	36	17
5	𝍸 𝍸 ||	12	60	29
6	𝍸 𝍸 ||||	14	84	43
7	𝍸 𝍸 𝍸 ||	17	119	60
8	𝍸 𝍸 𝍸	15	120	75
9	𝍸 𝍸 |	11	99	86
10	𝍸 |||	8	80	94
11	||||	4	44	98
12	||	2	24	100
Totals		100	687	

a mode = 7; mean = 6·87 **b** 7 **c** 43

10 **a** 3 **b** 13 **c** 7
d 6 **e** 7 **f** 5

Spreadsheet Investigation 13:04

1 =SUM(E2:E8); =SUM(H2:H8); =SUM(K2:K8); =SUM(N2:N8)
2 =SUM(B10,E10,H10,K10,N10)
3 Sunday $3930; Wednesday $1420

Exercise 13:05

1 **a** 8 **b** 18 **c** 35 **d** 6·5

2

Class	*c.c.*	*Tally*	*Frequency (f)*	*f* × *c.c.*	*c.f.*
1–5	3	|||	3	9	3
6–10	8	𝍸 ||	7	56	10
11–15	13	𝍸 𝍸	10	130	20
16–20	18	𝍸 |||	8	144	28
21–25	23	||	2	46	30

$\Sigma f = 30 \quad \Sigma (f \times \text{c.c.}) = 385$

Modal class = 11–15; mean = 12·8 (correct to 1 decimal place); median class = 11–15

3 **a**

Class	*c.c.*	*Tally*	*Frequency (f)*	*f* × *c.c.*	*c.f.*
141–145	143	||	2	286	2
146–150	148	||||	4	592	6
151–155	153	𝍸	5	765	11
156–160	158	𝍸 ||	7	1106	18
161–165	163	𝍸 |||	8	1304	26
166–170	168	𝍸 |	6	1008	32
171–175	173	𝍸 |	6	1038	38
176–180	178	||	2	356	40

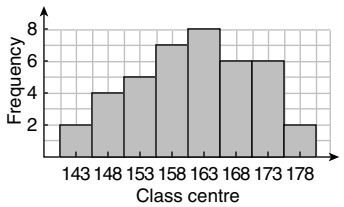
$\Sigma f = 40 \quad \Sigma (f \times \text{c.c.}) = 6455$

b highest score = 179,
lowest score = 143,
range = 36

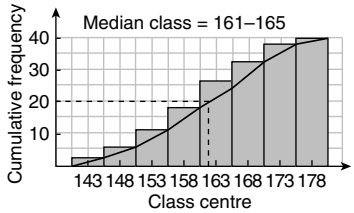
c modal class = 161–165

d $\bar{x} = 161{\cdot}4$
(correct to 1 decimal place)

e



f



g A class interval of 10 would have resulted in only 4 classes, and the patterns of cluster and spread in the data would have been more difficult to distinguish.

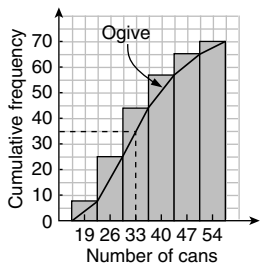
4 **a**

No. of cans sold (x)	*c.c.*	*Tally*	*Frequency (f)*	*f* × *c.c.*	*c.f.*
16–22	19	𝍸 ||	7	133	7
23–29	26	𝍸 𝍸 𝍸 |||	18	468	25
30–36	33	𝍸 𝍸 𝍸 |||	18	594	43
37–43	40	𝍸 𝍸 𝍸	15	600	58
44–50	47	𝍸 |||	8	376	66
51–57	54	||||	4	216	70

$\Sigma f = 70 \quad \Sigma (f \times \text{c.c.}) = 2387$

b 10 **c** 34·1

d median class = 30–36



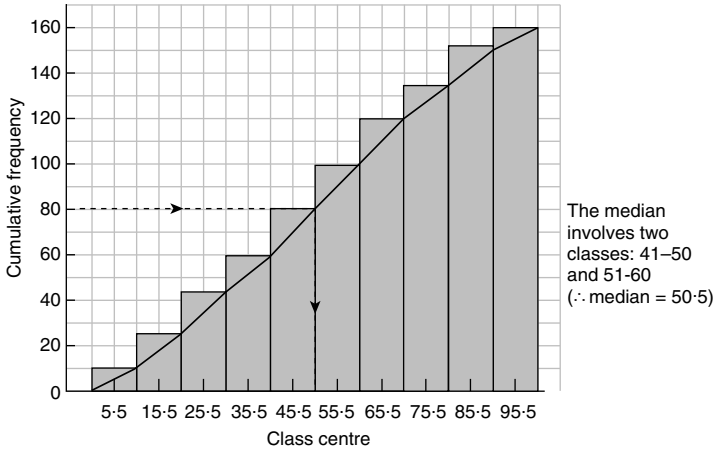
5 a

Class	*c.c.*	*Tally*	*Frequency (f)*	*f* × *c.c.*	*c.f.*
1–10	5·5	𝍸 ||||	9	49·5	9
11–20	15·5	𝍸 𝍸 𝍸	15	232·5	24
21–30	25·5	𝍸 𝍸 𝍸 ||||	19	484·5	43
31–40	35·5	𝍸 𝍸 𝍸 ||	17	603·5	60
41–50	45·5	𝍸 𝍸 𝍸 𝍸	20	910	80
51–60	55·5	𝍸 𝍸 𝍸 ||	17	943·5	97
61–70	65·5	𝍸 𝍸 𝍸 𝍸 |	21	1375·5	118
71–80	75·5	𝍸 𝍸 𝍸 |	16	1208	134
81–90	85·5	𝍸 𝍸 𝍸 |||	18	1539	152
91–100	95·5	𝍸 |||	8	764	160

$\Sigma f = 160 \quad \Sigma (f \times \text{c.c.}) = 8110$

b **i** 99 – 3 = 96 **ii** 61–70
iii 50·7 (1 decimal place)

c



d **i** 16·25%
ii 15%
iii 50%

Investigation 13:05

1 a 670 000 **b** 620 000 **c** 390 000
2 a 680 000 **b** 720 000 **c** 590 000
3 a **i** 910 000 **ii** 1 660 000
b 82%
4 87%
5 The number increased due to migration over the 20-year period.

Diagnostic Test 13

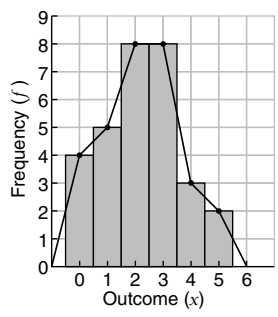
1 a

Outcome (x)	*Tally*	*f*	*c.f.*
0	||||	4	4
1	𝍸	5	9
2	𝍸 |||	8	17
3	𝍸 |||	8	25
4	|||	3	28
5	||	2	30

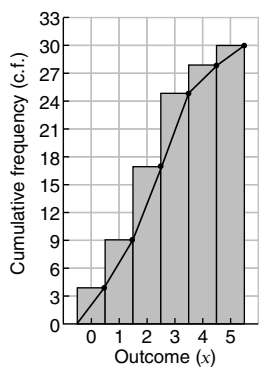
Total: 30

b 2 **c** 25

d



e



f Range is 5
g Modes are 2 and 3 (bimodal),
median is 2,
mean is 2·23 (to 2 decimal places)

2 a 28·23 **b** 4·36 (to 2 decimal places)
3 a 7 **b** 13 **c** 2
4 a 5 **b** 7 **c** 14

5 a

Class	*c.c.*	*Tally*	*f*	*f* × *c.c.*	*c.f.*
10–19	14·5	𝍸 ||	7	101·5	7
20–29	24·5	𝍸 𝍸	10	245	17
30–39	34·5	𝍸 𝍸 ||	12	414	29
40–49	44·5	|||	3	133·5	32
50–59	54·5	|	1	54·5	33
		Totals:	33	948·5	

Modal class 30–39
Mean 28·74 (to 2 decimal places)
Median class 20–29

b

Class	*c.c.*	*f*	*f* × *c.c.*	*c.f.*
1–5	3	11	33	11
6–10	8	18	144	29
11–15	13	22	286	51
16–20	18	10	180	61
21–25	23	7	161	68
	Totals:	68	804	

Modal class 11–15
Mean 11·82 (correct to 2 decimal places)
Median class 11–15

c

Class	*c.c.*	*f*	*f* × *c.c.*	*c.f.*
150–154	152	3	456	3
155–159	157	6	942	9
160–164	162	16	2592	25
165–169	167	9	1503	34
170–174	172	6	1032	40
	Totals:	40	6525	

Modal class 160–164
Mean 163·13 (correct to 2 decimal places)
Median class 160–164

13A Revision Assignment

1 a

Number of faulty products	*Tally*	*Frequency*	*Cum. frequency*
1	𝍸	5	5
2	𝍸 |||	8	13
3	𝍸 𝍸	10	23
4	𝍸 ||||	9	32
5	𝍸 |	6	38
6	|	1	39
7	|	1	40

b Mode = 3; Mean = 3·25; Median = 3

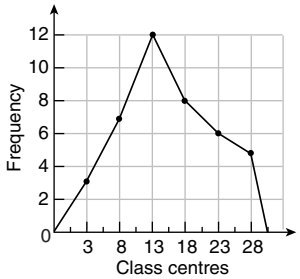
2 a 4 **b** 43 **c** 8
d 40 **e** 72 **f** 24

g

Class	*c.c.*	*f*
30–39	34·5	3
40–49	44·5	5
50–59	54·5	4
60–69	64·5	7
70–79	74·5	8
80–89	84·5	7
90–99	94·5	6

3 a i 11–15 **ii** 15·68

b

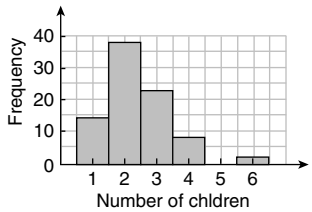


c No **d** 11–15

4 a 2 **b** 15 **c** 2

5 a 84 **b** 52

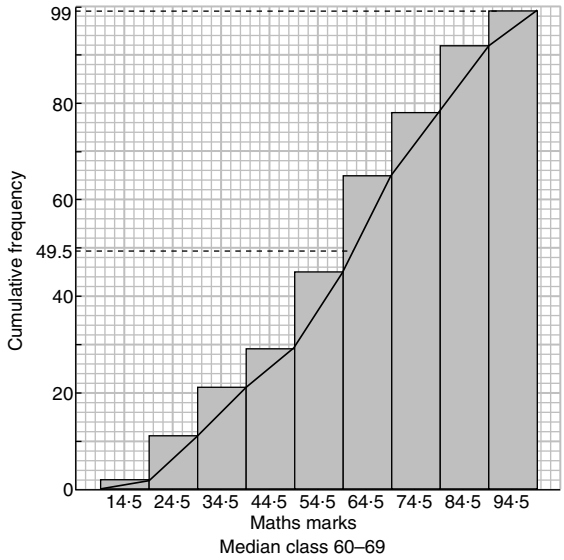
c



6 a

Class	*Class centre (c.c.)*	*Frequency (f)*	*Cum. frequency*	*f × c.c.*
10–19	14·5	2	2	29
20–29	24·5	9	11	220·5
30–39	34·5	10	21	345
40–49	44·5	8	29	356
50–59	54·5	16	45	872
60–69	64·5	20	65	1290
70–79	74·5	13	78	968·5
80–89	84·5	14	92	1183
90–99	94·5	7	99	661·5

i Modal class 60–69
ii Mean 59·9 (correct to 1 dec. pl.)



13B Working Mathematically

1 The window has 4 "green" squares. The clear glass has an octagon, 4 hexagons and 4 pentagons. Around the perimeter of the window are squares and rectangles.

2 **a** *parallel* lines **b** *perpendicular* lines **c** *vertical, horizontal*
d *concurrent* lines **e** *complementary* angles **f** *supplementary* angles
g *vertically opposite* angles **h** *360°* **i** *bisecting* and interval **j** *CD* is *perpendicular* to *AB*

3 48 **4** 120 minutes

5 Perimeter = 47·7 cm (correct to 1 dec. pl.); Area = 99·6 cm^2 (correct to 1 dec. pl.)

6 **a** 8° **b** 11:20 am **c** 16·5° **d** 3 h **e** 9–10 am, 11–12 am

Chapter 14: Probability

Prep Quiz 14:01

1 A **2** C **3** B **4** A **5** A **6** B **7** In most classes, C **8** B **9** A **10** Answers will vary.

Exercise 14:01

1 **a** (1 and 2), (1 and 3), (1 and 4), (1 and 5), (1 and 6), (2 and 3), (2 and 4), (2 and 5), (2 and 6), (3 and 4), (3 and 5), (3 and 6), (4 and 5), (4 and 6), (5 and 6)
b impossible **c** not likely **d** certain **e** even chance
f likely as 9 of the 15 outcomes have a sum that is odd.

2 **a** 1, 2, 3, 4, 5, 6, 7, 8 **b** 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
c (blue, blue), (blue, green), (blue, red), (green, red), (green, green)
d (5 and 5), (5 and 6), (5 and 9), (5 and 10), (6 and 9), (6 and 10), (9 and 10)

3 **a** very unlikely **b** very likely **c** unlikely **d** even chance
e impossible **f** certain

4 **a** very unlikely **b** very likely **c** unlikely (occurs about 28% of the time)
d likely **e** impossible **f** certain

5 **a** likely **b** unlikely **c** very unlikely
d certain; c(red), b(green), a(blue), d(coloured)

6 **a** very unlikely **b** unlikely (occurs 3 out of 10) **c** certain
d impossible; d(both are larger than 9), a(are both fives), b(have a sum equal to 15), c(have a sum greater than 9)

7 **a** Answers will vary, however, any 5 from the following might apply: who wins toss; recent form of the players; the country where it is played; the condition of the wicket; the weather; the ability of the captains.
b Answers will vary. **c** Yes **d** No, because it may be a draw.

8 **a** Unreasonable **b** Reasonable **c** Reasonable **d** Unreasonable
e Unreasonable **f** Reasonable **g** Unreasonable

9 a If the horse was truly a certainty, then it could not have been beaten.
b Difference between numbers is 100 000! (Mine was not the next ticket sold.) 'Closeness' to winning a lottery does not equate with proximity of numbers, but rather to physical position of balls or tickets in the barrel, etc.
c If it was a truly impossible position, then winning would not have been possible.
Note: All of these expressions are not meant to be interpreted literally. They involve emotive exaggerations (ie poetic licence) in order to stress a point of view.
10 This statement is false because some letters (such as the X) are rarely used to begin names.

Prep Quiz 14:02

1 10 **2** 3 **3** $\frac{3}{10}$ **4** $\frac{1}{5}$ **5** $\frac{1}{2}$ **6** 6 **7** $\frac{3}{5}$ **8** $\frac{1}{3}$ **9** $\frac{4}{5}$ **10** $\frac{12}{25}$

Exercise 14:02

1 a $\frac{7}{10}$ **b** $\frac{1}{10}$ **c** $\frac{1}{20}$ **d** $\frac{3}{10}$ **e** $\frac{3}{20}$ **2 a** Brown **b** Yellow **c** $\frac{1}{4}$
3 $\frac{1}{20}$, $\frac{19}{20}$ **4 a** $\frac{5}{21}$ **b** $\frac{1}{14}$ **c** $\frac{1}{2}$; 24%, 7%, 50% **5 a** $\frac{9}{50}$ **b** $\frac{1}{5}$ **c** $\frac{17}{20}$ **d** $\frac{1}{5}$
6 a i $\frac{1}{10}$ **ii** $\frac{13}{30}$ **iii** $\frac{1}{3}$ **b** no **7 a i** $\frac{12}{25}$ **ii** $\frac{1}{10}$ **iii** $\frac{21}{50}$ **b** no
8 a $\frac{16}{20}$, $\frac{4}{5}$ or 80% **b** $\frac{1}{20}$ or 5% **c** $\frac{3}{20}$ or 15%
9 a C, A, B < D, E (Answers may vary.)
b (Answers will vary.) One possibility is P(A) ≒ 0·1, P(B) ≒ 0·5, P(C) ≒ 0·5, P(D) ≒ 0·9, P(E) ≒ 0·95.
10 a impossible **b** even chance **c** certain **d** even chance **e** certain **f** even chance

Prep Quiz 14:03

1 $\frac{7}{8}$ **2** $\frac{2}{3}$ **3** $\frac{3}{7}$ **4** $\frac{3}{16}$ **5** $\frac{1}{10}$ **6** $\frac{1}{4}$ **7** $\frac{1}{3}$ **8** $\frac{2}{3}$ **9** $\frac{5}{26}$ **10** $\frac{3}{10}$

Exercise 14:03

1 a $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$ **2 a** $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{7}{10}$ **d** $\frac{4}{5}$ **e** 0 **f** 1
3 a getting an *even number* **b** getting a *head* **c** getting a *6* **d** drawing a *club*, *diamond or heart*
e seeing *green or orange* **f** *losing or playing a draw* **g** choosing a *consonant*
4 a $\frac{1}{52}$ **b** $\frac{1}{13}$ **c** $\frac{1}{2}$ **d** $\frac{1}{4}$ **e** $\frac{1}{26}$ **f** $\frac{2}{13}$ **g** $\frac{3}{13}$
5 a $\frac{1}{26}$ **b** $\frac{5}{26}$ **c** $\frac{1}{13}$ **d** $\frac{4}{13}$ **6 a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{6}$ **d** 1 **e** 0 **f** $\frac{5}{6}$
7 a $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$ **d** $\frac{1}{3}$ **8 a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{8}$ **d** 0
9 a $\frac{13}{25}$ **b** $\frac{1}{5}$ **c** $\frac{8}{25}$ **d** $\frac{8}{25}$

Diagnostic Test 14 Probability

1 a 1, 2, 3, 4, 5, 6 **b** head, tail **c** 3, 5, 7
2 a $\frac{3}{44}$ **b** $\frac{41}{44}$ **c** 0, It is possible for us to play better than before and so score more than 59 points.
3 a $\frac{1}{2}$ **b** $\frac{5}{6}$ **c** 0 **d** $\frac{2}{3}$
4 a $\frac{2}{13}$ **b** $\frac{3}{26}$ **c** $\frac{5}{52}$ **d** $\frac{16}{52}$ or $\frac{13}{26}$ **e** $\frac{28}{52}$ or $\frac{7}{13}$ **f** $\frac{24}{52}$ or $\frac{6}{13}$

14 Revision Assignment

1 a $\frac{1}{6}$ **b** 0 **c** $\frac{1}{3}$ **d** $\frac{1}{2}$ **e** $\frac{1}{3}$ **2 a** $\frac{7}{20}$ **b** $\frac{1}{10}$ **c** $\frac{13}{20}$
3 a $\frac{2}{5}$ **b** $\frac{1}{10}$ **c** $\frac{1}{2}$ **d** 0 **4 a** $\frac{12}{25}$ **b** $\frac{1}{5}$ **c** $\frac{8}{25}$ **d** $\frac{13}{25}$
5 a i $\frac{3}{10}$ **ii** $\frac{1}{2}$
b No; the scores recorded so far are a limited sample. He theoretically could score any result from zero to 180.
6 No; the result of the game depends on the relative strengths of each team. Depending on the type of football, a draw might be much less likely than either team winning.

14 Working Mathematically

1 **a** *midpoint* **b** *number line* **c** *diagonals* **d** *vertices* **e** *hypotenuse*
f *180°* **g** *360°* **h** *[b]* $\alpha = \beta$ **i** $\alpha = 60°$ **j** *circumference*

2 **a** (1) $3x - 2$ (2) $5x - 4$ (3) $7x + 7$ (4) $4x - 6$ (5) $4x + 10$ (6) $4x - 6$ (7) $2x + 7$
(8) $6x - 6$ (9) $3x + 9$ (10) $6x - 3$ (11) $3x + 13$ (12) $3x - 7$

b (1) $x + 4$ (2) $3x + 8$ (3) $5x - 3$ (4) $2x + 12$ (5) $2x + 6$ (6) $2x + 8$ (7) 9
(8) $4x + 10$ (9) $x - 1$ (10) $4x + 11$ (11) $x + 1$ (12) $x + 13$

c (1) $5x - 7$ (2) $5x + 1$ (3) $7x - 3$ (4) $5x + 7$ (5) $6x + 9$ (6) $4x + 8$ (7) $3x + 3$
(8) $6x - 8$ (9) $4x$ (10) $6x + 11$ (11) $3x + 1$ (12) $4x + 6$

d (1) 6 (2) 20 (3) 16 (4) −64 (5) −16 (6) 4 (7) 20
(8) −32 (9) 96 (10) 224 (11) 8 (12) 1024

3 1 260 000 degrees **4** 8 years **5** Estimates should be reasonably close to these. **a** 44% **b** 70% **c** 87%

6 **a** **i** 16 m **ii** 22 m **iii** 28 m **b** **i** 72 m **ii** 36 m **c** **i** 20 m **ii** 36 m **iii** 56 m **d** **i** 36 m **ii** 72 m **iii** at least 36 m

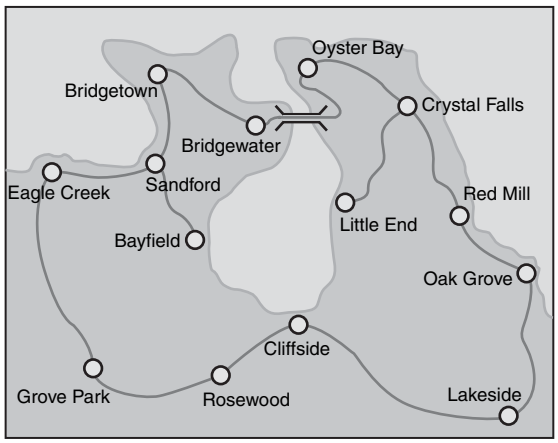
Chapter 15: Networks and Topology

Exercise 15:01

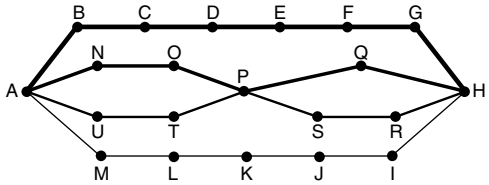
1 **a** Teacher to mark **b** No **c** An isomorphism
d You can travel between vertices (stations) more than one way.

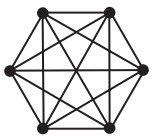
2 **a** Green **b** 16. They are where people can change from one line to another.
c The red line continues outside the Dublin area. **d** A subgraph of a greater network
e Not all the stations are named.

3 **a** One possible solution **b** The distance between stations and their actual location in the city
c Numbers showing either the distance between stations or the time taken to travel between stations



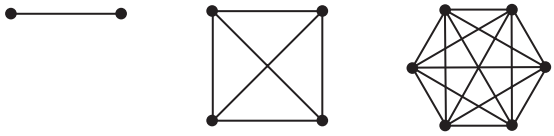
4 **a** One possible solution
b Isomorphism
c A, P and H. They are transport hubs.
d It would seem that travelling through 6 stops is the quickest. For example, through H and Q or A, N and O. However, there may not be the same time between stops so it is difficult to say which is the quickest route.



5 **a**  **b** 

6 **a** A tree, because there is only one way of getting from one vertex to another
b Geography – there may be mountains so people live in the valleys
c To cross the bay or this is where the city centre is
d 9 (including the AirBART shuttle)

7 **a** i, iii, iv, vi
b All vertices have an even degree, or there are 2 of odd degree.

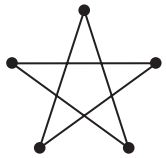
8 **a** 

b 1, only the graph of order 2
c No, there must only be 2 odd vertices but if there is an even number of vertices, they must all be odd.

9 **a** Yes

b 1 → A, 2 → C, 3 → E, 4 → G, 5 → B, 6 → D, 7 → F.

c one possible solution:



10 **a**

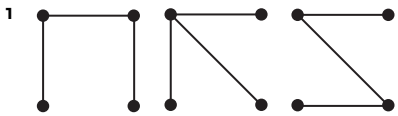
Graph	*Total of the degrees of all the vertices*	*Number of edges*
i	10	5
ii	12	6
iii	10	5
iv	20	10
v	22	11
vi	20	10

b The total of the degrees of all the vertices of a graph is twice the number of edges. $v = 2e$

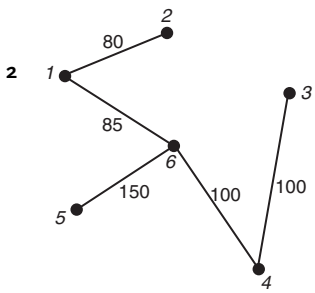
c Each edge has 2 ends. So each edge adds 2 to the total of all the vertices.

Exercise 15:01B

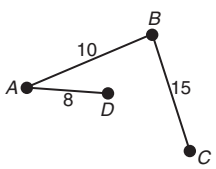
1



2



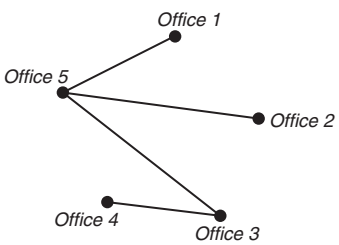
3 **a**



Minimum cost = 15 + 10 + 8
= 33 million dollars

b 13 million dollars

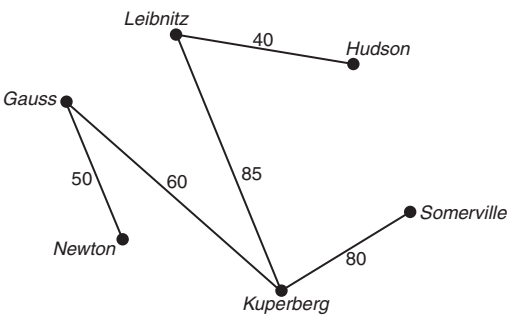
4 **a**



Minimum cost = 10 + 8 + 7 + 8
= 33
So $33 00

b No **c** In office 5

5 **a**



b The minimum flying times to join all the cities

c Kuperberg. It has the shortest flying times from the most cities.

d To get to some cities you have to fly through another one first.

e Newton → Gauss → Kuperberg → Leibnitz → Hudson. It takes 235 min instead of 120 min.

Prep Quiz 15:02

a *AB*
b No
c No. It is impossible to travel on both *CA* and *CD*.
d Yes, but only if it starts at C.

Exercise 15:02

1 a **i** 1 (D→B→A) **ii** 2 (C→D→B→A, C→A,)
iii 2 (B→A→E, B→A→C→D→E) **iv** 1 (B→A→C→D)
b Yes, C→D→B→A→E
c No **d** No **e** E

2 a Teacher to mark
b 1 skips because water can be boiled at the same time as getting a cup and adding coffee; step 4b skips because maybe the person doesn't take milk and sugar.
c If the wrong cup is chosen, or the wrong coffee
d You can't take the water or the milk and sugar out once they are added and you can't unstir the coffee.
e If the person didn't take milk and sugar in their coffee
f Yes, if the person didn't take milk and sugar.
g There is a problem if the person only has milk or sugar and not both.
h Teacher to mark

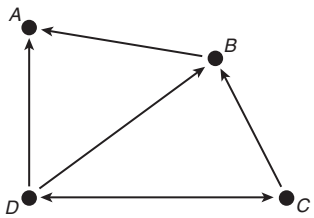
3 a

From \ To	A	B	C	D
A	0	1	1	0
B	0	0	1	0
C	1	0	0	1
D	1	0	0	0

b

From \ To	A	B	C	D	E
A	0	0	1	0	1
B	1	0	0	0	0
C	1	0	0	1	0
D	0	1	0	0	1
E	0	0	0	0	0

4 A possible solution



5 a At E, because you can't travel from E to any of the other landmarks.
b A→D→C→B→F→E

Exercise 15:03

1 Genus 0: 1, 2, 3, 5, 7
Genus 1: 0, 4, 6, 9
Genus 2: 8

2 Genus 0: C, E, F, G, H, I, J, K, L, M, N, S, T, U, V, W, X, Y, Z
Genus 1: A, D, O, P, Q, R
Genus 2: B

3 A, D, G are all genus 0; C, J, I are all genus 1; B, E, F, H are all genus 2

4 Genus 0: dinner plate, milk bottle, fork, bowl
Genus 1: bottle opener, teacup, doughnut, sewing needle, wooden spoon
Genus 2: trophy, 360º protractor, dinner tray

5 **a** They have been stretched.
b Yes
c So they look normal when viewed from an angle like the grandstand or the TV
d ANAMORPHOSIS
e Give Way. So that it looks like normal writing to drivers who look at it from an angle

6 All answers depend on the individual – they should be marked by the teacher.

Diagnostic Test 15

1 4 **2** 55 **3** 3 **4** **i** Yes *BDCA* **ii** Yes *BDCAB* **iii** No

5 Teacher to mark **6** Teacher to mark **7** Teacher to mark. 30

8

	A	B	C	D
A	0	1	1	1
B	0	0	1	0
C	1	1	1	1
D	0	0	0	0

9 **i** Yes **ii** No **iii** No

10 The same number of holes

Working Mathematically 15

1 600 m

2 80 m

3 **a** 85% **b** \$300

4 88%

5 80°

6 **a** $\frac{1}{11}$ **b** $\frac{7}{22}$

7 24

8 \$71.10

Revision Assignment 15A

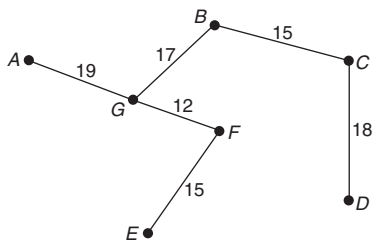
1 21 **2** 4 **3** Yes **4** **i** No **ii** No

5 **a** A weighted graph
c The minimum length of edges that joins all the vertices

6 **a** A graph in which the edges only go in particular directions
b To be marked by teacher

7 The number of holes they have

b



Answers to ID Cards

ID Card 1 (Metric Units) page xiv

1 *metres*	**2** *decimetres*	**3** *centimetres*	**4** *millimetres*
5 *kilometres*	**6** *square metres*	**7** *square centimetres*	**8** *square kilometres*
9 *hectares*	**10** *cubic metres*	**11** *cubic centimetres*	**12** *seconds*
13 *minutes*	**14** *hours*	**15** *metres per second*	**16** *kilometres per hour*
17 *grams*	**18** *milligrams*	**19** *kilograms*	**20** *tonnes*
21 *litres*	**22** *millilitres*	**23** *kilolitres*	**24** *degrees Celsius*

ID Card 2 (Symbols) page xiv

1 *is equal to*	**2** *is approximately equal to*	**3** *is not equal to*	**4** *is less than*
5 *is less than or equal to*	**6** *is not less than*	**7** *is greater than*	**8** *is greater than or equal to*
9 *4 squared*	**10** *4 cubed*	**11** *the square root of 2*	**12** *the cube root of 2*
13 *is perpendicular to*	**14** *is parallel to*	**15** *is congruent to*	**16** *is similar to*
17 *per cent*	**18** *therefore*	**19** *for example*	**20** *that is*
21 *pi*	**22** *the sum of*	**23** *the mean*	**24** *probability of event E*

ID Card 3 (Language) page xv

1 $6 - 2 = 4$	**2** $6 + 2 = 8$	**3** $6 \div 2 = 3$	**4** $6 - 2 = 4$	**5** $6 \div 2 = 3$
6 *2*	**7** *6*	**8** $6 \times 2 = 12$	**9** $6 - 2 = 4$	**10** $6 \times 2 = 12$
11 $2 + 6 = 8$	**12** $6 - 2 = 4$	**13** $6^2 = 36$	**14** $\sqrt{36} = 6$	**15** $6 - 2 = 4$
16 $6 \times 2 = 12$	**17** $(6 + 2) \div 2 = 4$	**18** $6 + 2 = 8$	**19** $6^2 = 36$	**20** $6 - 2 = 4$
21 $6 - 2 = 4$	**22** $6 + 2 = 8$	**23** $6 \div 2 = 3$	**24** $6 + 2 = 8$	

ID Card 4 (Language) page xvi

1 *square*	**2** *rectangle*	**3** *parallelogram*	**4** *rhombus*
5 *trapezium*	**6** *regular pentagon*	**7** *regular hexagon*	**8** *regular octagon*
9 *kite*	**10** *scalene triangle*	**11** *isosceles triangle*	**12** *equilateral triangle*
13 *circle*	**14** *oval (or ellipse)*	**15** *cube*	**16** *rectangular prism*
17 *triangular prism*	**18** *square pyramid*	**19** *rectangular pyramid*	**20** *triangular pyramid*
21 *cylinder*	**22** *cone*	**23** *sphere*	**24** *hemisphere*

ID Card 5 (Language) page xvii

1 *point A*	**2** *interval AB*	**3** *line AB*	**4** *ray AB*
5 *collinear* points	**6** *midpoint*	**7** *number line*	**8** *diagonals*
9 *acute-angled triangle*	**10** *right-angled triangle*	**11** *obtuse-angled triangle*	**12** *vertices*
13 ΔABC	**14** *hypotenuse*	**15** $180°$	**16** $a° + b°$
17 $360°$	**18** *[b]* $a° = b°$	**19** $a° = 60°$	**20** $3 \times 180° = 540°$
21 *AB is a diameter: OC is a radius.*		**22** *circumference*	**23** *semicircle*
24 *AB* is a *tangent*. *CD* is an *arc*. *EF* is a *chord*.			

ID Card 6 (Language) page xviii

1 *parallel* lines	**2** *perpendicular* lines	**3** *vertical, horizontal*	**4** *concurrent* lines
5 angle *ABC* or *CBA*	**6** *acute* angle	**7** *right* angle	**8** *obtuse* angle
9 *straight* angle	**10** *reflex* angle	**11** *revolution*	**12** *adjacent* angles
13 *complementary* angles	**14** *supplementary* angles	**15** *vertically opposite* angles	**16** $360°$
17 *transversal*	**18** *corresponding* angles	**19** *alternate* angles	**20** *co-interior* angles
21 *bisecting* an interval	**22** *bisecting* an angle	**23** $\angle CAB = 60°$	**24** *CD* is *perpendicular* to *AB*

ID Card 7 (Language) page xix

1 *anno Domini*	**2** *before Christ*	**3** *ante meridiem*	**4** *post meridiem*
5 *hectare*	**6** *regular* shapes	**7** *net* of a cube	**8** *cross-section*
9 *face*	**10** *vertex*	**11** *edge*	**12** *axes of symmetry*
13 *reflection (or flip)*	**14** *translation (or slide)*	**15** *rotation (or turn)*	**16** *tessellation*
17 *coordinates*	**18** *tally*	**19** *picture* graph	**20** *column* graph
21 *line* graph	**22** *sector (or pie)* graph	**23** *bar* graph	**24** *scatter diagram*

Index

Most references are to chapter topics, eg 1:03 refers to Chapter 1: Section 3 (Fractions).
A:01 … to D:01 refers to the appendix section of the CD.
The following abbreviations are used:

D & D	Drag and Drops
FS	Fun Spot
Inv	Investigation
MT	Mathematical Terms
RM	Reading Mathematics
Tech	Technology Applications
CW	Challenge Worksheet (CD)
Challenge	Challenge Activity (Book)
PA	Practical Activity

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Foundation Worksheet

1:05 | Decimals and Fractions

Name: _____ **Class:** _____

Examples

Change these fractions into decimals.

1 $\frac{7}{8} = 7 \div 8$
$= 0{\cdot}875$

2 $\frac{5}{12} = 5 \div 12$
$= 0{\cdot}4166\ldots$
$= 0{\cdot}41\dot{6}$

3 $\frac{7}{10} = 0{\cdot}7$

4 $\frac{64}{1000} = 0{\cdot}064$

1 place for over 10
2 places for over 100
3 places for over 1000

Change these decimals into simplest fractions.

5 $0{\cdot}8 = \frac{8}{10}$ $(\div 2)$
$= \frac{4}{5}$

6 $0{\cdot}25 = \frac{25}{100}$ $(\div 25)$
$= \frac{1}{4}$

7 $0{\cdot}437 = \frac{437}{1000}$

8 $0{\cdot}28 = \frac{28}{100}$ $(\div 4)$
$= \frac{7}{25}$

Note: Your calculators can help with these—check with your teacher.

Exercise

1 Write these fractions as decimals.

a $\frac{1}{5}$	**b** $\frac{17}{100}$	**c** $\frac{9}{10}$	**d** $\frac{53}{100}$	**e** $\frac{7}{20}$
f $\frac{41}{50}$	**g** $\frac{99}{100}$	**h** $\frac{1}{10}$	**i** $\frac{4}{25}$	**j** $\frac{1}{2}$
k $\frac{3}{40}$	**l** $\frac{1}{8}$	**m** $\frac{127}{1000}$	**n** $\frac{5}{6}$	**o** $\frac{2}{3}$

2 Write these decimals as fractions in simplest form.

a 0·6	**b** 0·95	**c** 0·24	**d** 0·355	**e** 0·08
f 0·19	**g** 0·3	**h** 0·473	**i** 0·73	**j** 0·007
k 0·75	**l** 0·025	**m** 0·55	**n** 0·32	**o** 0·175

Fun Spot 1:05 | What's a mistake?

Write each fraction as a decimal.
Place the letter in the box above the correct answer.

A $\frac{1}{4}$	**D** $\frac{6}{100}$	**E** $\frac{3}{1000}$	**G** $\frac{67}{100}$
H $\frac{13}{20}$	**K** $\frac{8}{10}$	**M** $\frac{43}{50}$	**R** $\frac{427}{1000}$
S $\frac{2}{5}$	**T** $\frac{3}{20}$	**U** $\frac{21}{25}$	

								,											
0·86	0·427	0·15	0·25	0·8	0·003	0·4	0·06	0·25	0·84	0·67	0·65	0·15	0·003	0·427

Answers can be found in the Interactive Student CD.

Foundation Worksheet

1:09 | Approximation

Name: _____ **Class:** _____

Examples

Approximate (round off):

1 5·732 to 2 decimal places
3 is in the 2nd decimal place
The next number is less than 5
∴ leave the 2nd place, ie 5·73

2 1790 to the nearest hundred
7 is in the hundreds place
The next number is more than 5
∴ round up the hundreds digit, ie 1800

3 165·852 to 1 decimal place
8 is in the first decimal place
The next number is 5 or more
∴ round up the first place, ie 165·9

4 76·059 to the nearest integer
6 is the first integer
The next number is less than 5
∴ leave the integer, ie 76

Exercise

1 Write 7463·9 to the nearest:
a integer **b** ten **c** hundred

2 Write 6·4937 correct to:
a 3 decimal places **b** 2 decimal places **c** 1 decimal place

3 Approximate 78·375 to:
a the nearest integer **b** 1 decimal place **c** 2 decimal places

4 Approximate 70 494 to the nearest:
a thousand **b** hundred **c** ten

5 Round off 62·905 84 correct to:
a 3 decimal places **b** 1 decimal place **c** 2 decimal places

6 Round off 76 538 to the nearest:
a ten **b** hundred **c** thousand

7 Approximate 128·0497 to:
a the nearest integer **b** the nearest ten **c** 3 decimal places

8 Approximate 96·8624 to:
a the nearest integer **b** 1 decimal place **c** 3 decimal places

Answers can be found in the Interactive Student CD.

Foundation Worksheet

1:10 | Estimation

Name: _____ **Class:** _____

Examples

1 $20{\cdot}23 \div 2{\cdot}71$
$\doteqdot 21 \div 3$
$\doteqdot 7$

Choose numbers that allow simple calculations. Remember, you are after rough answers only.

2 $186 + 720 + 413$
$\doteqdot 200 + 700 + 400$
$\doteqdot 1300$

3 $9{\cdot}7 \times 18{\cdot}5$
$\doteqdot 10 \times 20$
$\doteqdot 200$

4 $84 \div 23 \times 276 \doteqdot 80 \div 20 \times 300$
$\doteqdot 4 \times 300$
$\doteqdot 1200$

5 $15{\cdot}32 \times 11{\cdot}3^2 \doteqdot 15 \times 10^2$
$\doteqdot 15 \times 100$
$\doteqdot 1500$

Exercise

1 Estimate:

a $4{\cdot}9 + 10{\cdot}2$ **b** $6{\cdot}15 \times 9{\cdot}89$ **c** $53{\cdot}7 \times 10{\cdot}3$

d $68{\cdot}7 + 23{\cdot}4$ **e** $93{\cdot}04 - 18{\cdot}17$ **f** $8{\cdot}2^2$

g $42{\cdot}7 \times 38{\cdot}1$ **h** $192 + 426$ **i** 976×12

2 Find an approximate answer to the following.

a $16{\cdot}1 \div 7{\cdot}9$ **b** $7{\cdot}1^2 \div 9{\cdot}9$ **c** $57{\cdot}03 \div 12{\cdot}1$

d $52 \times 38 \div 21$ **e** $1603 - 186 - 712$ **f** $42 \times 31 \times 18$

g $23{\cdot}7 + 5{\cdot}6^2$

Fun Spot 1:10 | Why did the house have to call a doctor?

Write the letter above the approximation for each number.

A 9·27	**B** 57	**C** 101	**D** 3·03	**E** 76
H 784	**I** 7·14	**N** 186	**O** 4·64	**P** 880
S 41·1	**T** 490	**U** 313	**W** 18·4	

60	80	100	10	300	40	80	7	500	800	9	3		

20	7	200	3	5	20	900	9	200	80	40

Answers can be found in the Interactive Student CD.

Foundation Worksheet

4:01 | Collecting Like Terms

Name: _____ **Class:** _____

Examples

1 $5a + 3a = (5 + 3)a$
$= 8a$

2 $6c - 8c = (6 - 8)c$
$= -2c$

In algebra you do your counting with letters.

3 $7x + 4x - x = (7 + 4 - 1)x$
$= 10x$

Exercise

1 Simplify:

a $8 - 10$ **b** $-6 - 2$ **c** $-4 + 7$ **d** $7 + (-2)$

e $10 - 8 - 6$ **f** $-9 + 4$ **g** $6 - (-4)$ **h** $-3 + 7 + 9$

2 Complete:

a $3x + 4x = (\ldots + \ldots)x$
$= \ldots$

b $10d - 3d = (\ldots - \ldots)d$
$= \ldots$

c $5p + p = (\ldots + \ldots)p$
$= \ldots$

d $9y + 8y = (\ldots + \ldots)y$
$= \ldots$

e $-4k - 7k = (\ldots - \ldots)k$
$= \ldots$

f $5w - 8w = (\ldots - \ldots)w$
$= \ldots$

3 Simplify:

a $9a - 2a$ **b** $10b - b$ **c** $4x + 5x$

d $3y - 8y$ **e** $3a + 8a$ **f** $-2x + 5x$

g $-7t - 3t$ **h** $13n - 6n$ **i** $2p - 7p$

j $-4m + m$ **k** $9c - 9c$ **l** $11y - 10y$

m $4d - 5d - 6d$ **n** $5x + 8x - 6x$ **o** $3h - 6h + h$

Fun Spot 3:01 | How does a geologist relax?

Simplify each of the following and match the letters with the answers below.

C $5a - a$ **E** $2a + 7a$ **H** $4a - 8a$ **I** $a + a$

K $x - 3x$ **L** $6x + 2x$ **M** $15x - 5x$ **N** $-7x + 3x$

O $y + 2y + 5y$ **R** $-4y - 3y - 2y$ **S** $6y - 4y - 2y$ **T** $6y - 5y + 4y$ **U** $-8y + 10y - 9y$

$-4a$	$9a$	$8x$	$2a$	0	$5y$	$9a$	$-4x$	0	$5y$	$8y$	$-9y$	$8y$	$4a$	$-2x$	$10x$	$-7y$	0	$2a$	$4a$

Foundation Worksheet

4:02 | Multiplying Pronumerals

Name: _____ **Class:** _____

Examples

1 $5 \times c = 5c$

2 $3 \times 4m = 12m$

3 $a \times b = ab$

4 $w \times w = w^2$

5 $-8x \times 6 = -8 \times 6 \times x$
$= -48x$

6 $5p \times q = 5 \times p \times q$
$= 5pq$

7 $9h \times \frac{1}{3} = 9 \times \frac{1}{3} \times h$
$= 3h$

Multiply the numbers and the letters separately. Your answer should omit the × sign.

Exercise

1 Simplify:

a 5×8 **b** 7×4 **c** -9×2 **d** $3 \times (-7)$ **e** 6×9

f $\frac{1}{3} \times 12$ **g** $-3 \times (-8)$ **h** $20 \times \frac{1}{2}$ **i** 10×5 **j** 4×3

2 Write in simplified form.

a $6 \times a$ **b** $c \times d$ **c** $k \times 7$ **d** $-8 \times f$

e $m \times 10$ **f** $p \times p$ **g** $14 \times t$ **h** $p \times q \times r$

i $12 \times x$ **j** $15 \times n$ **k** $g \times 6$ **l** $\frac{1}{2} \times y$

3 Simplify:

a $5 \times 3a$ **b** $2x \times 4$ **c** $10 \times 4c$ **d** $3y \times 7$

e $-11 \times 2b$ **f** $8 \times 8q$ **g** $9m \times (-7)$ **h** $5u \times v$

i $10c \times c$ **j** $x \times 4x$ **k** $2m \times 3n$ **l** $4b \times 5c$

m $\frac{1}{2} \times 8t$ **n** $\frac{1}{3}a \times 6$ **o** $20d \times \frac{1}{4}$ **p** $\frac{1}{2}f \times 12$

Fun Spot 3:02 | What training do you need to be a garbologist?

Simplify each of the following and match the letter with the answers below.

A $7c \times 5$ **C** $6c \times 6$ **E** $-c \times 8$ **G** $\frac{1}{2}c \times 18$

I $-3 \times 5b$ **K** $b \times b$ **N** $2b \times 7$ **O** $12 \times \frac{1}{4}b$

P $3a \times b$ **S** $2a \times 4b$ **T** $5ab \times 6$ **U** $a \times 4b$ **Y** $-5b \times 2a$

$14b$	$3b$	$14b$	$-8c$	$-10ab$	$3b$	$4ab$	$3ab$	$-15b$	$36c$	b^2	$-15b$	$30ab$	$4ab$	$3ab$	$35c$	$8ab$	$-10ab$	$3b$	$4ab$	$9c$	$3b$

Answers can be found in the Interactive Student CD.

Foundation Worksheet

4:03 | Algebraic Division

Name: _____ **Class:** _____

Examples

Divide the numbers and letters separately.

1 $\frac{4x}{6} = \frac{2x}{3}$ (÷2 top and bottom)

2 $\frac{18c}{12} = \frac{3c}{2}$ (÷ 6 top and bottom)

3 $10m \div 25 = \frac{10m}{25}$ (fractions are divisions)

$= \frac{2m}{5}$ (÷ 5 top and bottom)

4 $24k \div 8 = \frac{24k}{8}$

$= 3k$ $(24 \div 8 = 3)$

5 $7a \div a = \frac{7a}{a}$ (÷ a top and bottom)

$= 7$

6 $9a \div 12a = \frac{9a}{12a}$ (÷ 3 and a top and bottom)

$= \frac{3}{4}$

Exercise

1 Reduce these fractions to their lowest terms.

a $\frac{6a}{12}$ **b** $\frac{3d}{9}$ **c** $\frac{10m}{14}$ **d** $\frac{8h}{12}$ **e** $\frac{12w}{15}$

f $\frac{4r}{10}$ **g** $\frac{6x}{16}$ **h** $\frac{2b}{18}$ **i** $\frac{8v}{6}$ **j** $\frac{15y}{12}$

2 Simplify:

a $a \div a$ **b** $4t \div t$ **c** $10q \div q$ **d** $6x \div 3x$

e $8p \div 2p$ **f** $12k \div k$ **g** $20g \div 10g$ **h** $28m \div 4m$

3 Simplify:

a $10x \div 5$ **b** $14a \div 2$ **c** $15b \div 3$ **d** $50n \div 5$

e $24r \div 8$ **f** $17y \div 17$ **g** $100f \div 20$ **h** $42x \div 6$

Fun Spot 3:03 | What happens when you ask an oyster a personal question?

Complete each division. Match the letter with the answers below.

A $\frac{4x}{14}$ **C** $5x \div 25$ **T** $\frac{2x}{6}$ **I** $6x \div x$ **L** $16x \div 20$

M $12x \div 3x$ **P** $\frac{8x}{18}$ **S** $\frac{6x}{12x}$ **U** $30x \div 20$

										.
6	$\frac{x}{3}$	$\frac{x}{5}$	$\frac{4x}{5}$	$\frac{2x}{7}$	4	$\frac{1}{2}$	$\frac{3x}{2}$	$\frac{4x}{9}$		

Answers can be found in the Interactive Student CD.

Foundation Worksheet

4:04A | Simplifying Algebraic Fractions (+/−)

Name: _____ **Class:** _____

Examples

To add or subtract fractions with the same denominators (bottom lines), we add or subtract the numerators (top lines), then simplify.

1 $\frac{2}{9} + \frac{3}{9} = \frac{2+3}{9}$
$= \frac{5}{9}$

2 $\frac{7}{13} - \frac{4}{13} = \frac{7-4}{13}$
$= \frac{3}{13}$

3 $\frac{7}{12} + \frac{1}{12} = \frac{8}{12}$ (÷ 4 top and bottom)
$= \frac{2}{3}$

4 $\frac{3x}{8} + \frac{x}{8} = \frac{4x}{8}$ (÷ 4)
$= \frac{x}{2}$

5 $\frac{13x}{10} - \frac{7x}{10} = \frac{6x}{10}$ (÷ 2)
$= \frac{3x}{5}$

6 $\frac{7c}{15} + \frac{11c}{15} = \frac{18c}{15}$ (÷ 3)
$= \frac{6c}{5}$

Exercise

1 Simplify:

a $\frac{1}{7} + \frac{3}{7}$ **b** $\frac{5}{11} + \frac{3}{11}$ **c** $\frac{3}{19} + \frac{5}{19}$ **d** $\frac{11}{15} + \frac{2}{15}$

e $\frac{1}{10} + \frac{3}{10}$ **f** $\frac{1}{6} + \frac{3}{6}$ **g** $\frac{7}{20} + \frac{9}{20}$ **h** $\frac{7}{16} + \frac{3}{16}$

2 Simplify:

a $\frac{7}{9} - \frac{1}{9}$ **b** $\frac{6}{13} - \frac{2}{13}$ **c** $\frac{6}{7} - \frac{2}{7}$ **d** $\frac{10}{11} - \frac{3}{11}$

e $\frac{17}{20} - \frac{9}{20}$ **f** $\frac{11}{12} - \frac{5}{12}$ **g** $\frac{13}{18} - \frac{3}{18}$ **h** $\frac{9}{10} - \frac{7}{10}$

3 Simplify:

a $\frac{x}{5} + \frac{2x}{5}$ **b** $\frac{2y}{9} + \frac{5y}{9}$ **c** $\frac{9k}{17} - \frac{k}{17}$ **d** $\frac{2a}{5} + \frac{4a}{5}$

e $\frac{9c}{10} - \frac{3c}{10}$ **f** $\frac{5p}{8} + \frac{3p}{8}$ **g** $\frac{11d}{12} + \frac{4d}{12}$ **h** $\frac{17r}{20} - \frac{3r}{20}$

i $\frac{3m}{16} + \frac{7m}{16}$ **j** $\frac{7x}{8} + \frac{5x}{8}$ **k** $\frac{19d}{10} - \frac{3c}{10}$ **l** $\frac{5y}{8} - \frac{y}{8}$

Fun Spot 3:04A | Where did Cyclops go after primary school?

Complete each question, then match the letters with the answers.

A $\frac{a}{4} + \frac{2a}{4}$ **C** $\frac{a}{4} + \frac{3a}{4}$ **E** $\frac{7a}{8} - \frac{3a}{8}$ **H** $\frac{5a}{9} + \frac{a}{9}$ **L** $\frac{11a}{12} - \frac{a}{12}$

N $\frac{7x}{5} + \frac{3x}{5}$ **O** $\frac{8x}{15} + \frac{3x}{15}$ **S** $\frac{8x}{15} - \frac{7x}{15}$ **T** $\frac{7x}{20} + \frac{8x}{20}$ **Y** $\frac{x}{6} + \frac{x}{6}$

$\frac{3x}{4}$	$\frac{11x}{15}$	$\frac{3a}{4}$	$2x$	$\frac{a}{2}$	$\frac{x}{3}$	$\frac{a}{2}$	$\frac{x}{15}$	a	$\frac{2a}{3}$	$\frac{11x}{15}$	$\frac{11x}{15}$	$\frac{5a}{6}$

Answers can be found in the Interactive Student CD.

Foundation Worksheet

4:04B | Simplifying Algebraic Fractions (×/÷)

Name: ______________________________ **Class:** ______________

Examples

- Cancel any common factors between top and bottom.
- Multiply the tops and bottoms.
- When dividing, invert the second fraction and multiply.

1 $\frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$

2 $\frac{{}^{3}\cancel{9}}{11} \times \frac{7}{\cancel{12}_{4}} = \frac{21}{44}$

3 $\frac{{}^{2}\cancel{4a}}{5} \times \frac{9}{\cancel{2a}_{1}} = \frac{18}{5}$

4 $\frac{{}^{1}\cancel{3x}}{{}_{1}\cancel{5}} \times \frac{{}^{2}\cancel{10y}}{{}_{3}\cancel{9}} = \frac{2xy}{3}$

5 $\frac{1}{2} \div \frac{7}{8} = \frac{1}{\cancel{2}_{1}} \times \frac{{}^{4}\cancel{8}}{7}$
$= \frac{4}{7}$

6 $\frac{16x}{19} \div \frac{4x}{5} = \frac{{}^{4}\cancel{16x}}{19} \times \frac{5}{\cancel{4x}_{1}}$
$= \frac{20}{19}$

7 $\frac{6}{b} \div \frac{18}{c} = \frac{{}^{1}\cancel{6}}{b} \times \frac{c}{\cancel{18}_{3}}$
$= \frac{c}{3b}$

Exercise

1 Simplify:

a $\frac{1}{3} \times \frac{4}{5}$ **b** $\frac{3}{10} \times \frac{5}{6}$ **c** $\frac{4}{5} \times \frac{15}{16}$ **d** $\frac{20}{21} \times \frac{7}{4}$

e $\frac{11}{12} \div \frac{3}{4}$ **f** $\frac{10}{21} \div \frac{2}{3}$ **g** $\frac{9}{10} \div \frac{3}{5}$ **h** $\frac{3}{5} \div \frac{9}{20}$

2 Simplify:

a $\frac{a}{2} \times \frac{b}{5}$ **b** $\frac{x}{3} \times \frac{2y}{5}$ **c** $\frac{14}{n} \times \frac{3n}{2}$ **d** $\frac{d}{4} \times \frac{2e}{7}$

e $\frac{10p}{13} \times \frac{3q}{40}$ **f** $\frac{5m}{9} \times \frac{18}{25m}$ **g** $\frac{3a}{5} \times \frac{10}{9}$ **h** $\frac{15}{8c} \times \frac{4c}{5}$

i $\frac{6}{5y} \times \frac{10x}{3}$ **j** $\frac{t}{6} \times \frac{2}{3t}$ **k** $\frac{4u}{5} \times \frac{10}{7u}$ **l** $\frac{3k}{20} \times \frac{5}{6k}$

3 Simplify:

a $\frac{a}{8} \div \frac{a}{3}$ **b** $\frac{6n}{5} \div \frac{3n}{5}$ **c** $\frac{4x}{15} \div \frac{2x}{5}$ **d** $\frac{9}{8x} \div \frac{3}{16x}$

e $\frac{9c}{10} \div \frac{27c}{50}$ **f** $\frac{20y}{21} \div \frac{5y}{7}$ **g** $\frac{6}{5a} \div \frac{3}{10b}$ **h** $\frac{15m}{22} \div \frac{3m}{11}$

Fun Spot 3:04B | How does a monster predict the future?

fun spot

Calculate the answers and match their letters below.

A $\frac{2}{3} \times \frac{3}{4}$ **C** $\frac{5x}{9} \times \frac{2}{5x}$ **E** $\frac{5x}{6} \times \frac{3}{2x}$ **H** $\frac{1}{3} \times \frac{1}{4}$ **O** $\frac{x}{3} \times \frac{6}{x}$

P $\frac{6}{13} \div \frac{2}{5}$ **R** $\frac{9x}{4} \div \frac{3x}{2}$ **S** $\frac{10}{7x} \div \frac{5}{x}$ **U** $\frac{12}{5x} \div \frac{6}{x}$

$\frac{1}{12}$	$\frac{5}{4}$		$\frac{2}{5}$	$\frac{2}{7}$	$\frac{5}{4}$	$\frac{2}{7}$		$\frac{1}{2}$		$\frac{1}{12}$	2	$\frac{3}{2}$	$\frac{3}{2}$	2	$\frac{3}{2}$	$\frac{2}{7}$	$\frac{2}{9}$

2	$\frac{15}{13}$	$\frac{5}{4}$

Foundation Worksheet

4:05 | Grouping Symbols

Name: _____ **Class:** _____

Examples

Multiply the term outside by each term inside.

1 $5(x+3) = 5 \times x + 5 \times 3$
$= 5x + 15$

2 $7(2c+5) = 7 \times 2c + 7 \times 5$
$= 14c + 35$

3 $2(m-4) = 2 \times m - 2 \times 4$
$= 2m - 8$

4 $8(4y-1) = 8 \times 4y - 8 \times 1$
$= 32y - 8$

5 $3(5-4t) = 3 \times 5 - 3 \times 4t$
$= 15 - 12t$

6 $a(a+3) = a \times a + 3 \times a$
$= a^2 + 3a$

Exercise

1 Simplify:

a $3 \times 4y$ **b** $4 \times c$ **c** $5 \times 2a$ **d** $9 \times 4m$ **e** $6 \times 10p$

f $x \times x$ **g** $5t \times 6$ **h** $6k \times 3$ **i** $w \times 3w$ **j** $h \times 7h$

2 Complete the following.

a $3(2m+5) = 3 \times 2m + 3 \times \ldots$
$= \ldots$

b $7(x+3) = 7 \times x + 7 \times \ldots$
$= \ldots$

c $5(a-7) = 5 \times \ldots - 5 \times 7$
$= \ldots$

d $4(2y-1) = 4 \times \ldots - 4 \times 1$
$= \ldots$

e $p(p-4) = p \times p - p \times \ldots$
$= \ldots$

f $c(5-2c) = c \times \ldots - c \times 2c$
$= \ldots$

3 Remove the grouping symbols (expand).

a $3(2a+5)$ **b** $6(d-7)$ **c** $10(5q+3)$

d $x(x-7)$ **e** $t(2t+3)$ **f** $4(3y+7)$

g $8(3-2x)$ **h** $7(2c-9)$ **i** $n(3n+1)$

j $9(3-5p)$ **k** $r(3-5r)$ **l** $2(7x+8)$

m $5(11c+10)$ **n** $2(4+3n)$ **o** $8(6-n)$

Fun Spot 3:05 | What animal with two humps is found at the south pole?

Remove the grouping symbols and match the letters with the answers below.

A $4(x-2)$ **C** $7(x+3)$ **E** $4(2-x)$ **L** $x(x+2)$

M $7(x-3)$ **O** $2(x+2)$ **S** $7(3x+1)$ **T** $x(3x+7)$

$4x-8$	x^2+2x	$2x+4$	$21x+7$	$3x^2+7x$	$7x+21$	$4x-8$	$7x-21$	$8-4x$	x^2+2x

Foundation Worksheet

4:06 | Simplifications with Expansions

Name: _____ **Class:** _____

Examples

1 Expand.
2 Collect like terms.
3 Take care with negatives.

1 $5(x+4)+7 = 5x+\underline{20+7}$
$= 5x+27$

2 $20+4(n-3) = \underline{20}+4n\underline{-12}$
$= 8+4n$

3 $3(2y+5)-5y+4 = 6y\underline{+15}-5y\underline{+4}$
$= y+19$

4 $9a-13+2(3a-4) = \underline{9a}-13+\underline{6a}-8$
$= 15a-21$

5 $2(6-t)+4(7-2t) = \underline{12}-2t\underline{+28}-8t$
$= 40-10t$

6 $7(2x-3)-3(x-5) = \underline{14x}-21\underline{-3x}+15$
$= 11x-6$

Exercise

1 Expand and simplify:

a $4(t+3)+2$ **b** $5(x+5)-9$ **c** $7(y-3)-2y$
d $10(2m-5)+17$ **e** $3(10-3k)-8$ **f** $8+2(3a+5)$
g $12-5(x+3)$ **h** $7x+2(3x-1)$ **i** $5p-2(2p-7)$

2 Expand and simplify:

a $4(t+3)+2+5t$ **b** $2(b+5)+5b+7$ **c** $3(n-4)+n+12$
d $5(2c+5)-6c+4$ **e** $2(4w+1)+5w-7$ **f** $6(2+5x)-10-7x$
g $9h+17+2(h-3)$ **h** $4y+9-2(2y+3)$ **i** $8+15a-5(a-6)$

3 Expand and simplify:

a $4(t+3)+2(t+7)$ **b** $3(x+5)+7(x+4)$
c $2(c+5)+5(2c+3)$ **d** $4(3k+5)+2(5k+7)$
e $3(5a+1)-2(3a-4)$ **f** $5(2y+5)-3(2y+7)$
g $2(6+5r)-3(2+3r)$ **h** $8(m-5)-4(m-3)$
i $4(7-3p)+5(2-p)$ **j** $3(2t-7)+4(t-5)$
k $3(4f+1)-2(5f-1)$ **l** $9(x+4)-3(2x-5)$

Answers can be found in the Interactive Student CD.

Foundation Worksheet

4:07 | Factorising

Name: _____ **Class:** _____

Examples

- Factorising is the opposite of expanding.
- Look for the highest number which divides into each term.
- You can check your answer by expanding or multiplying out.

1 $5x + 10 = 5(x + \ldots)$
2 is missing, as 5 into 10 goes 2.

2 $4y - 12 = 4(y - \ldots)$
3 is missing, as $12 \div 4 = 3$.

3 $7a + 21b = 7(\ldots + 3b)$
a is missing, as $7a \div 7 = a$.

4 $2p + 18 = 2(p + 9)$
2 is a common factor, so divide it into each term.

5 $ab + 4a = a(b + 4)$
a is a common factor, so we take it

6 $10m - 20 = 10(m - 2)$
2, 5 and 10 are common factors but 10 is the highest.

Exercise

1 What is the (highest) common factor of:

a 6 and 8? **b** 5 and 10? **c** 9 and 12? **d** 12 and 20?
e $4x$ and 6? **f** $3p$ and $7p$? **g** 8 and 18? **h** 21 and 14?

2 Complete the following.

a $6a + 12 = 6(\ldots + 2)$ **b** $2x - 6y = 2(x - \ldots)$ **c** $8x + 4 = 4(2x + \ldots)$
d $10e + 15 = 5(\ldots + 3)$ **e** $3a + 30 = 3(a + \ldots)$ **f** $9x - xy = x(\ldots - y)$
g $15 - 3x = 3(5 - \ldots)$ **h** $7 - 14x = 7(\ldots - 2x)$ **i** $20m + 8n = 4(5m + \ldots)$

3 Factorise:

a $6a + 6$ **b** $12x - 9$ **c** $xy + 2x$
d $10m - 15$ **e** $9 - 6q$ **f** $8y + 2$
g $4c + 16$ **h** $18k - 24$ **i** $10 - 30d$
j $3a - ab$ **k** $14e + 8f$ **l** $6m + 21n$
m $7 + 35t$ **n** $15p + 20q$ **o** $16w - 8$

Fun Spot 3:07 | What is black and white and talks to apes?

Match the correct factorising with the letters below.

A $6x + 3$ **D** $3x + 6$ **E** $3x + 3$ **I** $3x + 3y$ **N** $3x + xy$
R $2x - 6$ **S** $6x - 3$ **T** $2y + 4$ **U** $3y - 6$ **Z** $2x - 6y$

$2(y+2)$	$3(2x+1)$	$2(x-3)$	$2(x-3y)$	$3(2x+1)$	$x(3+y)$	$3(x+y)$	$x(3+y)$	$3(2x+1)$	$3(x+2)$	$3(x+y)$	$x(3+y)$	$x(3+y)$	$3(x+1)$	$2(x-3)$	$3(2x-1)$	$3(y-2)$	$3(x+y)$	$2(y+2)$

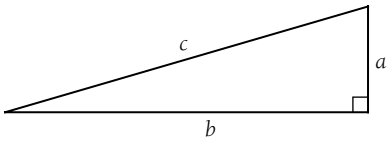
Foundation Worksheet

5:01 | Pythagoras' Theorem A

Name: _____ **Class:** _____

Example

Measure each side of the triangle.
Work out a^2, b^2 and c^2.
Check that $c^2 = a^2 + b^2$



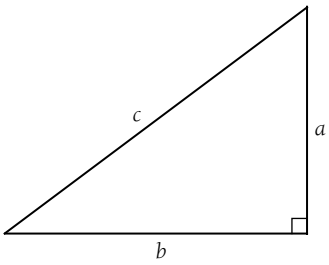
a	b	c	a^2	b^2	c^2
14	48	50	196	2304	2500

$2500 = 196 + 2304$ True.

Exercise

Measure each side of the triangle, in millimetres. Work out a^2, b^2 and c^2. Does $c^2 = a^2 + b^2$?

1



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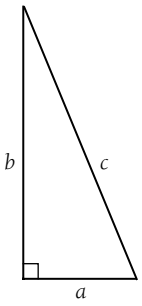

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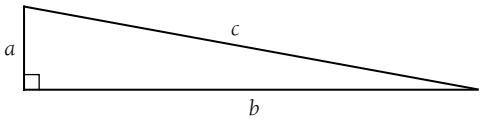

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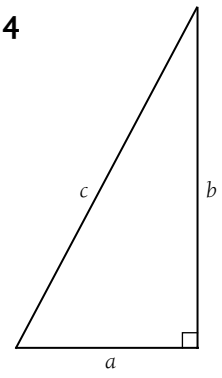

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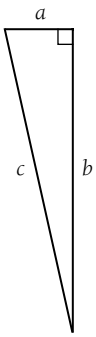

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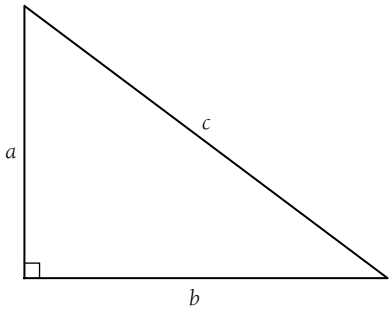

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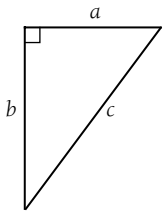

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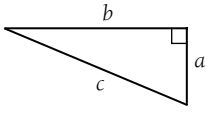

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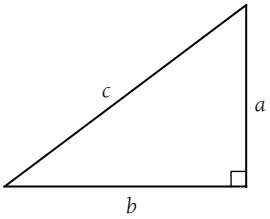

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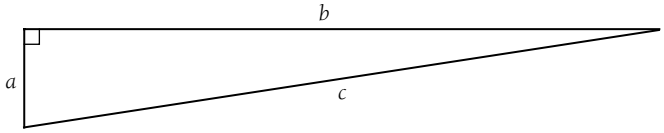


Answers can be found in the Interactive Student CD.

Foundation Worksheet

5:02 | Pythagoras' Theorem B

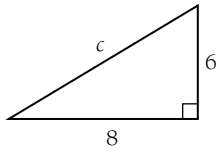
Name: _____ **Class:** _____

Examples

The hypotenuse is the longest side.

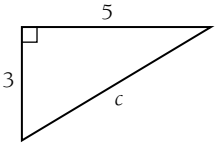
Find the length of the hypotenuse in each case.

1



$c^2 = 6^2 + 8^2$
$= 100$
$c = \sqrt{100}$
$= 10$ (exactly)

2

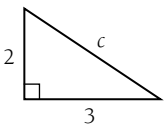


$c^2 = 3^2 + 5^2$
$= 34$
$c = \sqrt{34}$
$= 5{\cdot}83$ (2 dec. pl.)

Exercise

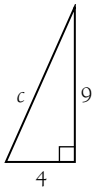
Find the length of the hypotenuse (exactly, or correct to two decimal places).

1



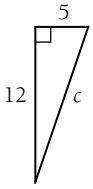
$c^2 = 2^2 + 3^2$
$= 13$
$\therefore c = \sqrt{13}$
$= \ldots$

2



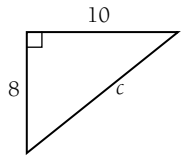
$c^2 = 4^2 + 9^2$
$= 97$
$\therefore c = \sqrt{97}$
$= \ldots$

3



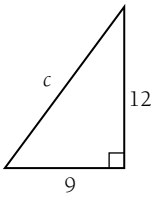
$c^2 = 5^2 + 12^2$
$= 169$
$\therefore c = \ldots$
$= \ldots$

4



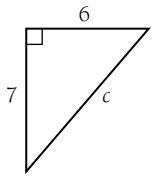
$c^2 = 8^2 + 10^2$
$= \ldots$
$\therefore c = \ldots$
$= \ldots$

5



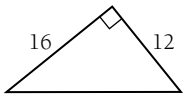
$c^2 = 9^2 + 12^2$
$= \ldots$
$\therefore c = \ldots$
$= \ldots$

6



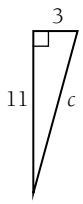
$c^2 = 6^2 + 7^2$
$= \ldots$
$\therefore c = \ldots$
$= \ldots$

7



$c^2 = \ldots$
$= \ldots$
$\therefore c = \ldots$
$= \ldots$

8



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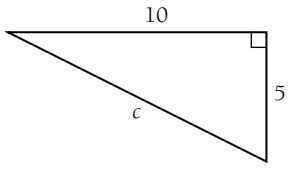

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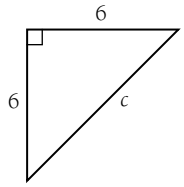

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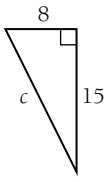

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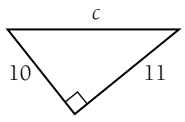


Fun Spot 3:02 | What do cats put in soft drinks?

Complete these calculations. Match the letters with the answers.

B $\sqrt{400}$ **C** 18^2 **E** 15^2 **I** $\sqrt{289}$ **M** 16^2 **S** $\sqrt{361}$ **U** 23^2

256	17	324	225		324	529	20	225	19

Answers can be found in the Interactive Student CD.

Foundation Worksheet

5:03 | Pythagoras' Theorem

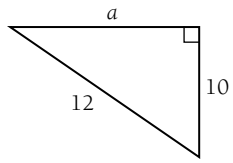
Name: _____ **Class:** _____

Examples

Find the value of the pronumeral.

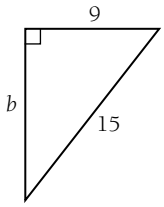
When finding a shorter side, subtract the squares.

1



$a^2 + 10^2 = 12^2$
$a^2 + 100 = 144$
$\therefore a^2 = 44$ (144 − 100)
$a = \sqrt{44}$
$= 6{\cdot}63$ (2 dec. pl.)

2

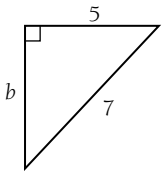


$b^2 + 9^2 = 15^2$
$b^2 + 81 = 225$
$\therefore b^2 = 144$ (225 − 81)
$b = \sqrt{144}$
$= 12$ (exactly)

Exercise

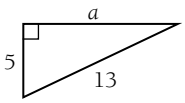
Find the value of the pronumeral.

1



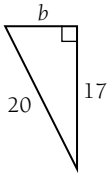
$b^2 + 5^2 = 7^2$
$b^2 + 25 = 49$
$\therefore b^2 = 24$
$b = \sqrt{24}$
$= \ldots$

2



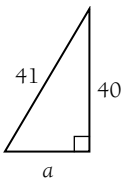
$a^2 + 5^2 = 13^2$
$a^2 + \ldots = \ldots$
$\therefore a^2 = \ldots$
$a = \sqrt{\ldots}$
$= \ldots$

3



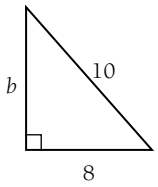
$b^2 + 17^2 = 20^2$
$b^2 + \ldots = \ldots$
$\therefore b^2 = \ldots$
$b = \sqrt{\ldots}$
$= \ldots$

4



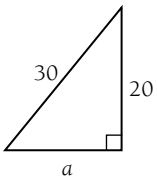
$a^2 + 40^2 = 41^2$
$a^2 + \ldots = \ldots$
$\therefore a^2 = \ldots$
$a = \sqrt{\ldots}$
$= \ldots$

5



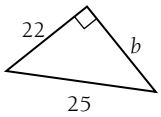
$b^2 + 8^2 = 10^2$

6



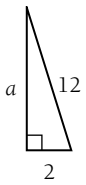
$a^2 + 20^2 = 30^2$

7

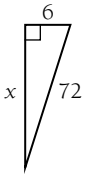


$b^2 + 22^2 = 25^2$

8



9



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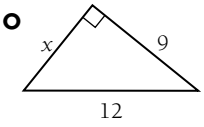
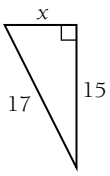
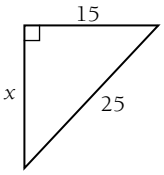


11



12



Fun Spot 3:03 | Which side of a chicken has the most feathers?

Complete these calculations. Match the letters with the answers.

D $10^2 - 5^2$ **E** $19^2 - 13^2$ **H** $15^2 + 17^2$ **I** $7^2 + 6^2$

O $3^2 + 5^2$ **S** $12^2 - 5^2$ **T** $23^2 - 13^2$ **U** $61^2 - 60^2$

360	514	192	34	121	360	119	85	75	192	

Foundation Worksheet

6:02 | The Index Laws

Name: ______________________ **Class:** ____________

Examples

1 Evaluate:

a 4^3
$= 4 \times 4 \times 4$
$= 64$

b 5^2
$= 5 \times 5$
$= 25$

c 2^5
$= 2 \times 2 \times 2 \times 2 \times 2$
$= 32$

2 Evaluate on your calculator:

a 8^3
Press 8 [x^y] 3 [=]
$= 512$

b 12^6
Press 12 [x^y] 6 [=]
$= 2\,985\,984$

Note: Some calculators have [y^x] as their index button.

Exercise

1 Evaluate:

a 2^3 **b** 3^4 **c** 5^3 **d** 7^2 **e** 7^3
f 10^2 **g** 2^4 **h** 1^5 **i** 4^4 **j** 6^2
k 3^3 **l** 10^4 **m** 9^2 **n** 9^3 **o** 1^4

2 Use your calculator to evaluate:

a 6^3 **b** 4^5 **c** 8^4 **d** 9^4 **e** 7^4
f 3^5 **g** 5^4 **h** 10^3 **i** 11^3 **j** 6^5
k 9^5 **l** 2^{10} **m** 8^6 **n** 6^8 **o** 11^5
p 7^7 **q** 5^{10} **r** 4^8 **s** 2^{16} **t** 3^7
u 10^6 **v** 18^4 **w** 14^5 **x** 20^4 **y** 13^6

Fun Spot 6:02 | Why didn't the boy want to work in the wool factory?

fun spot

Match the letters with the answers below.

A 3×3 **D** $7 \times 7 \times 7$ **E** $10 \times 10 \times 10 \times 10 \times 10$
G $5 \times 5 \times 5 \times 5 \times 5$ **H** $6 \times 6 \times 6 \times 6$ **N** $1 \times 1 \times 1$
O $13 \times 13 \times 13$ **S** 14^3 **T** 16^2
U 3^6 **W** 2^9 **Y** 15^4

[ ][ ]   [ ][ ][ ]   [ ][ ][ ]   [ ][ ][ ][ ][ ]   [ ][ ]   [ ][ ][ ] !

1296, 100 000 | 512, 9, 2744 | 256, 2197, 2197 | 50 625, 2197, 729, 1, 3125 | 256, 2197 | 343, 50 625, 100 000

Foundation Worksheet

6:03 | Negative Indices

Name: _____ **Class:** _____

Examples

The negative index means '1 over'.

1 $a^{-3} = \frac{1}{a^3}$

2 $7^{-2} = \frac{1}{7^2}$

$= \frac{1}{49}$

3 $4^{-3} = \frac{1}{4^3}$

$= \frac{1}{64}$

4 $\frac{1}{6^7} = 6^{-7}$

5 $\frac{1}{8} = \frac{1}{8^1}$

$= 8^{-1}$

Exercise

1 Write down the value of the following.

a 4^{-1}	**b** 2^{-3}	**c** 3^{-2}	**d** 5^{-1}	**e** 6^{-2}
f 10^{-2}	**g** 10^{-3}	**h** 5^{-2}	**i** 8^{-2}	**j** 4^{-2}
k 9^{-1}	**l** 2^{-5}	**m** 15^{-1}	**n** 3^{-3}	**o** 11^{-3}
p 20^{-1}	**q** 12^{-2}	**r** 5^{-3}	**s** 9^{-3}	**t** 16^{-2}

2 Write the following with a negative index.

a $\frac{1}{4}$	**b** $\frac{1}{2^3}$	**c** $\frac{1}{3^2}$	**d** $\frac{1}{7}$	**e** $\frac{1}{10}$
f $\frac{1}{5^6}$	**g** $\frac{1}{10^4}$	**h** $\frac{1}{11^2}$	**i** $\frac{1}{2^7}$	**j** $\frac{1}{3^5}$
k $\frac{1}{40}$	**l** $\frac{1}{8^2}$	**m** $\frac{1}{6^2}$	**n** $\frac{1}{2}$	**o** $\frac{1}{4^3}$
p $\frac{1}{3}$	**q** $\frac{1}{6^4}$	**r** $\frac{1}{13^2}$	**s** $\frac{1}{25^4}$	**t** $\frac{1}{12^3}$

Fun Spot 6:03 | What is the biggest ant in the world?

Find the value of each of these. Match the letter with the answer below to finish the riddle.

A 3^{-4} **E** 6^{-1} **H** 19^{-2} **L** 8^{-3} **N** 4^{-4} **P** 12^{-3} **T** 6^{-4}

								-			
$\frac{1}{81}$	$\frac{1}{256}$	$\frac{1}{6}$	$\frac{1}{512}$	$\frac{1}{6}$	$\frac{1}{1728}$	$\frac{1}{361}$			$\frac{1}{81}$	$\frac{1}{256}$	$\frac{1}{1296}$

Answers can be found in the Interactive Student CD.

Foundation Worksheet

7:01 | Solving Equations by Inspection

Name: _____ **Class:** _____

Examples

The simplest equations can be solved mentally. Try changing the equation into a question.

Solve:

1 $a + 8 = 11$
What number plus 8 equals 11?
$\therefore a = 3$

2 $4c = 20$
4 times what number gives 20?
Or 4 whats = 20?
$\therefore c = 5$

3 $y - 8 = 5$
What number take away 8 gives 5?
Or 8 from which number equals 5?
$\therefore y = 13$

Exercise

1 Solve:

a $y + 5 = 8$	**b** $m - 7 = 10$	**c** $x - 1 = -5$	**d** $t + 3 = 7$
e $x - 9 = 1$	**f** $k + 3 = 11$	**g** $a - 5 = 0$	**h** $m + 4 = 14$
i $p + 8 = 12$	**j** $q + 5 = 5$	**k** $x - 4 = -2$	**l** $y - 8 = 10$
m $11 - d = 4$	**n** $x - 10 = 20$	**o** $c + 5 = 0$	**p** $w + 3 = 2$
q $y + 20 = 30$	**r** $14 - x = 8$	**s** $a - 6 = 12$	**t** $h + 17 = 22$

2 Solve:

a $5m = 30$	**b** $6y = 42$	**c** $4x = 3$	**d** $3t = 18$
e $3a = 45$	**f** $2k = 24$	**g** $11x = 33$	**h** $9d = 54$
i $10x = 70$	**j** $8p = 32$	**k** $4y = 24$	**l** $2c = 22$
m $3d = 12$	**n** $7n = 35$	**o** $5x = 40$	**p** $7t = 63$
q $8x = 8$	**r** $3q = 12$	**s** $9y = 72$	**t** $8a = 4$

Fun Spot 8:01 | What is a runner's favourite subject?

Match the letters with the answers below.

A $x + 3 = 8$	**G** $7x = 42$	**H** $4x = 40$	**J** $x - 6 = 8$
O $9 - x = 2$	**P** $x \div 6 = 3$	**R** $x + 17 = 20$	**Y** $x \div 3 = 7$

			-						
14	7	6		3	5	18	10	21	

Answers can be found in the Interactive Student CD.

Foundation Worksheet

7:03 | Solving Simple Equations

Name: _____ **Class:** _____

Examples

The basic aim is to separate the pronumerals from the numbers.
The basic rule is to 'do the opposite'.

Solve:

1 $x + 18 = 33$
$(-18)\quad(-18)$
$x = 15$

2 $4a = 28$
$(\div 4)\quad(\div 4)$
$a = 7$

3 $3y + 7 = 22$
$(-7)\quad(-7)$
$3y = 15$
$(\div 3)\quad(\div 3)$
$y = 5$

4 $6m - 5 = 19$
$(+5)\quad(+5)$
$6m = 24$
$(\div 6)\quad(\div 6)$
$m = 4$

Exercise

1 Solve:

a $m + 15 = 32$ **b** $n - 12 = 27$ **c** $3m = 72$ **d** $x - 17 = 20$
e $5x = 75$ **f** $t + 26 = 43$ **g** $y - 26 = 42$ **h** $6p = 78$
i $f \div 2 = 5$ **j** $a - 35 = 9$ **k** $p - 15 = 53$ **l** $w + 17 = 13$
m $15q = 45$ **n** $4c = 64$ **o** $y - 34 = 16$ **p** $h \div 5 = 20$

2 Solve:

a $2a + 3 = 9$ **b** $4x - 1 = 3$ **c** $7n - 6 = 8$ **d** $5k + 8 = 28$
e $3t - 8 = 10$ **f** $6c + 11 = 47$ **g** $10w - 3 = 37$ **h** $2h - 7 = 23$
i $8m + 7 = 31$ **j** $11k + 9 = 20$ **k** $4a - 5 = 7$ **l** $3x + 5 = 20$
m $6b + 11 = 23$ **n** $5a - 9 = 16$ **o** $7y + 6 = 34$ **p** $10v - 2 = 98$
q $3m - 17 = 13$ **r** $9q + 15 = 78$ **s** $8c + 20 = 36$ **t** $4x - 15 = 29$

Fun Spot 8:03 | If you have a referee in football, what do you have in bowls?

Solve each equation. Complete the riddle by matching the letters with the answers below.

A $x + 8 = 20$ **C** $x - 4 = 12$ **E** $9x = 36$
F $x - 8 = -2$ **K** $2x + 7 = 13$ **L** $3x - 7 = 8$
N $5x - 2 = 3$ **O** $3x = 21$ **R** $6x = 60$ **S** $2x = 38$

16	7	10	1	6	5	12	3	4	19

Foundation Worksheet

7:05 | Equations with Pronumerals on Both Sides

Name: _____ **Class:** _____

Examples

Your aim is to separate the pronumerals to one side and the numbers to the other side of the equals sign. The rule is 'when you change sides, do the opposite'.

1 $5c + 7 = 4c + 12$
$5c - 4c = 12 - 7$
$\therefore c = 5$

2 $3x - 8 = 2x - 5$
$3x - 2x = -5 + 8$
$\therefore x = 3$

3 $4f - 3 = 2f + 17$
$4f - 2f = 17 + 3$
$2f = 20$
$\therefore f = 10$

4 $7m + 6 = 2m + 31$
$7m - 2m = 31 - 6$
$5m = 25$
$\therefore m = 5$

Exercise

1 Solve:

a $2x + 3 = x + 5$ **b** $5n - 2 = 4n - 3$ **c** $6c - 4 = 5c + 7$
d $4y + 7 = 3y + 15$ **e** $8t - 5 = 7t + 13$ **f** $3h - 7 = 2h + 3$
g $10d - 8 = 9d - 4$ **h** $7x + 13 = 6x + 8$ **i** $9a + 7 = 8a + 4$
j $4p - 7 = 3p + 5$ **k** $5y + 3 = 4y - 7$ **l** $2m - 5 = m - 8$
m $3x + 13 = 2x + 20$ **n** $8c - 3 = 7c + 15$ **o** $10q + 7 = 9q - 6$

2 Solve:

a $6a + 3 = 3a + 9$ **b** $7m - 2 = 3m + 6$ **c** $4t + 7 = 2t + 15$
d $7x - 9 = 2x + 21$ **e** $5y - 6 = 2y + 6$ **f** $6n + 8 = n + 28$
g $4t + 5 = t + 2$ **h** $8x - 7 = 2x + 17$ **i** $7c - 30 = c - 12$
j $5r - 9 = 2r + 18$ **k** $9w + 7 = 5w + 47$ **l** $10q - 2 = 2q + 30$
m $12c + 3 = 6c - 39$ **n** $6x - 3 = 4x - 11$ **o** $6y + 19 = 3y - 5$
p $8x - 5 = 3x + 15$ **q** $7m + 5 = 3m + 33$ **r** $10f - 4 = 3f - 25$
s $3a + 7 = a + 19$ **t** $4y - 8 = y - 2$ **u** $6t + 5 = t + 65$

Answers can be found in the Interactive Student CD.

Foundation Worksheet

7:06 | Equations with Fractions 1

Name: _____ **Class:** _____

Examples

- The basic rule is 'do the opposite'.
- Fractions are divisions.

1 Solve:

a $\frac{x}{6} = 4$

$x \div 6 = 4$

$(\times 6) \quad (\times 6)$

$\therefore x = 24$

b $\frac{a}{8} = -4$

$a \div 8 = -4$

$(\times 8) \quad (\times 8)$

$\therefore a = -32$

c $\frac{3c}{5} = 6$

$3c \div 5 = 6$

$(\times 5) \quad (\times 5)$

$3c = 30$

$\therefore c = 10$

2 Simplify:

a $\frac{3(x-6)}{3} = \cancel{3} \times \frac{(x-6)}{\cancel{3}}$

$= x - 6$

b $4\left(\frac{d}{4} + 7\right) = \cancel{4} \times \frac{d}{\cancel{4}} + 4 \times 7$

$= d + 28$

Exercise

1 Solve:

a $\frac{x}{3} = 2$ **b** $\frac{a}{5} = -2$ **c** $\frac{2m}{3} = 6$ **d** $\frac{c}{4} = 4$

e $\frac{5y}{4} = 10$ **f** $\frac{t}{7} = -3$ **g** $\frac{x}{8} = 1$ **h** $\frac{4k}{3} = 8$

i $\frac{q}{6} = 12$ **j** $\frac{6a}{5} = 12$ **k** $\frac{n}{7} = -1$ **l** $\frac{x}{14} = 2$

m $\frac{3y}{8} = 9$ **n** $\frac{w}{10} = -7$ **o** $\frac{2x}{7} = 6$ **p** $\frac{8v}{6} = 4$

q $\frac{m}{3} = -9$ **r** $\frac{t}{11} = 4$ **s** $\frac{4d}{3} = 8$ **t** $\frac{2p}{7} = -8$

2 Simplify:

a $5 \times \frac{(a+2)}{5}$ **b** $6\left(\frac{x}{6} - 3\right)$ **c** $4 \times \frac{(2m+1)}{4}$ **d** $8\left(\frac{c}{8} + 5\right)$

e $3 \times \frac{(2x-7)}{3}$ **f** $8\left(\frac{n}{8} + 1\right)$ **g** $7 \times \frac{(5a-4)}{7}$ **h** $3\left(\frac{y}{3} - 4\right)$

i $2\left(\frac{3c}{2} + 5\right)$ **j** $2 \times \frac{(4a-3)}{2}$ **k** $10\left(3 + \frac{y}{10}\right)$ **l** $9 \times \frac{(7+3a)}{9}$

m $5\left(\frac{2p}{5} + 3\right)$ **n** $4\left(\frac{3w}{4} - 1\right)$ **o** $8 \times \frac{(13m+4)}{8}$ **p** $6 \times \frac{(7q-2)}{6}$

Answers can be found in the Interactive Student CD.

Foundation Worksheet

7:08 | Solving Inequations

Name: _____ **Class:** _____

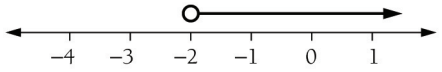
Examples

Rules for inequations are similar to rules for equations:
'Do the opposite'.
$>$ means greater than **$\geqslant$ means greater than or equal to**
$<$ means less than **$\leqslant$ means less than or equal to**

1 Show on a number line:

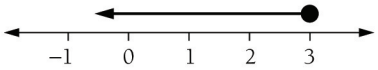
a $x > -2$

Show the numbers more than −2.



b $y \leqslant 3$

Show the numbers less than or equal to 3.



Note: Fill in the circle, as it could equal 3.

2 Solve:

a $x - 4 < 2$

$(+4) \quad (+4)$

$x < 6$

b $m + 3 > 1$

$(-3) \quad (-3)$

$m > -2$

Exercise

1 Show the following on a number line.

a $x > 3$	**b** $x \leqslant 2$	**c** $x < -1$	**d** $x \geqslant -4$
e $m \geqslant 0$	**f** $m < 5$	**g** $m \leqslant -3$	**h** $m > 1$
i $y \geqslant 4$	**j** $y > -1$	**k** $y < 0$	**l** $y \leqslant 6$

2 Solve:

a $x + 3 > 5$	**b** $m - 2 \leqslant 6$	**c** $n + 7 < 4$	**d** $y - 4 \leqslant 1$
e $a + 5 > 8$	**f** $x - 4 > -1$	**g** $h + 6 \leqslant 0$	**h** $p + 3 \geqslant -2$
i $3a < 12$	**j** $5y \geqslant 20$	**k** $6t > -12$	**l** $2c \leqslant -10$
m $w + 6 < 10$	**n** $x + 8 \geqslant -2$	**o** $y - 10 < 4$	**p** $f - 13 \geqslant 0$
q $10k > 20$	**r** $9 + d < 15$	**s** $x + 20 \leqslant -5$	**t** $c - 9 \geqslant -4$

Fun Spot 8:09 | Why should you be careful playing against The Big Cats?

Solve each equation and match the letters with the answers below.

A $5x = 20$	**B** $a + 10 = 15$	**C** $c - 4 = 3$	**E** $y \div 3 = 4$
G $6 - h = 4$	**H** $7x = 42$	**I** $t \div 4 = 5$	**M** $q + 7 = 7$
S $3n = 30$	**T** $m - 5 = -2$	**Y** $y + 17 = 25$	

3	6	12	8		0	20	2	6	3

										!
5	12		7	6	12	12	3	4	6	10

Answers can be found in the Interactive Student CD.

Foundation Worksheet

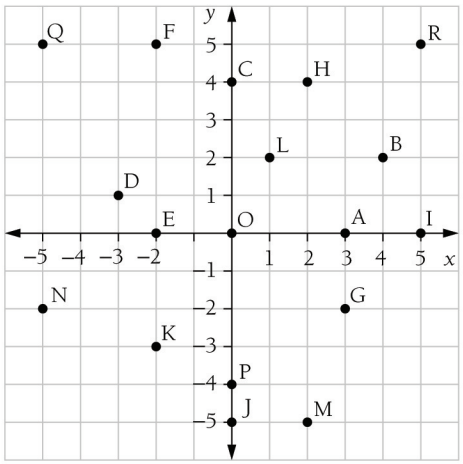
8:01 | Coordinates

Name: _____ **Class:** _____

- The *x*-coordinate is given first and tells you how many to count right (+) or left (–).
- The *y*-coordinate is given second and tells you how many to count up (+) or down (–).

Exercise

1 From the number plane shown, give the coordinates of these points.



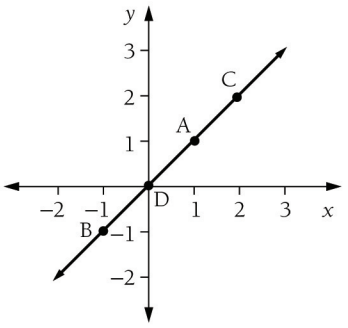
a A	**b** M	**c** O
d D	**e** F	**f** P
g K	**h** I	**i** R

2 Using the number plane shown, write the letter naming these points.

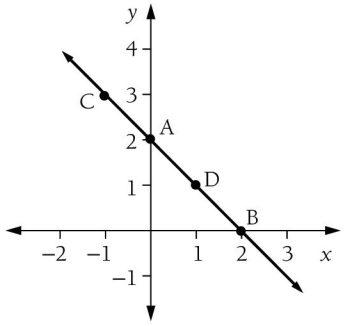
a (1, 2)	**b** (2, 4)	**c** (4, 2)
d (–2, 0)	**e** (–5, –2)	**f** (0, –5)
g (3, –2)	**h** (0, 4)	**i** (–5, 5)

3 Write down the coordinates of the points A, B, C and D on the following lines.

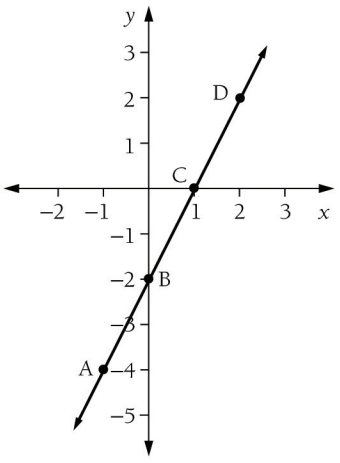
a



b



c



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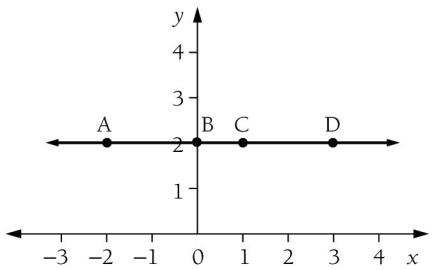


Answers can be found in the Interactive Student CD.

Foundation Worksheet

8:02 | Graphing Lines

Name: _____ **Class:** _____

Examples

1 Complete the tables for:

a $y = 2x$

x	0	1	2
y	0	2	4

(Multiply the x-value by 2.)

b $x + y = 5$

x	0	1	2
y	5	4	3

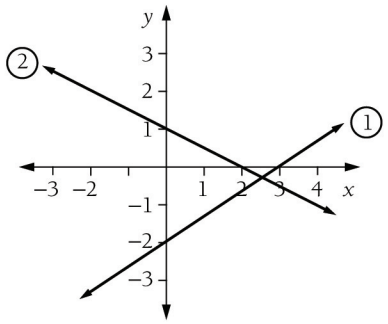
(The x- and y-values add to 5.)

c $y = 3x - 1$

x	0	1	2
y	−1	2	5

(Multiply the x-value by 3, then subtract 1.)

2 Read off the x- and y-intercepts for each line.



Line 1: x-intercept = 3, y-intercept = −2
Line 2: x-intercept = 2, y-intercept = 1

Exercise

1 Complete these tables for:

a $y = x$

x	0	1	2
y			

b $y = x + 2$

x	0	1	2
y			

c $x + y = 2$

x	0	1	2
y			

d $x - y = 2$

x	2	4	6
y			

e $y = x - 3$

x	0	1	2
y			

f $y = 3x + 2$

x	0	1	2
y			

g $y = -x$

x	0	1	2
y			

h $x + y = 5$

x	1	3	5
y			

i $y = 5x$

x	0	1	2
y			

j $y = 2x + 3$

x	0	1	2
y			

k $x - y = 4$

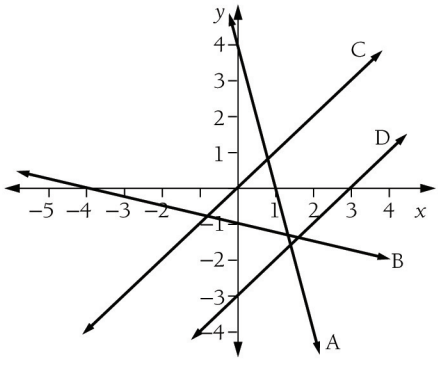
x	4	5	6
y			

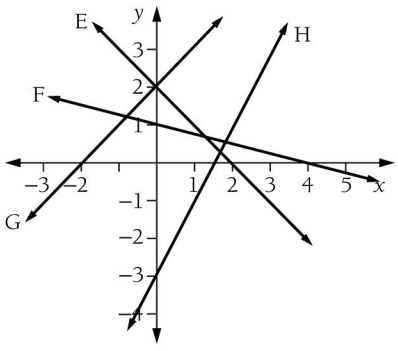
l $y = 3 - 2x$

x	0	1	2
y			

2 Read off the x- and y-intercepts for the following lines.

a A **b** B **c** C **d** D **e** E **f** F **g** G **h** H





Answers can be found in the Interactive Student CD.

Foundation Worksheet

8:03 | Horizontal and Vertical Lines

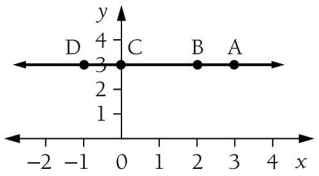
Name: _____ **Class:** _____

- The *x*-coordinate is counted across and is written first.
- The *y*-coordinate is counted up or down and is written second.
- To write a point, use brackets and a comma: (6, –1).

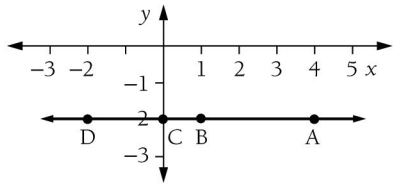
Exercise

1 Write down the coordinates of the points A, B, C and D on each line.

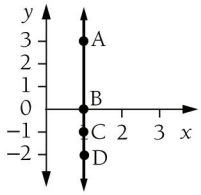
a



b



c



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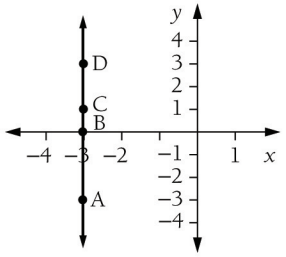
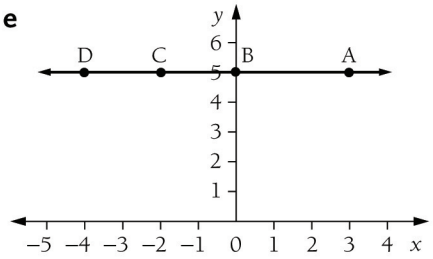


e



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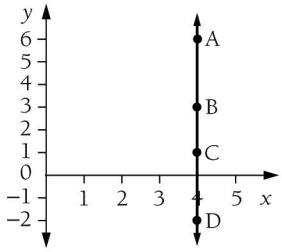
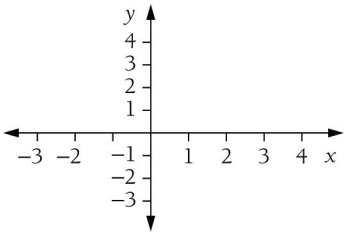


2 What do you notice about your answers in Question 1?

3 On the number plane, plot 3 points with:

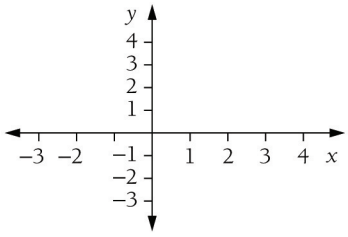
a an *x*-coordinate of 3

b a *y*-coordinate of 1



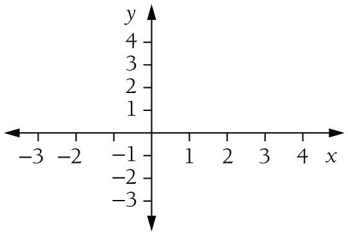
c an *x*-coordinate of –2

d a *y*-coordinate of 0



e an *x*-coordinate of 0

f a *y*-coordinate of –1



Fun Spot 10:03 | How do you stop your laptop's batteries running out?

Complete these tables of ordered pairs, using the given equations.

$y = x - 4$

x	0	1	2	4
y				

A D E H

$y + x = 4$

x	1	3	5
y			

I K N

$y = 3x$

x	–2	2	4
y			

R S T

Match the letters and answers from these tables to complete the riddle.

																.

0 3 –3 –2 12 0 –2 3 –6 6 –1 –2 –4 1 –2 –6 6

Answers can be found in the Interactive Student CD.

Foundation Worksheet

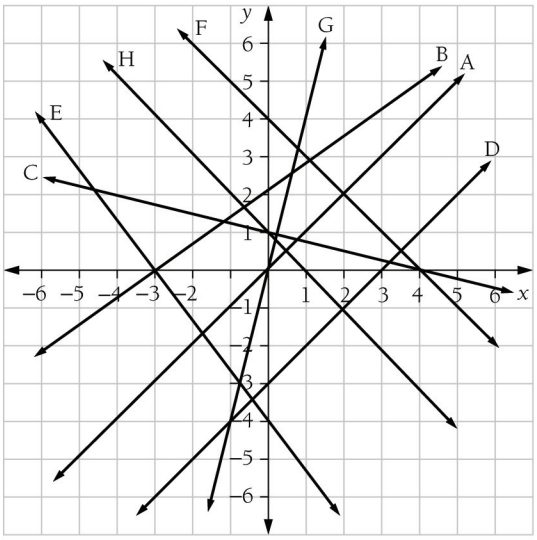
8:04 | Intersecting Lines

Name: _____ **Class:** _____

Exercise

1 Use the graph to find the point of intersection of these lines.

a A and F
b C and H
c E and B
d G and D
e D and H
f D and F
g B and F

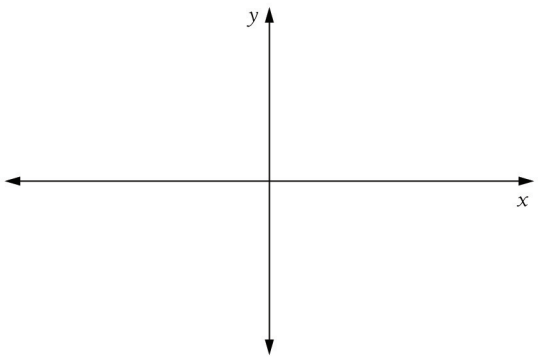


2 Graph the following pairs of lines to find their point of intersection.

a $y = x$ and $x + y = 2$

x				
y				

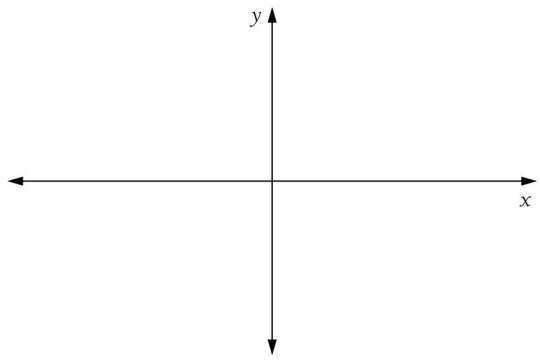
x				
y				



b $x + y = 3$ and $y = x - 3$

x				
y				

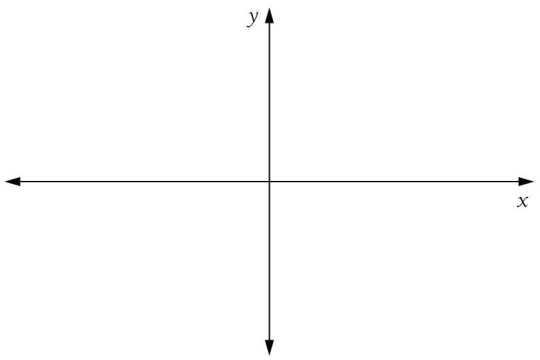
x				
y				



c $y = 2x$ and $y = x - 1$

x				
y				

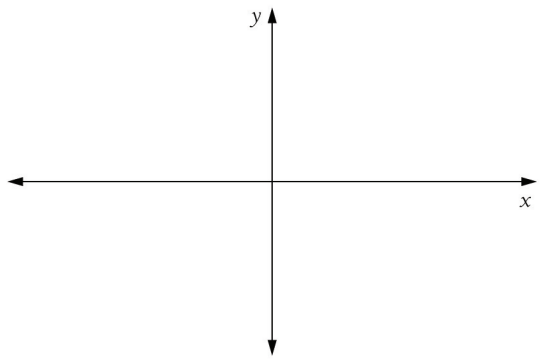
x				
y				



d $y = x + 4$ and $y = -x$

x				
y				

x				
y				



Answers can be found in the Interactive Student CD.

Foundation Worksheet

9:01 | Generalised Arithmetic

Name: _____ **Class:** _____

Mathematics uses symbols as quick ways of writing sentences:

- **sum, add, total, more than … can be replaced by +**
- **product, times, lots of … can be replaced by ×**
- **difference, minus, subtract, less, … use −**
- **quotient, divide, how many … use ÷ or a fraction**
- **average means divide the total by how many numbers there are**
- **squares, cubes, etc, are written as indices**

Exercise

1 Write an expression for:

a the sum of $3a$ and $2b$

b the average of m and n

c the quotient of x and 5

d the difference between p and q

e the cube of k

f the product of y and 40

g the sum of 15 times p and 18

h the square of w

i 20 more than a

j the product of 6 and a and b

k the total of u, v and w

l the average of p, q and r

m how many xs in y

n the quotient of $(x + 5)$ and $(x - 2)$

2 **a** Find the cost of x books at 75c each.

b Find the age of Bill, who is 25 years old, in another y years.

c Find the average of Angus's 4 race times: a, b, c and d.

d If Duncan is m years old, what was his age 4 years ago?

e A river's level was 10 metres deep. If the level fell by d metres, what is its depth now?

f A girl was given \$$d$ pocket money. If she spent \$$e$ at one shop and a further \$$f$ at another, how much has she left?

g Justine spends \$$x$ for a CD. What is her change from \$50?

h What is the number of minutes in t hours?

i Find the profit if a painting bought for \$$B$ is later re-sold for \$$R$.

j A cyclist travels at 30 km/h. How far will she go in h hours?

k Veronica was paid \$$w$ for a 5-day week. How much was this per day?

l Josie bought 5 items for \$$x$, \$$2x$, \$$y$, \$$3y$ and \$$2y$. What was the total cost?

m Kerrie drove y km before lunch and $2y$ km after lunch. How far did she go? Find her average speed if she drove for 8 hours.

n Jamie is m years old. His wife is 4 years younger, and his daughters are 30 and 32 years younger than he is. What is the total family age?

Answers can be found in the Interactive Student CD.

Foundation Worksheet

9:02 | Substitution

Name: _____ **Class:** _____

Examples

1 Find the value of:

a $3p$ if $p = 8$
$= 3 \times 8$
$= 24$

b $4x - 5y$ if $x = 5$, $y = 2$
$= 4 \times 5 - 5 \times 2$
$= 10$

2 If $a = 4$, $b = -3$, $c = 6$, find the value of:

a $abc = 4 \times (-3) \times 6$
$= -72$

b $a^2 - c^2 = 4^2 - 6^2$
$= 16 - 36$
$= -20$

c $\frac{3a}{c} = \frac{3 \times 4}{6}$
$= 2$

Exercise

1 Find the value of:

a $5m$ if $m = 7$

b $2a + 3b$ if $a = 3$, $b = 4$

c $10c$ if $c = -4$

d $7x - 2y$ if $x = 5$, $y = 8$

e a^2 if $a = 9$

f $\frac{24}{x}$ if $x = 6$

g $m - 5$ if $m = -3$

h $\frac{1}{4}c$ if $c = 100$

i $\sqrt{p + 6}$ if $p = 10$

j $3m + n$ if $m = 6$ and $n = 5$

k xy if $x = 4$ and $y = 7$

l $20 - 3k$ if $k = 9$

m $\frac{t + 5}{6}$ if $t = 25$

n $3(x + 2)$ if $x = 10$

2 If $x = 3$, $y = 5$, $z = -2$, find the value of:

a xy

b $x^2 + z^2$

c xyz

d $(x + y)^2$

e $2x + y$

f $y - 3z$

g $x + y + z$

h $2x^2$

i $x - y - z$

j $(z + 6)^3$

k $2xz$

l $3x + 5y$

m $x(y + z)$

n $\frac{6x}{z}$

o $\frac{7x - y}{8}$

p $2x + 3y + 4z$

Fun Spot 12:02 | Playing truant from school is like having a credit card.

Find the value of each expression by substituting the given values.
Match the letters with the answers below.

Use $a = 8$ to find: **A** $2a$ **E** $10 - a$ **F** a^2 **L** $a - 8$

Use $x = 10$ to find: **N** $\frac{1}{2}x$ **O** $5x$ **P** $2x + 5$

Use $m = 5$, $n = 3$ to find: **R** $m + n$ **S** $2m + 3n$ **T** mn

Use $p = 4$, $q = -3$ to find: **U** $p^2 - q^2$ **W** $3pq$ **Y** $2p - 8q$

0	50	15	19		50	64		64	7	5		5	50	−36	,	25	16	32	,	0	16	15	2	8	.

Answers can be found in the Interactive Student CD.

Foundation Worksheet

9:05 | Translating Problems into Equations

Name: _____ **Class:** _____

Examples

1 Write equations for the following.

a The difference between x and 10 is 20.

$$x + 10 = 20$$

b The product of 6 and y is 84.

$$6y = 84$$

2 Write equations for the following, and solve them.

a The sum of 5 times a number and 17 results in 62.

$$5n + 17 = 62$$
$$5n = 45$$
$$n = 9$$

b If I add 13 to a number and then divide by 4 the result is 6.

$$\frac{n + 13}{4} = 6$$
$$n + 13 = 24$$
$$n = 11$$

Exercise

1 Write equations for the following.

a The sum of a and 5 equals 8.

b The product of 7 and x equals 42.

c The difference between y and 13 equals 7.

d The sum of $4m$ and 7 equals 29.

e The product of 10 and k is -70.

f The quotient of x and 8 is 4.

g A number, n, minus 5 gives 17.

h The product of 4 and x, less 7, equals 23.

2 Write equations for the following, then solve them. (Use n for the number.)

a If I subtract 9 from 3 times a number the result is 27.

b If I divide a number by 6 the result is 5.

c When 15 is subtracted from a number the result is -3.

d A number is multiplied by 4, and then 7 is added, giving 57.

e I add 6 to a number then divide by 3. The result is 15.

f If I add 5 to a number then multiply by 3, the result is 39.

g Double a number, add 17 and the result is 53.

h Think of a number, multiply by 3, subtract 7; the result is 44.

i If I divide 5 times a number by 4, the result is 12.

j The product of a number and 6, less 5, gives 19.

k If I subtract 7 from a number, then divide by 2 the result is 18.

l If I divide a number by 4 the result is -6.

m Subtract 4 from a number, then multiply by 5; the result is 60.

n The sum of 8 times a number and 3 equals 19.

o If I subtract 4 from 7 times a number the result is 17.

Answers can be found in the Interactive Student CD.

Foundation Worksheet

10:01 | Adjacent Angles

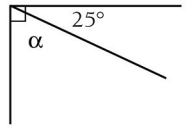
Name: _____ **Class:** _____

Examples

- Adjacent angles forming a right angle are complementary (add to 90°).
- Adjacent angles forming a straight angle are supplementary (add to 180°).

Find the value of the pronumerals.

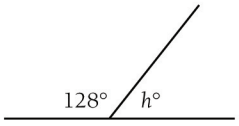
1



$\alpha + 25 = 90°$

$\alpha = 65°$

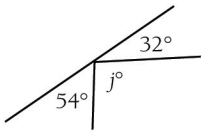
2



$h + 128 = 180$

$h = 52$

3



$j + 54 + 32 = 180$

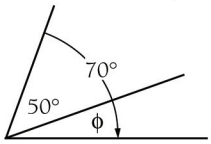
$j + 86 = 180$

$j = 94$

Exercise

1 Find the size of ϕ in each of the following.

a



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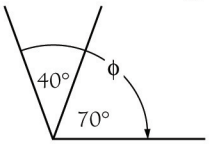

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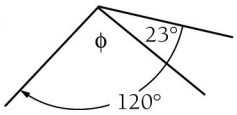

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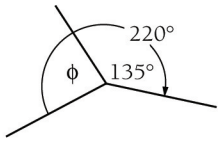
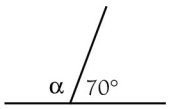


2 Find the size of α in each of the following.

a



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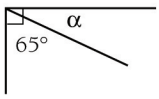

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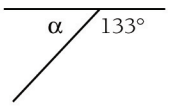

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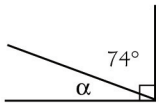
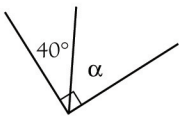


e



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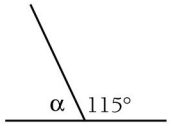
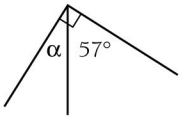
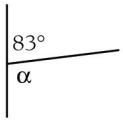


g

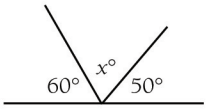


h

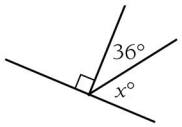


3 Find the value of the pronumeral.

a



b



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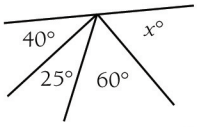

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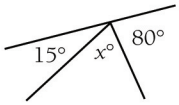

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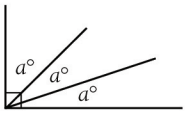

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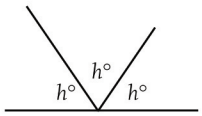

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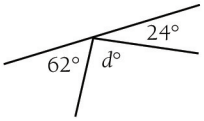

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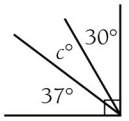


Fun Spot 5:01 | What do you get when you cross a vampire with a snowman?

Match the letters with the answers below.

Write the complement of: **B** 17° **E** 30° **F** 52° **I** 76°

Write the supplement of: **O** 100° **R** 65° **S** 22° **T** 140°

38°	115°	80°	158°	40°	73°	14°	40°	60°

Answers can be found in the Interactive Student CD.

Foundation Worksheet

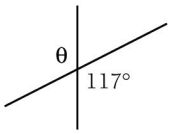
10:02 | Angles at a Point

Name: _____ **Class:** _____

Examples

- Vertically opposite angles are equal.
- The angles at a point (or in a revolution) add to 360°.

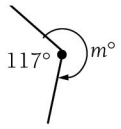
1 Find θ.



$\theta = 117°$
(vertically opposite ∠s)

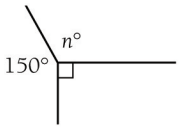
2 Find the value of the pronumeral.

a



$m + 117 = 360$
$m = 360 - 117$
$= 243$
(angles at a point)

b

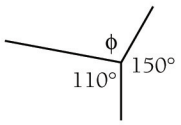


$n + 150 + 90 = 360$
$n + 240 = 360$
$n = 120$
(angles at a point)

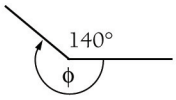
Exercise

1 Find the size of ϕ.

a



b



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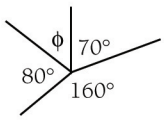

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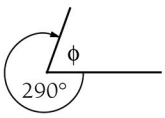
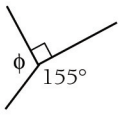


e



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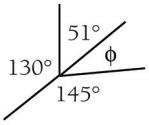
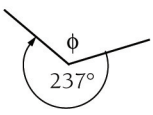
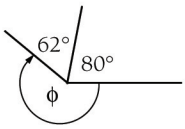


g



h



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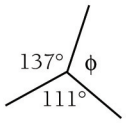
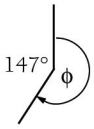
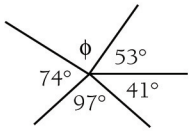


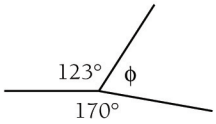
j



k

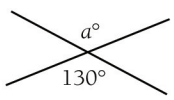


l

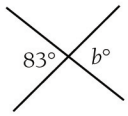


2 Find the value of the pronumeral.

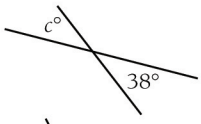
a



b



c



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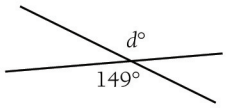
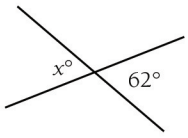
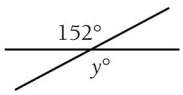


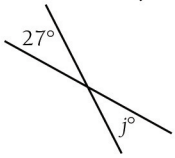
e



f



g



h

Fun Spot 5:02 | What do sea monsters eat for a snack?

Solve each equation and match the letters with answers below.

A $x + 52 = 360$ **H** $x + 222 = 360$ **I** $x + 123 = 360$ **O** $x + 98 = 360$

P $x + 57 + 63 = 360$ **S** $x + 120 + 110 = 360$ **T** $x + 72 + 154 = 360$

240	262	134	308	134	262

130	138	237	240	130

Answers can be found in the Interactive Student CD.

Foundation Worksheet

10:05 | Angles and Parallel Lines

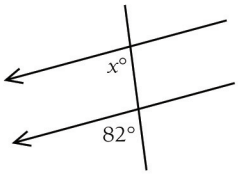
Name: _____ **Class:** _____

Examples

Find the value of the pronumerals, giving reasons.

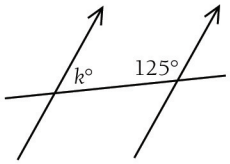
- Corresponding → F-shape
- Co-interior → U-shape
- Alternate → N-shape
- Remember that maths can be FUN!

1



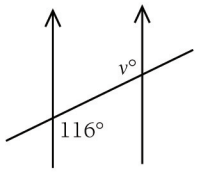
$x = 82$
(corresponding ∠s with parallel lines)

2



$k + 125 = 180$
$k = 55$
(co-interior ∠s with parallel lines)

3

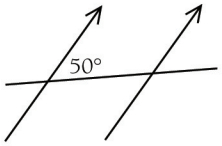


$v = 116$
(alternate ∠s with parallel lines)

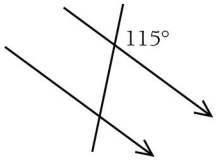
Exercise

1 Write down the size of all missing angles.

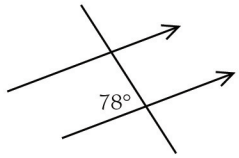
a



b

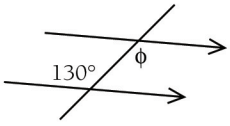


c



2 Find the size of ϕ.

a



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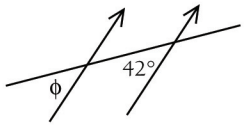
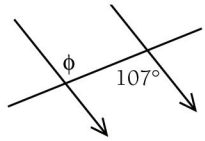


c



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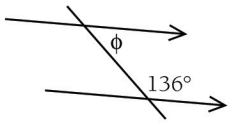

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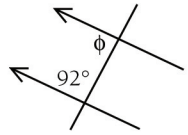

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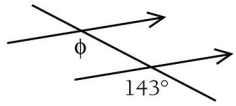
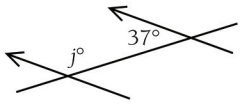
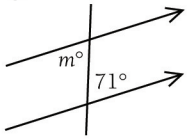


3 Find the value of the pronumerals, giving reasons.

a



b



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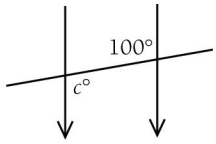

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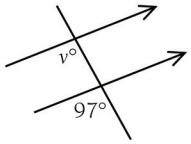

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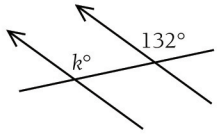

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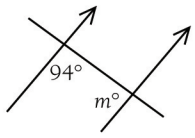
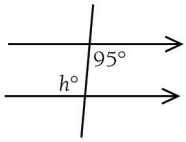
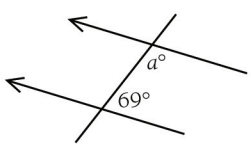


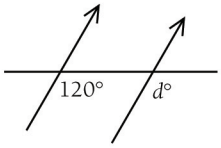
g



h



i



Answers can be found in the Interactive Student CD.

Foundation Worksheet

10:06 | Triangles

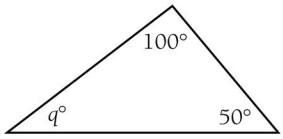
Name: _____ **Class:** _____

Examples

- The angle sum of a triangle is 180°.
- In an isosceles triangle two sides and two opposite angles are equal.

Find the value of the pronumerals.

1

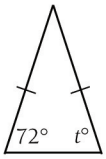


$q + 100 + 50 = 180$

$q + 150 = 180$

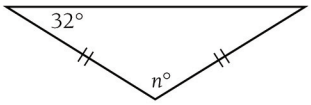
$\therefore q = 30$

2



$t = 72$

3



$n + 32 + 32 = 180$

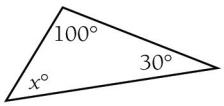
$n + 64 = 180$

$\therefore n = 116$

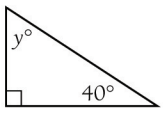
Exercise

1 Find the value of the pronumeral.

a



b



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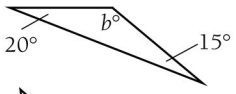

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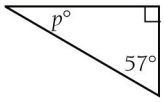

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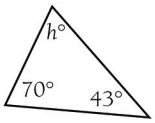

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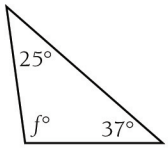

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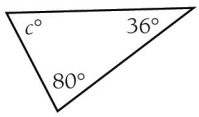

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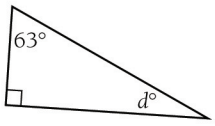
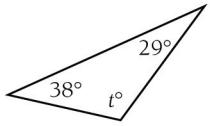
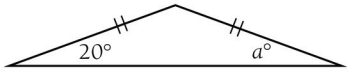


i



2 Find the value of the pronumeral.

a



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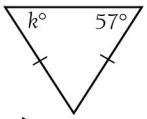

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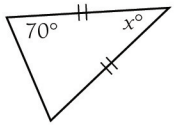

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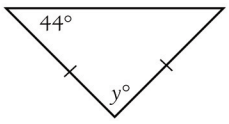

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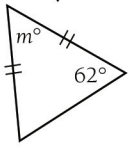

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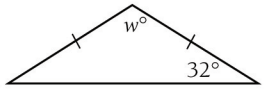


Fun Spot 5:06 | What was more useful than the invention of the first telephone?

Match the letters with the answers below.

C $x + 70 + 30 = 180$ **D** $m + 20 + 90 = 180$ **E** $y + 50 + 90 = 180$

H $p + 17 + 32 = 180$ **L** $a + 72 + 55 = 180$ **N** $t + 58 + 58 = 180$

O $c + 11 + 78 = 180$ **P** $x + 104 + 17 = 180$ **S** $k + 80 + 40 = 180$ **T** $h + 54 + 90 = 180$

36	131	40	60	40	80	91	64	70	36	40	53	40	59	131	91	64	40		

Answers can be found in the Interactive Student CD.

Foundation Worksheet

10:09 | Formulae

Name: _____ **Class:** _____

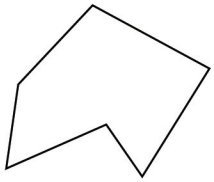
Examples

This polygon has 6 sides.

$\therefore$ angle sum $= (6 - 2) \times 180°$

$= 4 \times 180°$

$= 720°$



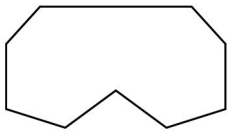
The angle sum of the polygon with n sides is found using the formula $(n - 2) \times 180°$.

Exercise

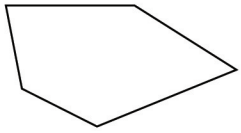
1 Count the number of sides, n, for each polygon.

a

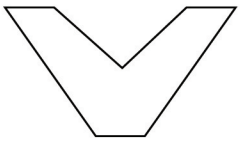
b



c



d



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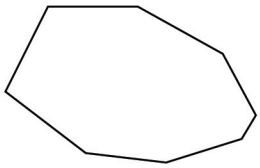

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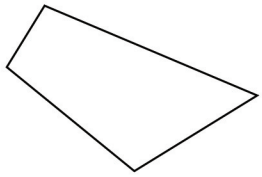
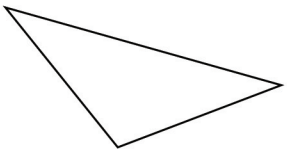


g



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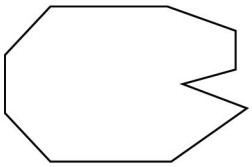
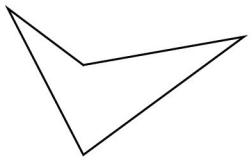


i



2 Evaluate $(n - 2) \times 180$ if n is:

a 5	**b** 8	**c** 4	**d** 6	**e** 10
f 3	**g** 7	**h** 9	**i** 12	**j** 20

3 Find the angle sum of the polygons in Question **1**.

Fun Spot 5:09 | What do you call a man with an elephant on his head?

Match the name of the polygon with the number of sides below.

A pentagon	**D** septagon	**E** quadrilateral	
H decagon	**Q** nonagon	**S** hexagon	**U** dodecagon

								!
6	9	12	5	6	10	4	7	

Answers can be found in the Interactive Student CD.

Foundation Worksheet

12:02 | Perimeter

Name: _____ **Class:** _____

Examples

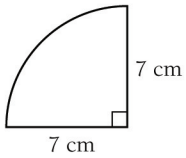
- **The perimeter is the total length around a shape.**
- **For a circle, perimeter = circumference = $\pi \times$ diameter $= 2\pi \times$ radius**

1 Find the perimeter of these shapes.

a $P = \frac{1}{4}$ circle $+ 7 \times 2$

$= \frac{1}{4} \times 2\pi \times 7 + 7 \times 2$

$\div 25$ cm



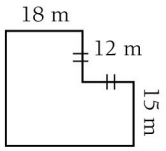
b Same markings = same length

Bottom = 18 + 12 = 30

Left side = 12 + 15 = 27

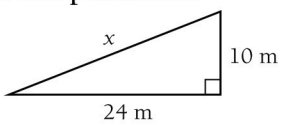
$P = 15 + 2 \times 12 + 18 + 27 + 30$

$= 114$ m



2 **a** Find x using Pythagoras' theorem.

b Find the perimeter.



a $x^2 = 10^2 + 24^2$

$= 676$

$x = \sqrt{676}$

$= 26$

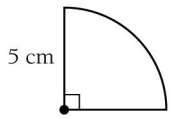
b $P = 10 + 24 + 26$

$= 60$ m

Exercise

1 Find the perimeter of these shapes, correct to the nearest whole.

a



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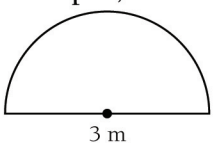

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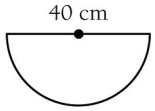
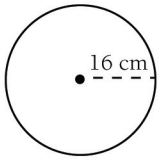


d



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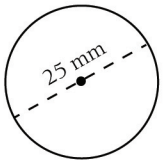

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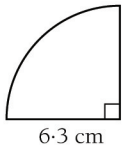

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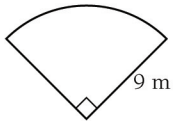
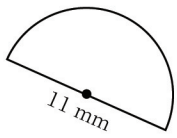
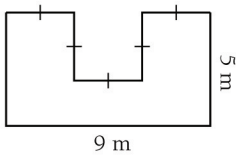


h



2 Find the perimeter of these shapes.

a



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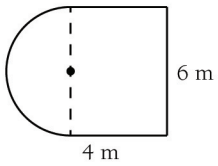

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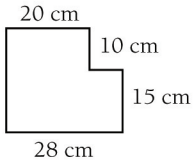

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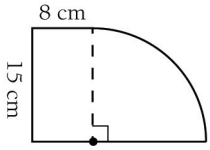

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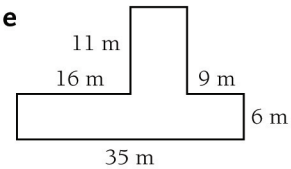

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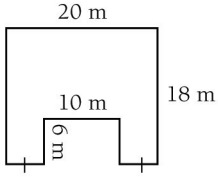
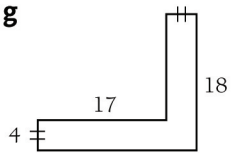
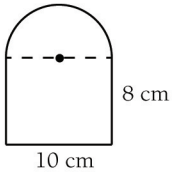


g



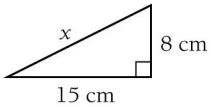
h



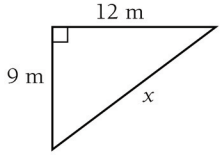
3 **a** Use Pythagoras' theorem to find x in the following figures.

b Find the perimeter of the figure.

i



ii



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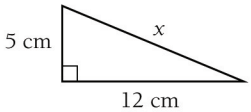
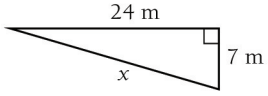


iv



Answers can be found in the Interactive Student CD.

Foundation Worksheet

12:04 | Area

Name: _____ **Class:** _____

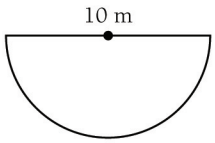
Examples

Find the area of the following shapes.

1 Area $= \frac{1}{2}$ circle

$= \frac{1}{2} \times \pi \times 5^2$

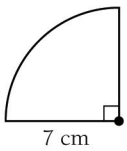
$\div 39{\cdot}3 \text{ m}^2$



2 Area $= \frac{1}{4}$ circle

$= \frac{1}{4} \times \pi \times 7^2$

$\div 38{\cdot}5 \text{ cm}^2$



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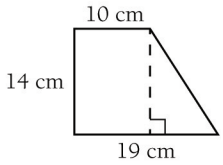
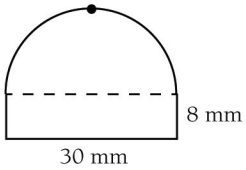


Area of rectangle $= 14 \times 10 = 140 \text{ cm}^2$

Area of triangle $= \frac{1}{2} \times 9 \times 14 = 63 \text{ cm}^2$

Total area $= 203 \text{ cm}^2$

4



Area of rectangle $= 30 \times 8 = 24 \text{ mm}$

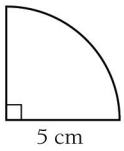
Area of semicircle $= \frac{1}{2} \times \pi \times 15^2 \div 353 \text{ mm}^2$

Total area $\div 377 \text{ mm}^2$

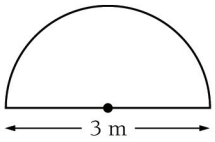
Exercise

1 Find the area of these shapes, correct to 1 decimal place.

a



b



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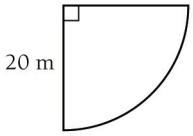

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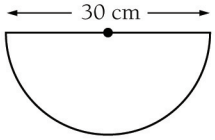

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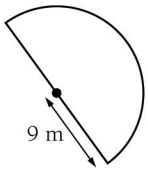

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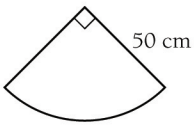

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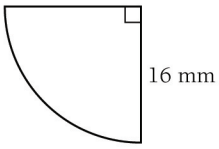

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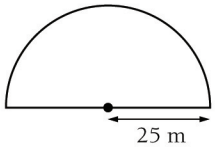
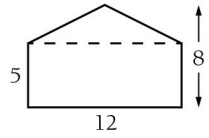


2 Find the area, correct to the nearest whole.

a



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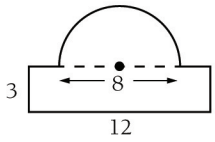

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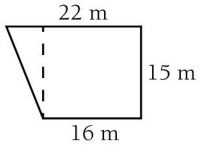

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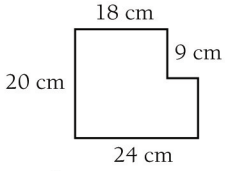

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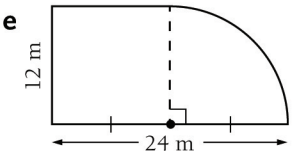

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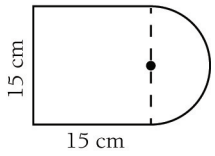

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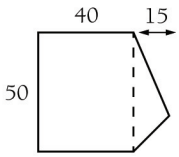

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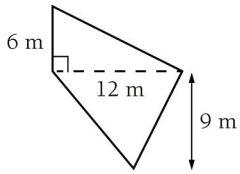

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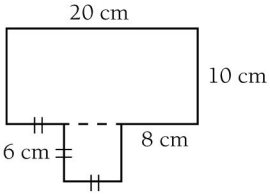

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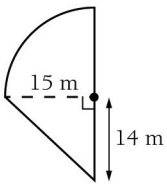
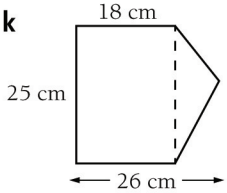


k



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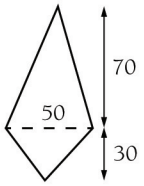


Answers can be found in the Interactive Student CD.

Foundation Worksheet

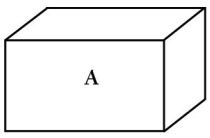
12:05 | Surface Area of Prisms

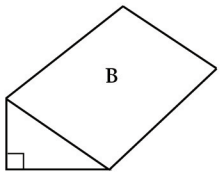
Name: _____ **Class:** _____

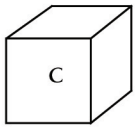
- The surface area is the sum of the areas of all the faces of a prism.
- If you imagine opening out a solid onto a flat surface you see the *net*. The area of the net gives the surface area.

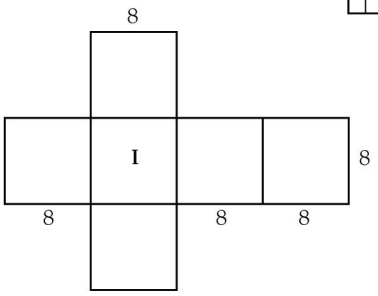
Exercise

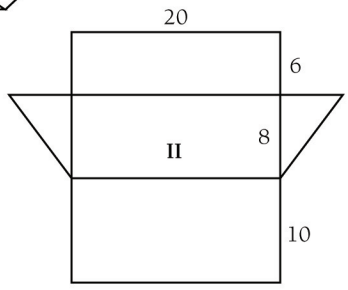
1 a Match each solid to its net.

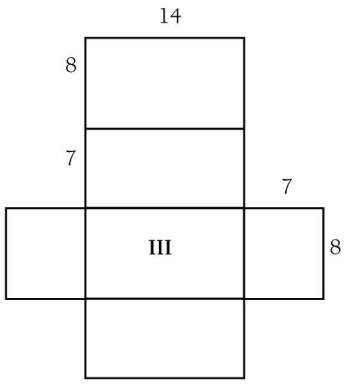










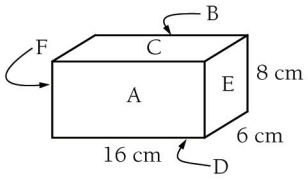


b Find the area of each net.

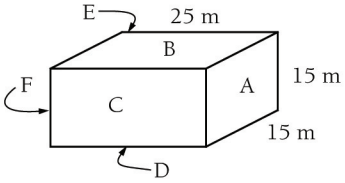
2 Complete the table for the given rectangular prism.

a

Face	A	B	C	D	E	F
Area						



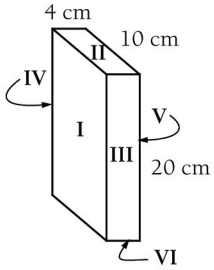
b



Face	A	B	C	D	E	F
Area						

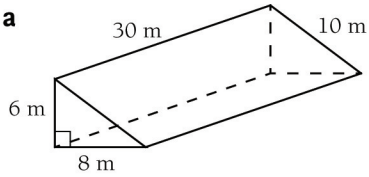
c

Face	I	II	III	IV	V	VI
Area						



3 Find the surface area of each triangular prism.

a



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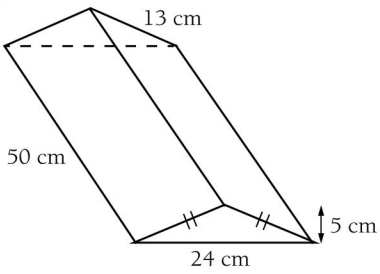

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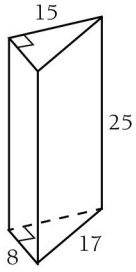


Answers can be found in the Interactive Student CD.

Foundation Worksheet

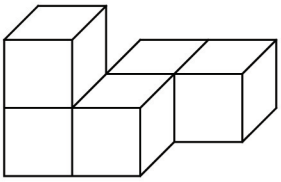
12:06 | Surface Area of Composite Solids

Name: _____ **Class:** _____

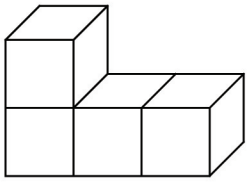
Exercise

1 Find the surface area of these solids, which are built from 1 cm cubes.

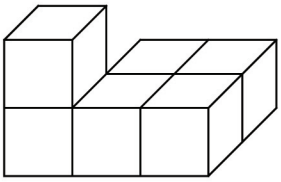
a



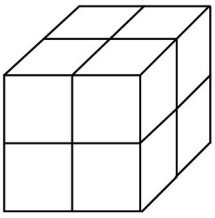
b



c



d



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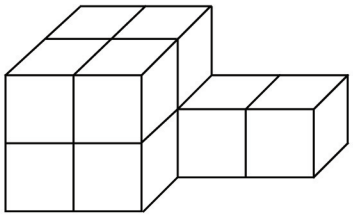

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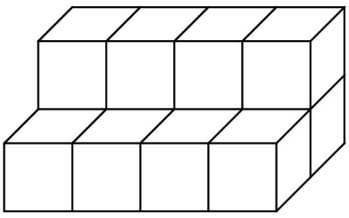
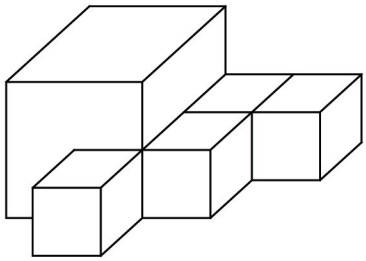
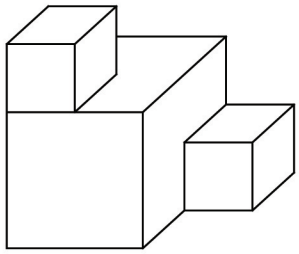


2 Cubes with sides of 1 cm or 2 cm have been used to build these solids. Find the surface area of each solid.

a



b



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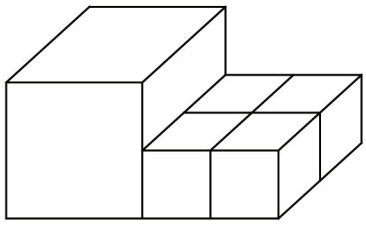
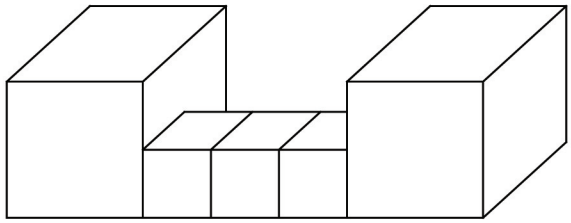
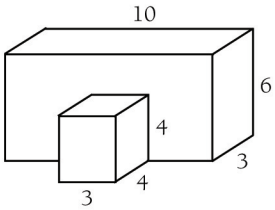


d



3 Find the surface area of the following composite solids.

a



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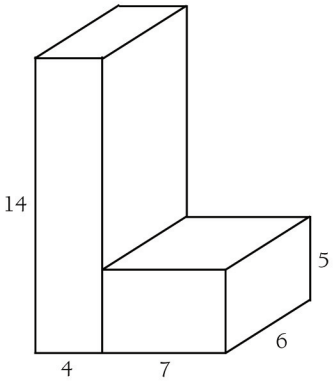

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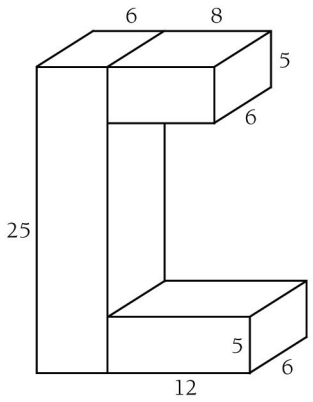


Answers can be found in the Interactive Student CD.

Foundation Worksheet

13:01 | Frequency and Cumulative Frequency

Name: _____ **Class:** _____

Example

Arrange these scores into a frequency distribution table:
0, 1, 1, 0, 3, 2, 1, 0, 1, 2, 2, 0, 1, 4, 1, 3, 2, 1, 2, 3, 3, 2, 0, 4, 3

Score	*Tally*	*Frequency*	*Cumulative frequency*
0	𝍸	5	5
1	𝍸 ||	7	12
2	𝍸 |	6	18
3	𝍸	5	23
4	||	2	25

Total = 25

Tally: Mark off each score in turn.
Frequency: Total of tally.
Cumulative frequency: Add the frequency line by line to keep a running total of the scores.

Exercise

1 Arrange these scores into a frequency distribution table.

a 5, 4, 3, 4, 5, 5, 4, 3, 5, 6, 7, 4, 3, 5, 5, 5, 5, 4, 3, 6

Score	*Tally*	*Frequency*

b 22, 23, 22, 21, 24, 23, 24, 22, 25, 23, 22, 21, 22, 23, 22, 24, 25, 21, 23, 23, 24, 24, 22, 21, 22, 23, 24, 21, 22, 23

Score	*Tally*	*Frequency*

c 40, 42, 41, 42, 43, 41, 44, 43, 42, 40, 41, 44, 43, 40, 40, 42, 41, 42, 43, 43, 42, 41, 44, 42, 41, 43, 40, 41, 42, 42

Score	*Tally*	*Frequency*

2 Referring to the scores above find:

a i how many scores of 3 there were
ii how many scores were above 5
b i how many 23s there were
ii how many scores were less than 24
c i how many scores were below 42
ii how many scores were 42 or more

3 Add a cumulative frequency column to each of the following tables.

a

Score	*Frequency*
1	3
2	2
3	4
4	3
5	2
6	1

b

Score	*Frequency*
0	3
10	5
20	7
30	8
40	6
50	1

c

Score	*Frequency*
10	4
11	7
12	12
13	8
14	5

d

Score	*Frequency*
74	8
75	12
76	15
77	17
78	13
79	8
80	7

e

Score	*Frequency*
20	4
22	7
24	8
26	9
28	6
30	3
32	0
34	3

f

Score	*Frequency*
1	3
2	4
3	6
4	10
5	9
6	7
7	8
8	0
9	4
10	4

Answers can be found in the Interactive Student CD.

Foundation Worksheet

13:02 | Mean, Median and Mode

Name: _____ **Class:** _____

Examples

- Mean = average $= \dfrac{\text{total of scores}}{\text{number of scores}}$
- Median = middle score when they are in order (or the average of the 2 middle scores)
- Mode = most common score

Find the mean, median and mode of each set of scores.

1 1, 1, 2, 2, 3, 3, 3, 4, 5

$\text{Mean} = \dfrac{1+1+2+2+\ldots}{9}$

$= \dfrac{24}{9} = 2\dfrac{2}{3}$

1 1 2 2 (3) 3 3 4 5

Median = 3

Mode = 3

2 6, 4, 5, 8, 2, 6, 7

$\text{Mean} = \dfrac{6+4+5+\ldots}{7}$

$= \dfrac{38}{7} = 5\dfrac{3}{7}$

2 4 5 (6) 6 7 8

Median = 6

Mode = 6

3 4, 10, 12, 5, 6, 7, 16, 6, 14, 6

$\text{Mean} = \dfrac{4+10+12+5+\ldots}{10}$

$= \dfrac{86}{10} = 8{\cdot}6$

4 5 6 6 (6 7) 10 12 14 16

$\text{Median} = \dfrac{6+7}{2} = 6{\cdot}5$

Mode = 6

Exercise

1 For the following sets of scores, find the mean (1 dec. pl. where necessary).

- **a** Batsman's scores: 48, 57, 20, 0, 15, 48, 72
- **b** Golf hole scores: 4, 5, 6, 4, 4, 5, 4, 4, 3, 5
- **c** Quiz marks: 8, 9, 7, 6, 7, 9, 10, 7, 10
- **d** Number of children: 0, 1, 0, 2, 3, 5, 2, 2, 2, 1, 4, 3, 2, 3, 1
- **e** Hannah's test results: 62, 67, 68, 74, 76, 78, 89, 92
- **f** Rainy days per week: 3, 1, 0, 4, 0, 0, 2, 2, 3
- **g** Overnight lowest temperature: 9, 9, 10, 4, 10, 5, 9

2 For each set of scores in Question **1**, what is: **i** the median? **ii** the mode?

3 Which of the mean, median or mode is of most use in summarising:

- **a** the batsman's scores?
- **b** the golfer's scores?
- **c** the quiz marks?
- **d** the number of children?

Fun Spot 11:02 | What goes through water but doesn't get wet?

Find the mean, median or mode as indicated for the sets of scores.
Match the appropriate letter with the answer below.

6, 7, 8, 10, 14 **A** mean **G** median

1, 2, 2, 3, 5, 5, 5 **H** mode **I** median

10, 12, 14, 20 **L** median **R** mean

4, 4, 7, 15, 20 **S** mean **T** median **Y** mode

13	3	8	5	7		14	9	4	10

Answers can be found in the Interactive Student CD.

Foundation Worksheet

13:04 | Mean and Median

Name: _____ **Class:** _____

Exercise

1 Calculate the mean, median and mode for each of the following.

a 3, 3, 3, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 7

b 10, 10, 10, 11, 11, 11, 12, 12, 12, 12, 12, 12

c 1, 1, 2, 3, 3, 5, 5, 5, 7, 8, 9, 9, 9, 9, 10

d 25, 23, 27, 22, 21, 22, 25, 25, 28

2 Calculate the mean, median and mode from these frequency distribution tables.

a

Score (x)	*Frequency (f)*	$f \times x$	*Cumulative frequency (cf)*
0	3		
1	6		
2	9		
3	5		
4	2		
	Total =	Total =	

b

x	f	$f \times x$	cf
5	2		
6	8		
7	11		
8	13		
9	7		
10	9		
	Total =	Total =	

c

x	f	$f \times x$	cf
0	1		
10	5		
20	7		
30	8		
40	3		
50	1		
	Total =	Total =	

d

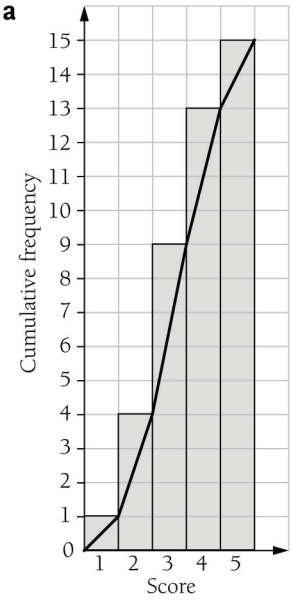
x	f	$f \times x$	cf
15	2		
16	3		
17	5		
18	1		
19	4		
	Total =	Total =	

e

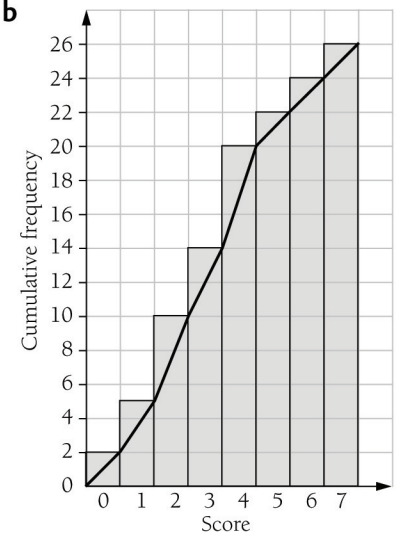
x	f	$f \times x$	cf
97	6		
98	13		
99	7		
100	3		
	Total =	Total =	

3 Use the following cumulative frequency diagrams to find the median.

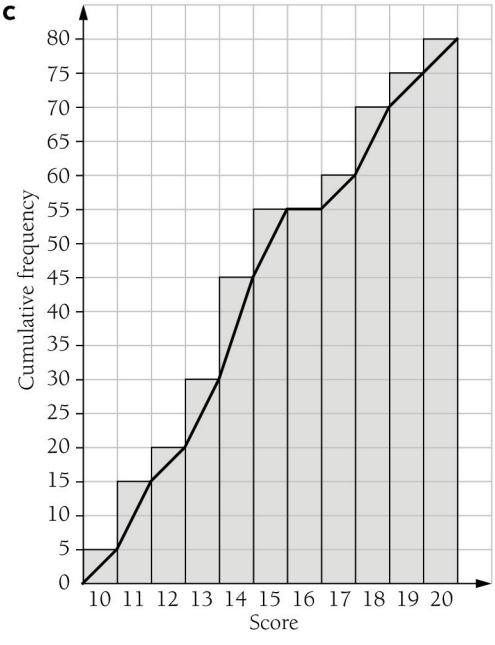
a



b



c



Answers can be found in the Interactive Student CD.

Foundation Worksheet

14:02 | Experimental Probability

Name: _____ **Class:** _____

Examples

$$\text{Probability} = \frac{\text{number of times occurred}}{\text{total number}}$$

200 people were surveyed to find which TV station they watched at 7 pm one evening. The results were:

Channel 2 **35** Channel 7 **52** Channel 9 **63** Channel 10 **40** SBS **10**

Find the probability that one of these people chosen at random:

1 watched Channel 10

Probability $= \frac{40}{200}$

$= \frac{1}{5}$

(Simplify.)

2 didn't watch 2

All except 35 = 200 − 35

Probability $= \frac{165}{200}$

$= \frac{33}{40}$

3 watched 7 or 10

Total = 52 + 40

Probability $= \frac{92}{200}$

$= \frac{23}{50}$

Exercise

1 In a class of 24, 12 were blonde, 10 were brunette, 2 were red-heads. Find the probability that one student chosen at random will be:

a blonde **b** not brunette

2 A survey of 60 Year 7 students showed that 12 agreed that calculators shouldn't be used in Mathematics tests, 40 disagreed and the rest were unsure. Find the probability that a student chosen at random would:

a disagree **b** be unsure

3 A spinning wheel has 5 sectors marked 1 to 5. The results of 100 spins were recorded. The results were:

sector 1: 16 times, sector 2: 25 times, sector 3: 21 times, sector 4: 19 times, sector 5: 19 times

a Find the probability of spinning:

i 1 **ii** an even-numbered sector **iii** sector 4 or higher

b Which number appears least likely to turn up?

4 Our class rolled 2 dice 1000 times between them and recorded the total.

Total on dice	2	3	4	5	6	7	8	9	10	11	12
Number of rolls	20	54	93	102	130	189	143	117	80	47	25

a Based on these results, what is the probability of rolling a total of:

i 4? **ii** 4 or less? **iii** 10? **iv** 6 or 7? **v** more than 7?

b Why do you think it's easier to roll a total of 10 than to roll 12?

5 The 40 households on our block were checked for their number of pets. 16 had 3 pets, 13 had 2, 8 had 1, 3 had none. Find the probability that a household chosen at random had:

a no pets **b** at least 1 pet **c** 1 or 3 pets

6 After checking 100 CD cases, a store noticed that 3 were cracked.

a What is the probability you'd find an uncracked one?

b How many cracked cases would you expect on checking 2000 CDs?

Answers can be found in the Interactive Student CD.

Foundation Worksheet

14:03 | Theoretical Probability

Name: _____ **Class:** _____

Examples

$$\text{Probability} = \frac{\text{number of times possible}}{\text{total number}}$$

In a jar there are 8 chocolates, 3 toffees and 4 caramels. If one is chosen at random, what is the probability it will be:

1 chocolate?
8 chocolates out of 15
Probability $= \frac{8}{15}$

2 chocolate or caramel?
8 chocolates and 4 caramels
Probability $= \frac{12}{15} = \frac{4}{5}$

3 not caramel?
4 caramels, 11 not
Probability $= \frac{11}{15}$

Exercise

1 A coin is tossed. Find the probability of:
a a head **b** a tail

2 Four cards are labelled A, B, C, D. Find the probability of selecting the card with:
a A **b** B or C **c** E

3 On a shelf there are 7 CDs by males, 6 CDs by females, 9 by groups and 2 compilations. If one is chosen at random to play, find the probability that it is:
a by a male **b** by a male or female **c** not a compilation

4 A letter is chosen at random from the word PARRAMATTA. What is the probability that the letter is:
a A? **b** not A? **c** R? **d** R or T?

5 A raffle has 50 tickets numbered 1 to 50. What is the probability:
a Josie wins if she buys all the tickets containing a 4?
b Veronica wins if she buys any ticket with a 7 on it?
c Kerrie wins if she buys any ticket which is a multiple of 3?

6 A bag contains 20 marbles: 10 blue, 2 white, 3 green, 5 red. What is the probability that a marble taken at random will be:
a white? **b** white or blue? **c** green or red? **d** not red?

7 A six-sided die is rolled. Find the probability the top face is:
a a 5 **b** a factor of 6 **c** a prime **d** less than 7

8 The names Kerrie, John, Mark, Paul and Lindy are placed on gift tags ready for Christmas wrapping. If I choose one without looking, find the chance I select:
a the correct one for the gift **b** a boy's tag
c a 4-letter name **d** the wrong one

Answers can be found in the Interactive Student CD.

Challenge Worksheet

4:03 | Algebraic Simplifications

Name: _____ **Class:** _____

Examples

1 $(2x + 3x) \times 5 = 5x \times 5$
$= 25x$

2 $10a - 3 \times 2a = 10a - 6a$
$= 4a$

3 $(5a + 7a) \times (3b - 2b) = 12a \times b$
$= 12ab$

4 $3m \times 2n \div mn = 6mn \div mn$
$= \frac{6\cancel{mn}}{\cancel{mn}}$
$= 6$

5 $\frac{7p + 8p - 3p}{2p \times 3q} = \frac{{}^{2}\cancel{12p}}{{}_{1}\cancel{6pq}}$
$= \frac{2}{q}$

Remember, the order in which operations should be done.

↓

GROUPING SYMBOLS

↓

× ÷

↓

+ −

Exercise

Write the simplest expression for:

1 $(2a + 3a) \times 4$

2 $(10x - 3x) \div 7$

3 $(9b - 3b) \times 2$

4 $(3m + 9m) \div 4$

5 $12x \div (2x + x)$

6 $5a \times (10a + 2a)$

7 $3m \times (10m - 9m)$

8 $15y \div (9y - 2y)$

9 $5a \times 7 \div a$

10 $8x \times 4y \div 2xy$

11 $10a \div 5 \times 3a$

12 $9xy \div 3x \times 2y$

13 $2x + 3x \times 4$

14 $5x \times 3x + 10x^2$

15 $20y - 5 \times 2y$

16 $18m - 12m \div 6$

17 $3 \times 2n + 5n \times 4$

18 $7x + 3 \times 2x - 10x$

19 $8x \div 4 - x$

20 $11m + 18m \div 2$

21 $\frac{6 \times 3x}{2x \times 5}$

22 $\frac{3p + 2p - p}{2 \times 2p}$

23 $\frac{11y - y}{6y + 4y}$

24 $\frac{5a \times 4b \times 2c}{10c \times b \times 8c}$

Answers can be found in the Interactive Student CD.

Challenge Worksheet

4:04 | Algebraic Fractions

Name: _____ **Class:** _____

Examples

1 $\frac{x}{2} \times \frac{x}{3} + \frac{x}{4} \times \frac{x}{5}$

$= \frac{x^2}{6} + \frac{x^2}{20}$

$= \frac{10x^2}{60} + \frac{3x^2}{60}$

$= \frac{13x^2}{60}$

2 $\frac{a}{3} \div \left(\frac{a}{4} \times \frac{a}{6}\right)$

$= \frac{a}{3} \div \frac{a^2}{24}$

$= \frac{a}{3} \times \frac{24}{a^2}$

$= \frac{24a}{3a^2}$

$= \frac{8}{a}$

Exercise

Simplify each of the following.

1 a $\frac{x}{2} + \frac{x}{3} + \frac{x}{4}$ **b** $\frac{3p}{2} - \frac{p}{4} - \frac{3p}{5}$

2 a $\frac{5m}{6} \div \frac{10m}{3} \times \frac{m}{5}$ **b** $\frac{5m}{6} \div \left(\frac{10m}{3} \times \frac{m}{5}\right)$

3 a $\frac{ab}{4} \div \frac{a}{3} - \frac{b}{2}$ **b** $\frac{n}{6} + \frac{2m}{5} \times \frac{15n}{4m}$

4 a $\left(\frac{m}{3} + \frac{m}{2}\right) - \left(\frac{m}{4} - \frac{m}{5}\right)$ **b** $\left(\frac{2a}{3} + \frac{3a}{2}\right) - \left(\frac{5a}{4} - \frac{4a}{5}\right)$

5 a $\frac{x}{2} \times \frac{y}{3} + \frac{x}{4} \times \frac{y}{2}$ **b** $\frac{x}{2} \div \frac{y}{3} + \frac{x}{4} \div \frac{y}{2}$

6 a $\left(5x - \frac{3x}{4}\right) \times \frac{y}{2} - \frac{3xy}{8}$ **b** $\left(\frac{a}{3} + \frac{a}{4}\right) \div \left(\frac{a}{3} - \frac{a}{4}\right)$

Answers can be found in the Interactive Student CD.

Challenge Worksheet

6:04 | Algebraic Expressions and Indices

Name: _____ **Class:** _____

Examples

1 Write each of the following without using negative or fractional indices.

a $(x+1)^{-2}$

$= [(x+1)^2]^{-1}$

$= \dfrac{1}{(x+1)^2}$

b $(x+1)^{-\frac{1}{2}}$

$= \left[(x+1)^{\frac{1}{2}}\right]^{-1}$

$= \dfrac{1}{\sqrt{(x+1)}}$

c 2^{-x}

$= (2^x)^{-1}$

$= \dfrac{1}{2^x}$

2 Write each of the following in index form.

a $\dfrac{1}{(2x)^3}$

Let $p = 2x$

$\dfrac{1}{(2x)^3} = \dfrac{1}{p^3}$

$= p^{-3}$

$= (2x)^{-3}$

b $\dfrac{1}{(x+3)^3}$

Let $p = x + 3$

$\dfrac{1}{(x+3)^3} = \dfrac{1}{p^3}$

$= p^{-3}$

$= (x+3)^{-3}$

c $\dfrac{1}{\sqrt{e^x+1}}$

Let $p = e^x + 1$

$\dfrac{1}{\sqrt{e^x+1}} = \dfrac{1}{p^{\frac{1}{2}}}$

$= p^{-\frac{1}{2}}$

$= (e^x+1)^{-\frac{1}{2}}$

Exercise

1 Write each of the following without using negative or fractional indices.

a $(x+2)^{-3}$ **b** $(x+2)^{\frac{1}{2}}$ **c** $(x+2)^{-\frac{1}{2}}$

d 3^{-x} **e** 3^{-2x} **f** $e^{-(x)}$

g $(e^x+2)^{-1}$ **h** $(e^{x+2})^{-1}$ **i** $(e^x+2)^{\frac{1}{2}}$

2 Write each of the following in index form.

a $\sqrt{2x+1}$ **b** $\sqrt{x^2+1}$ **c** $\sqrt{e^x}$

d $\dfrac{1}{\sqrt{x}}$ **e** $\dfrac{1}{(x+2)^2}$ **f** $\dfrac{1}{\sqrt{x+2}}$

g $x\sqrt{x+1}$ **h** $\dfrac{x}{\sqrt{x+1}}$ **i** $\dfrac{x^2}{x+1}$

Answers can be found in the Interactive Student CD.

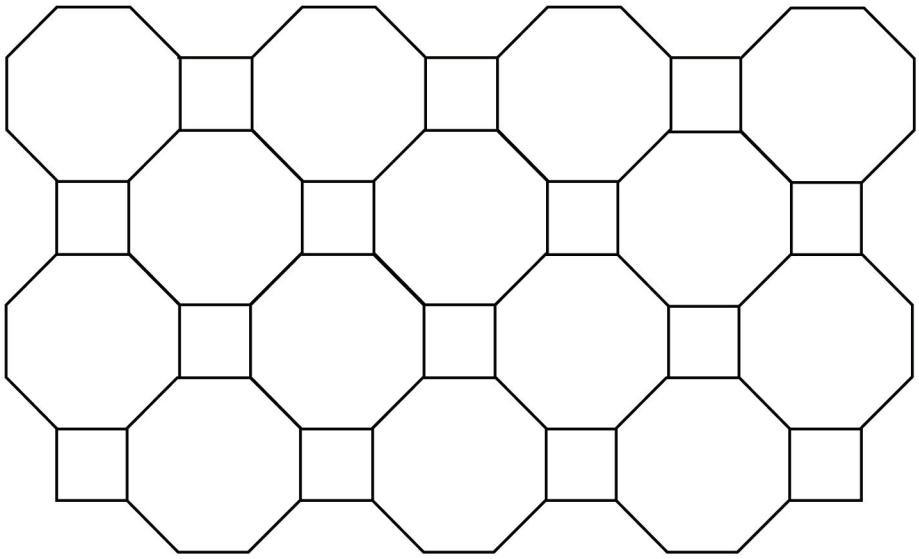
Challenge Worksheet

10:09 | Regular Polygons and Tessellations

Name: _____ **Class:** _____

When a tessellation (tiling pattern) is made up of regular polygons of only one shape it is said to be a regular tessellation.

In Investigation 5:09C it was found that there are only 3 types of regular tessellations.
Semi-regular tessellations are formed by using one or more types of regular polygons in such a way that the order of polygons at each vertex is the same. For instance, in the tessellation pictured, at each vertex the polygons are arranged clockwise in the order square-octagon-octagon or (4-8-8).
There are only 8 types of semi-regular tessellations.



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Exercise

Use the table below to find the other 7 semi-regular tessellations.

Angles of regular polygons

Polygon	*Sides*	*Angle size*
Triangle	3	60°
Square	4	90°
Pentagon	5	108°
Hexagon	6	120°
Heptagon	7	$128\frac{4}{7}°$
Octagon	8	135°
Nonagon	9	140°
Decagon	10	144°
Undecagon	11	$147\frac{3}{11}°$
Dodecagon	12	150°

Answers can be found in the Interactive Student CD.

Challenge Worksheet

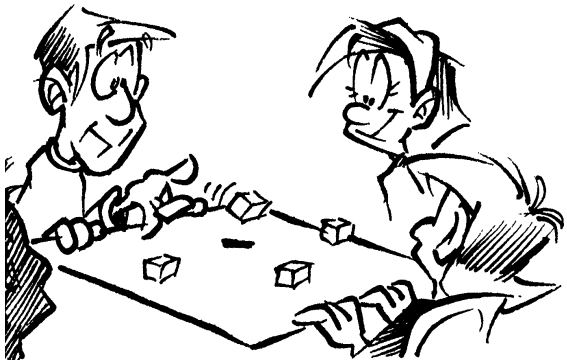
14 : 03 | Probability: An Unusual Case

Name: _____ **Class:** _____

Bradley Efron, a mathematician at Stanford University, invented a dice game that involves unexpected probabilities.

The faces of four dice are numbered as shown below.

Die A (used by Andrew) 1, 2, 3, 9, 10, 11
Die B (used by Ben) 0, 1, 7, 8, 8, 9
Die C (used by Cassie) 5, 5, 6, 6, 7, 7
Die D (used by Diane) 3, 4, 4, 5, 11, 12

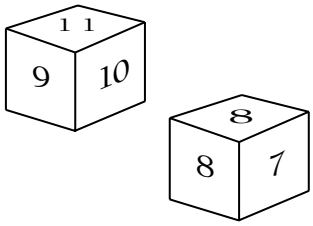


- Each player throws their die once.
- Andrew beats Ben if the number on his die is higher than the number on Ben's die.
- The table of outcomes on the right can be used to determine who has the greater probability of success.

$P(\text{A wins}) = \frac{22}{36}$

$P(\text{B wins}) = \frac{12}{36}$

$P(\text{tie}) = \frac{2}{36}$



Clearly, Andrew has the greater probability of winning.

Die B \ Die A	1	2	3	9	10	11
0	A	A	A	A	A	A
1	T	A	A	A	A	A
7	B	B	B	A	A	A
8	B	B	B	A	A	A
8	B	B	B	A	A	A
9	B	B	B	T	A	A

- A means die A wins.
- B means die B wins.
- T means a tie occurs.

Exercise

1 Complete the tables of outcomes below.

a

Die C \ Die B	0	1	7	8	8	9
5						
5						
6						
6						
7						
7						

b

Die D \ Die C	5	5	6	6	7	7
3						
4						
4						
5						
11						
12						

c

Die A \ Die D	3	4	4	5	11	12
1						
2						
3						
9						
10						
11						

2 **a** Should Andrew beat Ben? **b** Should Ben beat Cassie?
c Should Cassie beat Diane? **d** Should Diane beat Andrew?

3 Explain why this situation is so unusual.

4 **a** Should Andrew beat Cassie? **b** Should Ben beat Diane?

Answers can be found in the Interactive Student CD.

Foundation Worksheets

Answers

1:05 Decimals and Fractions

1 **a** 0·2 **b** 0·17 **c** 0·9 **d** 0·53 **e** 0·35 **f** 0·82 **g** 0·99 **h** 0·1
i 0·16 **j** 0·5 **k** 0·075 **l** 0·125 **m** 0·127 **n** $0{\cdot}8\dot{3}$ **o** $0{\cdot}\dot{6}$

2 **a** $\frac{3}{5}$ **b** $\frac{19}{20}$ **c** $\frac{6}{25}$ **d** $\frac{71}{200}$ **e** $\frac{2}{25}$ **f** $\frac{19}{100}$ **g** $\frac{3}{10}$ **h** $\frac{473}{1000}$
i $\frac{73}{100}$ **j** $\frac{7}{1000}$ **k** $\frac{3}{4}$ **l** $\frac{1}{40}$ **m** $\frac{11}{20}$ **n** $\frac{8}{25}$ **o** $\frac{7}{40}$

1:09 Approximation

1 **a** 7464 **b** 7460 **c** 7500
2 **a** 6·494 **b** 6·49 **c** 6·5
3 **a** 78 **b** 78·4 **c** 78·38
4 **a** 70 000 **b** 70 500 **c** 70 490
5 **a** 62·906 **b** 62·9 **c** 62·91
6 **a** 76 540 **b** 76 500 **c** 77 000
7 **a** 128 **b** 130 **c** 128·050
8 **a** 97 **b** 96·9 **c** 96·862

1:10 Estimation

1 **a** 15 **b** 60 **c** 540 **d** 90 **e** 70 **f** 64 **g** 1600 **h** 600 **i** 10 000
2 **a** 2 **b** 5 **c** 5 **d** 100 **e** 700 **f** 24 000 **g** 56

4:01 Collecting Like Terms

1 **a** -2 **b** -8 **c** 3 **d** 5 **e** -4 **f** -5 **g** 10 **h** 13
2 **a** $7x$ **b** $7d$ **c** $6p$ **d** $17y$ **e** $-11k$ **f** $-3w$
3 **a** $7a$ **b** $9b$ **c** $9x$ **d** $-5y$ **e** $11a$ **f** $3x$ **g** $-10t$ **h** $7n$
i $-5p$ **j** $-3m$ **k** 0 **l** y **m** $-7d$ **n** $7x$ **o** $-2h$

4:02 Multiplying Pronumerals

1 **a** 40 **b** 28 **c** -18 **d** -21 **e** 54 **f** 4 **g** 24 **h** 10
i 50 **j** 12
2 **a** $6a$ **b** cd **c** $7k$ **d** $-8f$ **e** $10m$ **f** p^2 **g** $14t$ **h** pqr
i $12x$ **j** $15n$ **k** $6g$ **l** $\frac{1}{2}y$
3 **a** $15a$ **b** $8x$ **c** $40c$ **d** $21y$ **e** $-22b$ **f** $64q$ **g** $-63m$ **h** $5uv$
i $10c^2$ **j** $4x^2$ **k** $6mn$ **l** $20bc$ **m** $4t$ **n** $2a$ **o** $5d$ **p** $6f$

4:03 Algebraic Division

1 **a** $\frac{a}{2}$ **b** $\frac{d}{3}$ **c** $\frac{5m}{7}$ **d** $\frac{2h}{3}$ **e** $\frac{4w}{5}$ **f** $\frac{2r}{5}$ **g** $\frac{3x}{8}$ **h** $\frac{b}{9}$
i $\frac{4v}{3}$ **j** $\frac{5y}{4}$
2 **a** 1 **b** 4 **c** 10 **d** 2 **e** 4 **f** 12 **g** 2 **h** 7
3 **a** $2x$ **b** $7a$ **c** $5b$ **d** $10n$ **e** $3r$ **f** y **g** $5f$ **h** $7x$

4:04A Simplifying Algebraic Fractions (+/−)

1 **a** $\frac{4}{7}$ **b** $\frac{8}{11}$ **c** $\frac{8}{19}$ **d** $\frac{13}{15}$ **e** $\frac{2}{5}$ **f** $\frac{2}{3}$ **g** $\frac{4}{5}$ **h** $\frac{5}{8}$
2 **a** $\frac{2}{3}$ **b** $\frac{4}{13}$ **c** $\frac{4}{7}$ **d** $\frac{7}{11}$ **e** $\frac{2}{5}$ **f** $\frac{1}{2}$ **g** $\frac{5}{9}$ **h** $\frac{1}{5}$
3 **a** $\frac{3x}{5}$ **b** $\frac{7y}{9}$ **c** $\frac{8k}{17}$ **d** $\frac{6a}{5}$ **e** $\frac{3c}{5}$ **f** p **g** $\frac{5d}{4}$ **h** $\frac{7r}{10}$
i $\frac{5m}{8}$ **j** $\frac{3x}{2}$ **k** $\frac{8c}{5}$ **l** $\frac{y}{2}$

4:04B Simplifying Algebraic Fractions (×/÷)

1 **a** $\frac{4}{15}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{5}{3}$ **e** $\frac{11}{9}$ **f** $\frac{5}{7}$ **g** $\frac{3}{2}$ **h** $\frac{4}{3}$

2 **a** $\frac{ab}{10}$ **b** $\frac{2xy}{15}$ **c** 21 **d** $\frac{de}{14}$ **e** $\frac{3pq}{52}$ **f** $\frac{2}{5}$ **g** $\frac{2a}{3}$ **h** $\frac{3}{2}$

i $\frac{4x}{y}$ **j** $\frac{1}{9}$ **k** $\frac{8}{7}$ **l** $\frac{1}{8}$

3 **a** $\frac{3}{8}$ **b** 2 **c** $\frac{2}{3}$ **d** 6 **e** $\frac{5}{3}$ **f** $\frac{4}{3}$ **g** $\frac{4b}{a}$ **h** $\frac{5}{2}$

4:05 Grouping Symbols

1 **a** $12y$ **b** $4c$ **c** $10a$ **d** $36m$ **e** $60p$ **f** x^2 **g** $30t$ **h** $18k$

i $3w^2$ **j** $7h^2$

2 **a** $6m + 15$ **b** $7x + 21$ **c** $5a - 35$ **d** $8y - 4$ **e** $p^2 - 4p$ **f** $5c - 2c^2$

3 **a** $6a + 15$ **b** $6d - 42$ **c** $50q + 30$ **d** $x^2 - 7x$ **e** $2t^2 + 3t$ **f** $12y + 28$

g $24 - 16x$ **h** $14c - 63$ **i** $3n^2 + n$ **j** $27 - 45p$ **k** $3r - 5r^2$ **l** $14x + 16$

m $55c + 50$ **n** $8 + 6n$ **o** $48 - 8n$

4:06 Simplifications with Expansions

1 **a** $4t + 14$ **b** $5x + 16$ **c** $5y - 21$ **d** $20m - 33$ **e** $22 - 9k$

f $18 + 6a$ **g** $-3 - 5x$ **h** $13x - 2$ **i** $p + 14$

2 **a** $9t + 14$ **b** $7b + 17$ **c** $4n$ **d** $4c + 29$ **e** $13w - 5$

f $2 + 23x$ **g** $11h + 11$ **h** 3 **i** $38 + 10a$

3 **a** $6t + 26$ **b** $10x + 43$ **c** $12c + 25$ **d** $22k + 34$ **e** $9a + 11$

f $4y + 4$ **g** $6 + r$ **h** $4m - 28$ **i** $38 - 17p$ **j** $10t - 41$

k $2f + 5$ **l** $3x + 51$

4:07 Factorising

1 **a** 2 **b** 5 **c** 3 **d** 4 **e** 2 **f** p **g** 2 **h** 7

2 **a** a **b** $3y$ **c** 1 **d** $2e$ **e** 10 **f** 9 **g** x **h** 1 **i** $2n$

3 **a** $6(a + 1)$ **b** $3(4x - 3)$ **c** $x(y + 2)$ **d** $5(2m - 3)$ **e** $3(3 - 2q)$

f $2(4y + 1)$ **g** $4(c + 4)$ **h** $6(3k - 4)$ **i** $10(1 - 3d)$ **j** $a(3 - b)$

k $2(7e + 4f)$ **l** $3(2m + 7n)$ **m** $7(1 + 5t)$ **n** $5(3p + 4q)$ **o** $8(2w - 1)$

6:02 The Index Laws

1 **a** 8 **b** 81 **c** 125 **d** 49 **e** 343 **f** 100 **g** 16 **h** 1

i 256 **j** 36 **k** 27 **l** 10 000 **m** 81 **n** 729 **o** 1

2 **a** 216 **b** 1024 **c** 4096 **d** 6561 **e** 2401 **f** 243

g 625 **h** 1000 **i** 1331 **j** 7776 **k** 59 049 **l** 1024

m 262 144 **n** 1 679 616 **o** 161 051 **p** 823 543 **q** 9 765 625 **r** 65 536

s 65 536 **t** 2187 **u** 1 000 000 **v** 104 976 **w** 537 824 **x** 160 000

y 4 826 809

6:03 Negative Indices

1 **a** $\frac{1}{4}$ **b** $\frac{1}{8}$ **c** $\frac{1}{9}$ **d** $\frac{1}{5}$ **e** $\frac{1}{36}$ **f** $\frac{1}{100}$ **g** $\frac{1}{1000}$ **h** $\frac{1}{25}$

i $\frac{1}{64}$ **j** $\frac{1}{16}$ **k** $\frac{1}{9}$ **l** $\frac{1}{32}$ **m** $\frac{1}{15}$ **n** $\frac{1}{27}$ **o** $\frac{1}{1331}$ **p** $\frac{1}{20}$

q $\frac{1}{144}$ **r** $\frac{1}{125}$ **s** $\frac{1}{729}$ **t** $\frac{1}{256}$

2 **a** 4^{-1} **b** 2^{-3} **c** 3^{-2} **d** 7^{-1} **e** 10^{-1} **f** 5^{-6} **g** 10^{-4} **h** 11^{-2}

i 2^{-7} **j** 3^{-5} **k** 40^{-1} **l** 8^{-2} **m** 6^{-2} **n** 2^{-1} **o** 4^{-3} **p** 3^{-1}

q 6^{-4} **r** 13^{-2} **s** 25^{-4} **t** 12^{-3}

7:01 Solving Equations by Inspection

1 **a** 3 **b** 17 **c** −4 **d** 4 **e** 10 **f** 8 **g** 5 **h** 10
i 4 **j** 0 **k** 2 **l** 18 **m** 7 **n** 30 **o** −5 **p** −1
q 10 **r** 6 **s** 18 **t** 5

2 **a** 6 **b** 7 **c** $\frac{3}{4}$ **d** 6 **e** 15 **f** 12 **g** 3 **h** 6
i 7 **j** 4 **k** 6 **l** 11 **m** 4 **n** 5 **o** 8 **p** 9
q 1 **r** 4 **s** 8 **t** $\frac{1}{2}$

7:03 Solving Simple Equations

1 **a** 17 **b** 39 **c** 24 **d** 37 **e** 15 **f** 17 **g** 68 **h** 13
i 10 **j** 44 **k** 68 **l** −4 **m** 3 **n** 16 **o** 50 **p** 100

2 **a** 3 **b** 1 **c** 2 **d** 4 **e** 6 **f** 6 **g** 4 **h** 15
i 3 **j** 1 **k** 3 **l** 5 **m** 2 **n** 5 **o** 4 **p** 10
q 10 **r** 7 **s** 2 **t** 11

7:05 Equations with Pronumerals on Both Sides

1 **a** 2 **b** −1 **c** 11 **d** 8 **e** 18 **f** 10 **g** 4 **h** −5
i −3 **j** 12 **k** −10 **l** −3 **m** 7 **n** 18 **o** −13

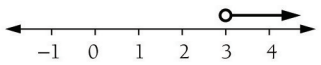
2 **a** 2 **b** 2 **c** 4 **d** 6 **e** 4 **f** 4 **g** −1 **h** 4
i 3 **j** 9 **k** 10 **l** 4 **m** −7 **n** −4 **o** −8 **p** 4
q 7 **r** −3 **s** 6 **t** 2 **u** 12

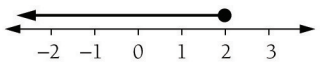
7:06 Equations with Fractions 1

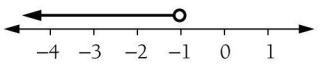
1 **a** 6 **b** −10 **c** 9 **d** 16 **e** 8 **f** −21 **g** 8 **h** 6
i 72 **j** 10 **k** −7 **l** 28 **m** 24 **n** −70 **o** 21 **p** 3
q −27 **r** 44 **s** 6 **t** −28

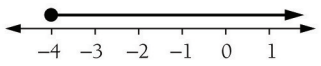
2 **a** $a+2$ **b** $x-18$ **c** $2m+1$ **d** $c+40$ **e** $2x-7$ **f** $n+8$
g $5a-4$ **h** $y-12$ **i** $3c+10$ **j** $4a-3$ **k** $30+y$ **l** $7+3a$
m $2p+15$ **n** $3w-4$ **o** $13m+4$ **p** $7q-2$

7:08 Solving Inequations

1 **a**


b


c


d


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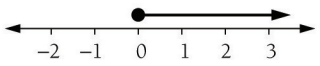

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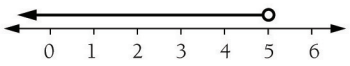

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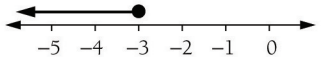

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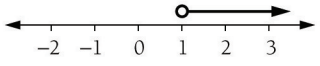

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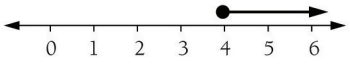

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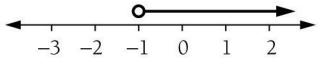

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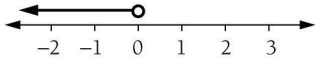
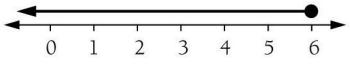


l


2 **a** $x > 2$ **b** $m \leqslant 8$ **c** $n < -3$ **d** $y \leqslant 5$ **e** $a > 3$ **f** $x > 3$
g $h \leqslant -6$ **h** $p \geqslant -5$ **i** $a < 4$ **j** $y \geqslant 4$ **k** $t > -2$ **l** $c \leqslant -5$
m $w < 4$ **n** $x \geqslant -10$ **o** $y < 14$ **p** $f \geqslant 13$ **q** $k > 2$ **r** $d < 6$
s $x \leqslant -25$ **t** $c \geqslant 5$

8:01 Coordinates

1 **a** (3, 0) **b** (2, –5) **c** (0, 0) **d** (–3, 1) **e** (–2, 5) **f** (0, –4)
g (–2, –3) **h** (5, 0) **i** (5, 5)

2 **a** L **b** H **c** B **d** E **e** N **f** J **g** G **h** C **i** Q

3 **a** A = (1, 1) B = (–1, –1) C = (2, 2) D = (0, 0) **b** (0, 2) (2, 0) (–1, 3) (1, 1)
c (–1, –4) (0, –2) (1, 0) (2, 2) **d** (–2, 2) (0, 2) (1, 2) (3, 2)

8:02 Graphing Lines

1 **a** 0, 1, 2 **b** 2, 3, 4 **c** 2, 1, 0 **d** 0, 2, 4 **e** –3, –2, –1 **f** 2, 5, 8
g 0, –1, –2 **h** 4, 2, 0 **i** 0, 5, 10 **j** 3, 5, 7 **k** 0, 1, 2 **l** 3, 1, –1

2 **a** x-intercept = 1, y-intercept = 4 **b** –4, –1 **c** 0, 0 **d** 3, –3 **e** 2, 2 **f** 4, 1
g –2, 2 **h** $1\frac{1}{2}$, –3

8:03 Horizontal and Vertical Lines

1 **a** A = (3, 3) B = (2, 3) C = (0, 3) D = (–1, 3) **b** (4, –2) (1, –2) (0, –2) (–2, –2)
c (1, 3) (1, 0) (1, –1) (1, –2) **d** (–3, –3) (–3, 0) (–3, 1) (–3, 3)
e (3, 5) (0, 5) (–2, 5) (–4, 5) **f** (4, 6) (4, 3) (4, 1) (4, –2)

2 The horizontal lines have points with the same y coordinate, and the vertical lines have points with the same x coordinate.

3 Check with your teacher.

8:04 Intersecting Lines

1 **a** (2, 2) **b** (0, 1) **c** (–3, 0) **d** (–1, –4) **e** (2, –1) **f** $(3\frac{1}{2}, \frac{1}{2})$ **g** (1, 3)

2 **a** (1, 1) **b** (3, 0) **c** (–1, –2) **d** (–2, 2)

9:01 Generalised Arithmetic

1 **a** $3a + 2b$ **b** $\frac{m+n}{2}$ **c** $\frac{x}{5}$ **d** $p - q$ **e** k^3 **f** $40y$
g $15p + 18$ **h** w^2 **i** $a + 20$ **j** $6ab$ **k** $u + v + w$ **l** $\frac{p+q+r}{3}$
m $\frac{y}{x}$ **n** $\frac{x+5}{x-2}$

2 **a** $75x$ cents **b** $y + 25$ **c** $\frac{a+b+c+d}{4}$ **d** $m - 4$ **e** $10 - d$ metres
f $\$(d - e - f)$ **g** $\$(50 - x)$ **h** $60t$ **i** $\$(R - B)$ **j** $30h$ km
k $\$\frac{w}{5}$ **l** $\$(3x + 6y)$ **m** $3y$ km, $\frac{3y}{8}$ km/h **n** $4m - 66$ years

9:02 Substitution

1 **a** 35 **b** 18 **c** –40 **d** 19 **e** 81 **f** 4 **g** –8 **h** 25
i 4 **j** 23 **k** 28 **l** –7 **m** 5 **n** 36

2 **a** 15 **b** 13 **c** –30 **d** 64 **e** 11 **f** 11 **g** 6 **h** 18
i 0 **j** 64 **k** –12 **l** 34 **m** 9 **n** –9 **o** 2 **p** 13

9:05 Translating Problems into Equations

1 **a** $a + 5 = 8$ **b** $7x = 42$ **c** $y - 13 = 7$ **d** $4m + 7 = 29$ **e** $10k = -70$
f $\frac{x}{8} = 4$ **g** $n - 5 = 17$ **h** $4x - 7 = 23$

2 **a** $3n - 9 = 27$; 12 **b** $\frac{n}{6} = 5$; 30 **c** $n - 15 = -3$; 12 **d** $4n + 7 = 57$; 12·5 **e** $\frac{n+6}{3} = 15$; 39
f $3(n + 5) = 39$; 8 **g** $2n + 17 = 53$; 18 **h** $3n - 7 = 44$; 17 **i** $\frac{5n}{4} = 12$; 9·6 **j** $6n - 5 = 19$; 4
k $\frac{n-7}{2} = 18$; 43 **l** $\frac{n}{4} = -6$; –24 **m** $5(n - 4) = 60$; 16 **n** $8n + 3 = 19$; 2 **o** $7n - 4 = 17$; 3

10:01 Adjacent Angles

1 **a** 20° **b** 110° **c** 97° **d** 85°

2 **a** 110° **b** 25° **c** 47° **d** 16° **e** 50° **f** 65° **g** 33° **h** 97°

3 **a** 70 **b** 54 **c** 55 **d** 85 **e** 30 **f** 60 **g** 94 **h** 23

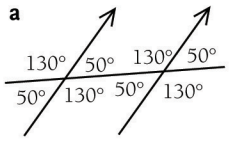
10:02 Angles at a Point

1 **a** 100° **b** 220° **c** 50° **d** 70° **e** 115° **f** 34° **g** 123° **h** 218°
i 112° **j** 213° **k** 95° **l** 67°

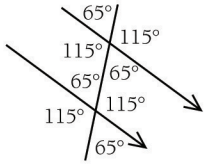
2 **a** 130 **b** 83 **c** 38 **d** 149 **e** 62 **f** 152 **g** 27 **h** 15

10:05 Angles and Parallel Lines

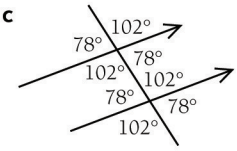
1 **a**



b



c



2 **a** 130° **b** 42° **c** 107° **d** 44° **e** 88° **f** 143°

3 **a** 143 **b** 71 **c** 100 **d** 97 **e** 132 **f** 86 **g** 95 **h** 111 **i** 120

10:06 Triangles

1 **a** 50 **b** 50 **c** 145 **d** 33 **e** 67 **f** 118 **g** 64 **h** 27 **i** 113

2 **a** 20 **b** 57 **c** 40 **d** 92 **e** 56 **f** 116

10:09 Formulae

1 **a** 6 **b** 9 **c** 5 **d** 7 **e** 8 **f** 4 **g** 3 **h** 10 **i** 4

2 **a** 540 **b** 1080 **c** 360 **d** 720 **e** 1440 **f** 180 **g** 900 **h** 1260 **i** 1800 **j** 3240

3 **a** 720 **b** 1260 **c** 540 **d** 900 **e** 1080 **f** 360 **g** 180 **h** 1440 **i** 360

12:02 Perimeter

1 **a** 18 cm **b** 8 m **c** 103 cm **d** 101 cm **e** 79 mm **f** 22 cm
g 32 m **h** 28 mm

2 **a** 34 m **b** 23·4 m **c** 106 cm **d** 69·6 cm **e** 104 m **f** 88 m
g 78 **h** 41·7 cm

3 **i** **a** 17 cm **b** 40 cm **ii** **a** 15 m **b** 36 m
iii **a** 13 cm **b** 30 cm **iv** **a** 25 m **b** 56 m

12:04 Area

1 **a** 19·6 cm^2 **b** 3·5 m^2 **c** 314·2 m^2 **d** 353·4 cm^2 **e** 127·2 m^2 **f** 1963·5 cm^2
g 201·1 mm^2 **h** 981·7 m^2

2 **a** 78 u^2 **b** 61 u^2 **c** 285 m^2 **d** 426 cm^2 **e** 257 m^2 **f** 313 cm^2
g 2375 u^2 **h** 90 m^2 **i** 236 cm^2 **j** 282 m^2 **k** 550 cm^2 **l** 2500 u^2

12:05 Surface Area of Prisms

1 a A—III B—II C—I

b I 384 u^2 II 528 u^2 III 532 u^2

2 a

Face	A	B	C	D	E	F
Area	128	128	96	96	48	48

b

Face	A	B	C	D	E	F
Area	225	375	375	375	375	225

c

Face	I	II	III	IV	V	VI
Area	200	40	80	80	200	40

3 a 768 m^2 **b** 2620 cm^2 **c** 1120 u^2

12:06 Surface Area of Composite Solids

1 a 22 cm^2 **b** 18 cm^2 **c** 24 cm^2 **d** 24 cm^2 **e** 32 cm^2 **f** 38 cm^2

2 a 38 cm^2 **b** 32 cm^2 **c** 36 cm^2 **d** 58 cm^2

3 a 272 u^2 **b** 482 u^2 **c** 1112 u^2

13:01 Frequency and Cumulative Frequency

1 a

Score	*Tally*	*Frequency*
3	𝍸	4
4	𝍸	5
5	𝍸 |||	8
6	||	2
7	|	1

b

Score	*Tally*	*Frequency*
21	𝍸	5
22	𝍸 ||||	9
23	𝍸 |||	8
24	𝍸 |	6
25	||	2

c

Score	*Tally*	*Frequency*
40	𝍸	5
41	𝍸 ||	7
42	𝍸 ||||	9
43	𝍸 |	6
44	|||	3

2 a i 4 **ii** 3 **b i** 8 **ii** 22 **c i** 12 **ii** 18

3 a

Cumulative frequency
3
5
9
12
14
15

b

Cumulative frequency
3
8
15
23
29
30

c

Cumulative frequency
4
11
23
31
36

d

Cumulative frequency
8
20
35
52
65
73
80

e

Cumulative frequency
4
11
19
28
34
37
37
40

f

Cumulative frequency
3
7
13
23
32
39
47
47
51
55

13:02 Mean, Median and Mode

1 a 37·1 **b** 4·4 **c** 8·1 **d** 2·1 **e** 75·8 **f** 1·7 **g** 8

2 a i 48 **ii** 48 **b i** 4 **ii** 4 **c i** 8 **ii** 7 **d i** 2 **ii** 2

e i 75 **ii** none **f i** 2 **ii** 0 **g i** 9 **ii** 9

3 (You may wish to discuss these.) **a** mean **b** none **c** mean or median **d** mean or mode

13:04 Mean and Median

1 a mean ≒ 4·7, median = 5, mode = 5 **b** 11·25, 11·5, 12 **c** ≒ 5·7, 5, 9 **d** ≒ 24·2, 25, 25

2 a 1·88, 2, 2 **b** 7·84, 8, 8 **c** 24, 20, 30 **d** ≒ 17·1, 17, 17 **e** ≒ 98·2, 98, 98

3 a 3 **b** 3 **c** 14

14:02 Experimental Probability

1 **a** $\frac{1}{2}$ **b** $\frac{7}{12}$ **2** **a** $\frac{2}{3}$ **b** $\frac{2}{15}$

3 **a** **i** $\frac{4}{25}$ **ii** $\frac{11}{25}$ **iii** $\frac{19}{50}$ **b** 1

4 **a** **i** $\frac{93}{1000}$ **ii** $\frac{167}{1000}$ **iii** $\frac{2}{25}$ **iv** $\frac{319}{1000}$ **v** $\frac{103}{250}$

b There are more ways to roll 10 (5, 5 4, 6 6, 4) than 12 (6, 6)

5 **a** $\frac{3}{40}$ **b** $\frac{37}{40}$ **c** $\frac{3}{5}$ **6** **a** $\frac{97}{100}$ **b** 60

14:03 Theoretical Probability

1 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **2** **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** 0

3 **a** $\frac{7}{24}$ **b** $\frac{13}{24}$ **c** $\frac{11}{12}$ **4** **a** $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{1}{5}$ **d** $\frac{2}{5}$

5 **a** $\frac{7}{25}$ **b** $\frac{1}{10}$ **c** $\frac{8}{25}$ **6** **a** $\frac{1}{10}$ **b** $\frac{3}{5}$ **c** $\frac{2}{5}$ **d** $\frac{3}{4}$

7 **a** $\frac{1}{6}$ **b** $\frac{2}{3}$ **c** $\frac{1}{2}$ **d** 1 **8** **a** $\frac{1}{5}$ **b** $\frac{3}{5}$ **c** $\frac{3}{5}$ **d** $\frac{4}{5}$

Challenge Worksheets

4:03 Algebraic Simplifications

1 $20a$ **2** x **3** $12b$ **4** $3m$ **5** 4 **6** $60a^2$ **7** $3m^2$ **8** $2\frac{1}{7}$ or $\frac{15}{7}$

9 35 **10** 16 **11** $6a^2$ **12** $6y^2$ **13** $14x$ **14** $25x^2$ **15** $10y$ **16** $16m$

17 $26n$ **18** $3x$ **19** x **20** $20m$ **21** $\frac{9}{5}$ **22** 1 **23** 1 **24** $\frac{a}{2c}$

4:04 Algebraic Fractions

1 a $\frac{13x}{12}$ **b** $\frac{13p}{20}$ **2 a** $\frac{m}{20}$ **b** $\frac{5}{4m}$

3 a $\frac{b}{4}$ **b** $\frac{5n}{3}$ **4 a** $\frac{47m}{60}$ **b** $\frac{103a}{60}$

5 a $\frac{7xy}{24}$ **b** $\frac{2x}{y}$ **6 a** $\frac{7xy}{4}$ **b** 7

4:03 Probability: An Unusual Case

1 a

Die C \ Die B	0	1	7	8	8	9
5	C	C	B	B	B	B
5	C	C	B	B	B	B
6	C	C	B	B	B	B
6	C	C	B	B	B	B
7	C	C	T	B	B	B
7	C	C	T	B	B	B

b

Die D \ Die C	5	5	6	6	7	7
3	C	C	C	C	C	C
4	C	C	C	C	C	C
4	C	C	C	C	C	C
5	T	T	C	C	C	C
11	D	D	D	D	D	D
12	D	D	D	D	D	D

c

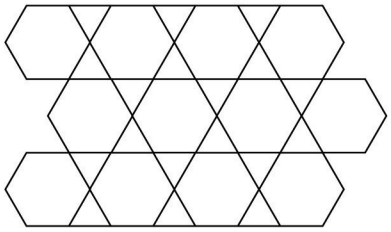
Die A \ Die D	3	4	4	5	11	12
1	D	D	D	D	D	D
2	D	D	D	D	D	D
3	T	D	D	D	D	D
9	A	A	A	A	D	D
10	A	A	A	A	D	D
11	A	A	A	A	T	D

2 a Yes **b** Yes **c** Yes **d** Yes

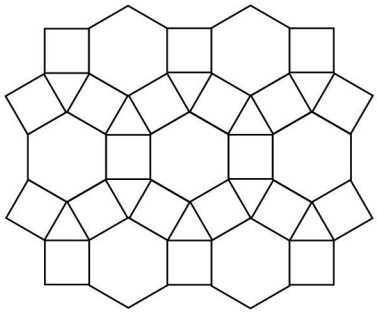
3 The question could be investigated experimentally. Theoretically, it can be shown that the probabilities of A, B, C, D winning are $\frac{408}{1296}$, $\frac{264}{1296}$, $\frac{132}{1296}$, $\frac{396}{1296}$ respectively. The probability of a tie is $\frac{96}{1296}$. Hence, A is most likely to win.

10:09 Regular Polygons and Tessellations

The seven other semi-regular tessellations are:



3–6–3–6



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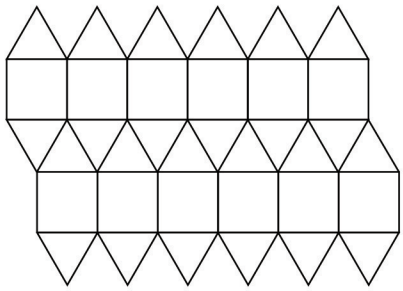
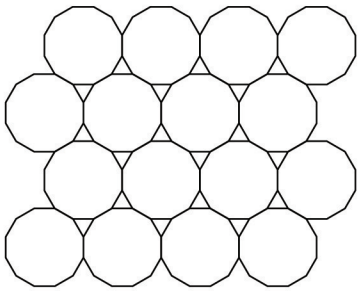
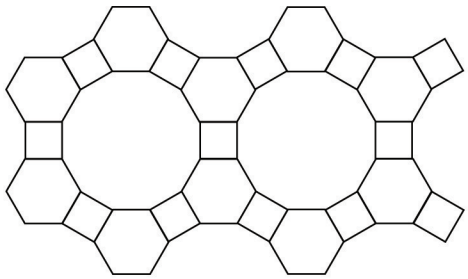


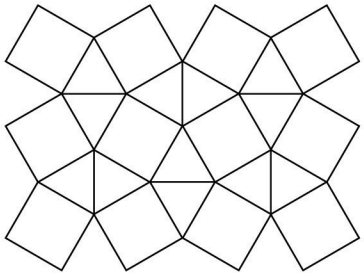
3–3–3–4–4



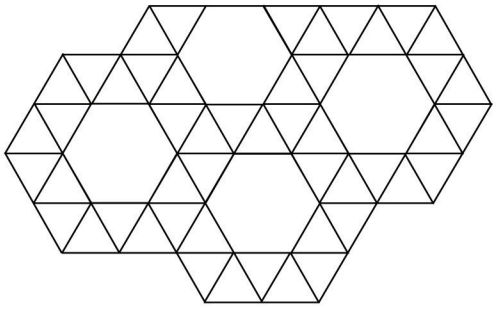
3–12–3–12



4–12–4–6



3–3–4–3–4



3–3–3–3–6

6:04 Algebraic Expressions and Indices

1 **a** $\frac{1}{(x+2)^3}$ **b** $\sqrt{x+2}$ **c** $\frac{1}{\sqrt{x+2}}$ **d** $\frac{1}{3^x}$ **e** $\frac{1}{3^{2x}}$ **f** $\frac{1}{e^x}$

g $\frac{1}{e^x+2}$ **h** $\frac{1}{e^{x+2}}$ **i** $\sqrt{e^x+2}$

2 **a** $(2x+1)^{\frac{1}{2}}$ **b** $(x^2+1)^{\frac{1}{2}}$ **c** $e^{\frac{1}{2}x}$ or $e^{\frac{x}{2}}$ **d** $x^{-\frac{1}{2}}$ **e** $(x+2)^{-2}$ **f** $(x+2)^{-\frac{1}{2}}$

g $x(x+1)^{\frac{1}{2}}$ **h** $x(x+1)^{-\frac{1}{2}}$ **i** $x^2(x+1)^{-1}$

Worksheets

Answers

5:01 Pythagoras' Theorem A (measurements are in mm)

1 30, 40, 50 Yes
2 15, 36, 39 Yes
3 11, 60, 61 Yes
4 24, 45, 51 Yes
5 9, 40, 41 Yes
6 36, 48, 60 Yes
7 18, 24, 30 Yes
8 10, 24, 26 Yes
9 24, 32, 40 Yes
10 13, 84, 85 Yes

5:02 Pythagoras' Theorem B

1 3·61
2 9·85
3 13
4 12·81
5 15
6 9·22
7 20
8 11·40
9 11·18
10 8·49
11 17
12 14·87

5:03 Pythagoras' Theorem

1 4·90
2 12
3 10·54
4 9
5 6
6 22·36
7 11·87
8 11·83
9 71·75
10 7·94
11 8
12 20

Challenge

6:04 Probability: An Unusual Case

1 a

Die C \ Die B	0	1	7	8	8	9
5	C	C	B	B	B	B
5	C	C	B	B	B	B
6	C	C	B	B	B	B
6	C	C	B	B	B	B
7	C	C	T	B	B	B
7	C	C	T	B	B	B

b

Die D \ Die C	5	5	6	6	7	7
3	C	C	C	C	C	C
4	C	C	C	C	C	C
4	C	C	C	C	C	C
5	T	T	C	C	C	C
11	D	D	D	D	D	D
12	D	D	D	D	D	D

c

Die A \ Die D	3	4	4	5	11	12
1	D	D	D	D	D	D
2	D	D	D	D	D	D
3	T	D	D	D	D	D
9	A	A	A	A	D	D
10	A	A	A	A	D	D
11	A	A	A	A		D

2 a Yes **b** Yes **c** Yes **d** Yes

3 It doesn't seem logical that Diane (D) is more likely to win against Andrew (A), since A should beat B, B should beat C and C should beat D.

4 a No. They have exactly the same chance of winning. **b** No